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Dissipationless spin-Hall current contribution in the extrinsic spin-Hall effect*

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This paper shows that a substantial amount of dissipationless spin-Hall current contribution may exist in the extrinsic spin-Hall effect, which originates from the spin-orbit coupling induced by the applied external electric field itself that drives the extrinsic spin-Hall effect in a nonmagnetic semiconductor (or metal). By assuming that the impurity density is in a moderate range such that the total scattering potential due to all randomly distributed impurities is a smooth function of the space coordinate, it is shown that this dissipationless contribution shall be of the same orders of magnitude as the usual extrinsic contribution from spin-orbit dependent impurity scatterings (or may even be larger than the latter one). The theoretical results obtained are in good agreement with recent relevant experimental results.

Keywords: extrinsic spin-Hall effect, spin-orbit coupling induced by external electric fields, dissipationless spin current

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1. Introduction

Generation of spin currents in nonmagnetic semiconductors by purely electric means is one of the principal challenges in the emerging field of semiconductor spintronics.[1,2] Several proposals have been put forward for achieving this goal. Among them, the spin-Hall effect has attracted recently much theoretical and experimental interest both for the possible applications in spintronics and also for the diverse and interesting physics contained in it.[3−13] This effect arises from spin-orbit interaction of charge carriers, and it consists of the generation of a spin current in the direction perpendicular to an applied external electric field that drives a charge current to circulate through a sample. Two different versions of the effect had been distinguished theoretically, namely extrinsic and intrinsic spin-Hall effect, respectively. Extrinsic spin-Hall effect results from spin-orbit dependent scatterings of charge carriers by impurities and hence shall vanish in the clean limit.[3−5] In contrast to extrinsic spin-Hall effect, intrinsic spin-Hall effect is a band structure effect, which results from the combined action of an external electric field and the intrinsic spin-orbit splitting of the band structures in some semiconducting electronic systems lacking bulk or structural inversion symmetry.[6,7] Since intrinsic spin-Hall effect is a band structure effect, it was argued in some recent studies that the spin-Hall current due to intrinsic spin-Hall effect is dissipationless, i.e., it shall be independent of disorder scattering effects and all states below the Fermi level shall contribute to it.[6,9]

It should be noted that, though significant progress has been achieved recently in the experimental detection of the spin-Hall effect,[8−13] a number of controversial issues still exist in the recent literatures concerning some fundamental aspects of this effect or concerning the theoretical interpretations of some recent relevant experimental data.[14−38] Due to the existence of such controversial issues, in addition to the usual extrinsic or the usual intrinsic contribution, other possible effects and/or contributions should also be examined in order to get a clearer understanding of this extraordinary effect. In this paper we will show that, in addition to the usual extrinsic contribution from spin-orbit dependent impurity scatterings, the spin-orbit coupling induced by the applied external electric field itself may also make a substantial contribution to the extrinsic spin-Hall effect. We will show

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that, similar to the dissipationless spin-Hall current due to intrinsic spin-Hall effect, the spin-Hall current contribution due to the spin-orbit coupling induced by the external electric field itself may also be dissipationless, i.e., it may be independent of disorder scattering effects and all states below the Fermi surface may contribute to it with equal weights. At first glance, since the spin-orbit coupling induced by impurity scatterings may be substantially stronger than that induced by external electric fields, one may think intuitively that the spin-Hall conductivity contribution due to the spin-orbit coupling induced by external electric fields should be negligible in comparison with the usual extrinsic contribution from spin-orbit dependent impurity scatterings. But counterintuitively, in this paper we will show that this is actually not the case. We will show that the spin-Hall conductivity contribution due to the spin-orbit coupling induced by external electric fields shall be of the same orders of magnitude as the usual extrinsic contribution from spin-orbit dependent impurity scatterings, or may even be larger than the latter one. Due to this fact, this contribution cannot be neglected in the studies of the extrinsic spin-Hall effect. The theoretical results obtained in the present paper are in good agreement with recent relevant experimental results.

The paper will be organized as follows. In Section 2 we will introduce the theoretical model considered and develop a semiclassical formalism to describe the extrinsic spin-Hall effect in such systems. The main results obtained and the essential physical reasons behind them will be discussed in some detail in Section 3.

2. Model and theoretical formalism

To illustrate the facts mentioned in the introduction, the following single-particle effective Hamiltonian will be considered,

$$\hat{H} = \frac{\hbar^2 k^2}{2m} + [e\boldsymbol{E}\cdot r + e\alpha_E(\boldsymbol{E}\times k)\cdot\sigma] + \left[\sum_i V_i(\boldsymbol{r}) + \sum_i \alpha_{\rm im}[\boldsymbol{\nabla}_{\boldsymbol{r}} V_i(\boldsymbol{r})\times\boldsymbol{k}]\cdot\boldsymbol{\sigma}\right], \quad (1)$$

where m is the effective mass of electrons and $\boldsymbol{k}$ is the wave vector, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ denotes the usual Pauli matrices of spin $\frac{1}{2}$, $V_i(\boldsymbol{r})$ denotes the scattering potential due to the ith impurity and $\boldsymbol{E}$ is the applied external electric field. The second term in the first square bracket is the spin-orbit coupling induced by the external electric field and the second term in the second square bracket is the usual extrinsic spin-orbit coupling induced by impurity scatterings. To discern the contributions from the two types of spin-orbit interaction, we denote the coupling constants of the two types of spin-orbit interaction as α_E and $\alpha_{\rm im}$, respectively. For a non-relativistic electron in vacuum, one has $\alpha_E = \alpha_{\rm im} = \hbar^2/4m_0^2c^2$ (m_0 is the vacuum electron mass and c the velocity of light).[39] In real semiconductors (or metals), the spin-orbit coupling constants may be several orders of magnitude larger than the vacuum value due to the influence of the crystal field.[40] For example, for n-doped GaAs, the magnitude of the spin-orbit coupling constants is around 5.3 Å^2, which is about six orders of magnitude larger than the vacuum value. Since the values of $\alpha_{\rm im}$ may also depend on the fine structures of impurity atoms,[41] the values of $\alpha_{\rm im}$ may be different from that of α_E, but they will be of the same order of magnitude.

From the fundamental theoretical points of view, the extrinsic spin-Hall effect has basically two physical origins: the spin-orientation dependence of the velocities of charge carriers and the spin-orbit dependent scatterings of charge carriers by impurities. For a system described by the Hamiltonian (1), the spin-dependent velocity of an electron with a given wave vector $\boldsymbol{k}$ shall be given by $\hat{v} = \dot{r} = \frac{1}{\mathrm{i}\hbar}[\boldsymbol{r}, \hat{H}]$, from which one gets

$$\hat{\boldsymbol{v}} = \frac{\hbar\boldsymbol{k}}{m} + e\hbar^{-1}\alpha_E\nabla_{\boldsymbol{k}}[(\boldsymbol{E}\times\boldsymbol{k})\cdot\boldsymbol{\sigma}] + \sum_i \hbar^{-1}\alpha_{\rm im}\nabla_{\boldsymbol{k}}\left[\boldsymbol{\sigma}\cdot(\nabla_{\boldsymbol{r}} V_i(\boldsymbol{r})\times\boldsymbol{k})\right]. \quad (2)$$

From Eq.(2) one can see that, due to the spin-orbit interaction, the velocities of electrons will contain a spin-orientation dependent part, which is along opposite directions for electrons with the same wave vectors but opposite spin-orientations. For an external electric field applied in the x direction ($\boldsymbol{E} = E_x\hat{\boldsymbol{x}}$), the spin-orientation dependence of the velocities of electrons combining with the spin-orbit dependent scattering effects by impurities may lead to the generation of a spin-Hall current in the y direction with spin parallel to the z direction. In the semiclassical description, the density of the induced spin-Hall current shall be given by

$$j_y^z(\boldsymbol{r}) = \sum_{\lambda,\boldsymbol{k}} j_y^z(\boldsymbol{k},\lambda) f_\lambda(\boldsymbol{k},\boldsymbol{r}), \quad (3)$$

where $f_\lambda(\boldsymbol{k},\boldsymbol{r})$ is the distribution function of electrons and $j_y^z(\boldsymbol{k},\lambda)$ is the expectation value of the spin-Hall current operator determined by the spin-orbit dependent scattering wave functions of electrons:

$$j_y^z(\boldsymbol{k},\lambda) \equiv \frac{\hbar}{4}\langle \Psi^+_{\lambda\boldsymbol{k}}|\{\hat{v}_y\sigma_z + \sigma_z\hat{v}_y\}|\,\Psi_{\lambda\boldsymbol{k}}\rangle_{\rm sc}, \qquad (4)$$

where $\hat{v}_y$ is the y component of the velocity operator and $|\,\Psi_{\lambda\boldsymbol{k}}\rangle_{\rm sc}$ is the spin-dependent scattering wave function of electron with the given wave vector $\boldsymbol{k}$ and the spin index λ ($\lambda = +$ for spin-up and $\lambda = -$ for spin down). Here we have used the conventional definition for the spin-Hall current operator, the natural one which was widely used in most previous studies.[14−30] It was generally believed that the conventional definition can be used without ambiguities in the absence of intrinsical spin-orbit split bands. In the presence of intrinsical spin-orbit split bands, some ambiguities may exist with this conventional definition, and several alternative definitions had been proposed recently based on different theoretical arguments in order to avoid these ambiguities.[35−38] Since the system considered in the present paper does not have intrinsical spin-orbit split bands, the conventional definition of spin current can be used reliably without ambiguities.

To calculate the electrically-induced spin-Hall current based on Eqs.(3) and (4), one needs to find out first the spin-orbit dependent scattering wave functions $|\,\Psi_{\lambda\boldsymbol{k}}\rangle_{\rm sc}$, the Hamiltonian (1) can be rewritten in the following form, of electrons and the nonequilibrium distribution function $f_\lambda(\boldsymbol{k},\boldsymbol{r})$. To find the spin-orbit dependent scattering wave functions $|\,\Psi_{\lambda\boldsymbol{k}}\rangle_{sc}$,

$$\hat{H} = \hat{H}_0 + \hat{H}', \qquad (5)$$

where $\hat{H}_0$ and $\hat{H}'$ are defined, respectively, by

$$\hat{H}_0 = \frac{\hbar^2k^2}{2m} + e\alpha_E\hat{\sigma}\cdot(\boldsymbol{E}\times k), \qquad (6)$$

$$\hat{H}' = e\boldsymbol{E}\cdot\boldsymbol{r} + \sum_i V_i(\boldsymbol{r}) + \sum_i \alpha_{\rm im}[\boldsymbol{\nabla}_{\boldsymbol{r}}V_i(\boldsymbol{r})\times k]\cdot\sigma. \qquad (7)$$

The Hamiltonian $\hat{H}_0$ can be diagonalized exactly, whose eigenfunctions shall be denoted as $|\psi^{(0)}_{\lambda\boldsymbol{k}}\rangle = \frac{1}{\sqrt{A}}{\rm e}^{{\rm i}\boldsymbol{k}\cdot r}|u_{\lambda\boldsymbol{k}}\rangle$, where A is the area of the two-dimensional system and $|u_{\lambda\boldsymbol{k}}\rangle$ is the spinor eigenfunction ($\lambda = +$ for spin-up and $\lambda = -$ for spin-down). Assuming that the external electric field is applied in the x direction ($\boldsymbol{E} = E_x\hat{x}$), we find that the eigenvalues of $\hat{H}_0$ shall be given by

$$\varepsilon_{\lambda\boldsymbol{k}} = \frac{\hbar^2k^2}{2m} + \lambda e\alpha_E k_y E_x. \qquad (8)$$

We assume that the impurity density is in a moderate range such that the total scattering potential due to all randomly distributed impurities is a smooth function of the space coordinate. In such cases one can treat the Hamiltonian $\hat{H}'$ as a perturbation. Then the scattering state of an eigenfunction $|\psi^{(0)}_{\lambda\boldsymbol{k}}\rangle$ of the Hamiltonian $\hat{H}_0$ due to the perturbation of the Hamiltonian $\hat{H}'$ can be determined from the Lippman–Schwinger equation,[42] which reads

$$|\,\Psi_{\lambda\boldsymbol{k}}\rangle_{\rm sc} = |\psi^{(0)}_{\lambda\boldsymbol{k}}\rangle + \hat{G}^{(+)}_0\hat{H}'|\,\Psi_{\lambda\boldsymbol{k}}\rangle_{\rm sc}, \qquad (9)$$

where $\hat{G}^{(+)}_0 = (\varepsilon_{\lambda\boldsymbol{k}} - \hat{H}_0 + {\rm i}0^+)^{-1}$ is the unperturbed retarded Green's function. Substituting the scattering wave function given by Eq.(9) into Eq.(4) and making use of the usual Born approximation and then keeping only the terms linear in the spin-orbit coupling constants, we can obtain the following result:

$$j_y^z(\boldsymbol{k},\lambda) = \lambda\frac{\hbar^2k_y}{2m} + \frac{1}{2}e\alpha_E E_x - \sum_i\frac{\hbar}{2}\frac{\alpha_{\rm im}k_x}{\tau^{(i)}_{\lambda\boldsymbol{k}}}, \qquad (10)$$

where $\tau^{(i)}_{\lambda\boldsymbol{k}}$ is a relaxation time defined by

$$\frac{1}{\tau^{(i)}_{\lambda\boldsymbol{k}}} = \sum_{\boldsymbol{k}'\lambda'} w^{(i)}_{\lambda\boldsymbol{k},\lambda'\boldsymbol{k}'}(1 - \frac{k'_x}{k_x})\delta(\varepsilon_{\lambda\boldsymbol{k}} - \varepsilon_{\lambda'\boldsymbol{k}'}), \qquad (11)$$

where $w^{(i)}_{\lambda\boldsymbol{k},\lambda'\boldsymbol{k}'} \equiv \frac{2\pi}{\hbar}|\langle\psi^{(0)}_{\lambda\boldsymbol{k}}|V_i(\boldsymbol{r})|\psi^{(0)}_{\lambda'\boldsymbol{k}'}\rangle|^2$ denotes the transition probability of an electron from the state $|\psi^{(0)}_{\lambda\boldsymbol{k}}\rangle$ to the state $|\psi^{(0)}_{\lambda'\boldsymbol{k}'}\rangle$ due to the scattering by the ith impurity.

The nonequilibrium distribution function of electrons in the presence of the external electric field can be obtained by solving the Boltzmann transport equation,

$$\begin{aligned}&\boldsymbol{v}_{\lambda\boldsymbol{k}}\cdot\nabla_{\boldsymbol{r}}f_\lambda(\boldsymbol{k},\boldsymbol{r}) - e\boldsymbol{E}\cdot v_{\lambda\boldsymbol{k}}\frac{\partial f_\lambda(\boldsymbol{k},\boldsymbol{r})}{\partial\epsilon_{\lambda\boldsymbol{k}}}\\ &= \sum_{\boldsymbol{k}',\lambda'}[w_{\lambda'\boldsymbol{k}',\lambda\boldsymbol{k}}f_{\lambda'}(\boldsymbol{k}',\boldsymbol{r}) - w_{\lambda\boldsymbol{k},\lambda'\boldsymbol{k}'}f_\lambda(\boldsymbol{k},r)]\\ &\quad\times\delta(\varepsilon_{\lambda\boldsymbol{k}} - \varepsilon_{\lambda'\boldsymbol{k}'}),\end{aligned} \qquad (12)$$

where $\boldsymbol{v}_{\lambda\boldsymbol{k}} = \hbar\boldsymbol{k}/m$ and $w_{\lambda\boldsymbol{k},\lambda'\boldsymbol{k}'} \equiv \sum_i w^{(i)}_{\lambda\boldsymbol{k},\lambda'\boldsymbol{k}'}$ is the total transition probability of an electron from the state $|\psi^{(0)}_{\lambda\boldsymbol{k}}\rangle$ to the state $|\psi^{(0)}_{\lambda'\boldsymbol{k}'}\rangle$ due to all disorder scattering events. For a homogeneous system and a uniform external electric field, the distribution function shall be independent of the spatial coordinate $\boldsymbol{r}$ and can be denoted as $f_\lambda(\boldsymbol{k})$. In the linear response

regime, the deviation of the nonequilibrium distribution function $f_\lambda(\boldsymbol{k})$ from the equilibrium Fermi distribution function f_0, $\delta f_\lambda(\boldsymbol{k}) \equiv f_\lambda(\boldsymbol{k}) - f_0$, can be described by the following ansatz:

$$\delta f_\lambda(\boldsymbol{k}) = -e\,(\partial f_0/\partial\epsilon_{\rm F})\,[A_{\lambda\boldsymbol{k}}(\boldsymbol{E}\cdot\boldsymbol{v}_{\lambda\boldsymbol{k}}) + B_{\lambda\boldsymbol{k}}(\boldsymbol{E}\times\hat{z})\cdot\boldsymbol{v}_{\lambda\boldsymbol{k}}], \quad (13)$$

where $\epsilon_{\rm F}$ is the Fermi energy and $A_{\lambda\boldsymbol{k}}$ and $B_{\lambda\boldsymbol{k}}$ are two coefficients which need to be determined self-consistently from Eq.(12). The second term in the square bracket of Eq.(13) characterizes the anisotropy of impurity scatterings. Substituting Eq.(13) into Eq.(12) and by some tedious algebra, one can show that the solution for the coefficients $A_{\lambda\boldsymbol{k}}$ and $B_{\lambda\boldsymbol{k}}$ can be expressed as

$$A_{\lambda\boldsymbol{k}} = \frac{\tau_{\lambda\boldsymbol{k}}}{1+(\tau_{\lambda\boldsymbol{k}}/\tau'_{\lambda\boldsymbol{k}})^2}, \quad (14)$$

$$B_{\lambda\boldsymbol{k}} = \frac{\tau'_{\boldsymbol{k}\lambda}}{1+(\tau'_{\lambda\boldsymbol{k}}/\tau_{\lambda\boldsymbol{k}})^2}, \quad (15)$$

where $\tau_{\lambda\boldsymbol{k}}$ and $\tau'_{\lambda\boldsymbol{k}}$ are two relaxation times defined by

$$\frac{1}{\tau_{\lambda\boldsymbol{k}}} = \sum_{\boldsymbol{k}',\lambda'} w_{\lambda\boldsymbol{k},\lambda'\boldsymbol{k}'}(1-k'_x/k_x)\delta(\varepsilon_{\lambda\boldsymbol{k}}-\varepsilon_{\lambda'\boldsymbol{k}'}), \quad (16)$$

$$\frac{1}{\tau'_{\lambda\boldsymbol{k}}} = \sum_{\boldsymbol{k}',\lambda'} w_{\lambda'\boldsymbol{k}',\lambda\boldsymbol{k}}(k'/k)\sin(\theta_{\boldsymbol{k}}-\theta_{\boldsymbol{k}'})\delta(\varepsilon_{\lambda\boldsymbol{k}}-\varepsilon_{\lambda'\boldsymbol{k}'}), \quad (17)$$

where $\theta_{\boldsymbol{k}}$ ($\theta_{\boldsymbol{k}'}$) denotes the angle between $\boldsymbol{k}$ ($\boldsymbol{k}'$) and the external electric field, respectively. By use of Eqs.(13)–(17), the nonequilibrium distribution function of electrons can be determined.

3. Results and discussions

After the expectation value $j_y^z(\boldsymbol{k},\lambda)$ and the nonequilibrium distribution function $f_\lambda(\boldsymbol{k})$ are obtained, the spin-Hall current density can be calculated through Eq.(3), from which one can show that the spin-Hall conductivity (defined by $\sigma_{\rm s} \equiv \lim_{E_x\to 0}(j_y^z/E_x)$) can be expressed as

$$\sigma_{\rm s} = \sigma_{{\rm s}(E)} + \sigma_{\rm s(im)}, \quad (18)$$

where $\sigma_{{\rm s}(E)}$ and $\sigma_{\rm s(im)}$ are given, respectively, by

$$\sigma_{{\rm s}(E)} = \frac{1}{2}N_{\rm c}e\alpha_E, \quad (19)$$

$$\sigma_{\rm s(im)} = \sum_{\boldsymbol{k}\lambda}(\partial f_0/\partial\epsilon_{\rm F})\,\frac{e\hbar^2\alpha_{\rm im}k_x^2}{2m[1+(\tau_{\lambda\boldsymbol{k}}/\tau'_{\lambda\boldsymbol{k}})^2]}, \quad (20)$$

where $N_{\rm c}$ is the charge carrier density.

From Eqs.(18)–(20) one can see that, after the influence of the spin-orbit coupling induced by the external electric field itself is taken into account, the spin-Hall conductivity shall be a sum of two distinct contributions. One contribution ($\sigma_{\rm s(im)}$) is from spin-orbit dependent impurity scatterings and the other ($\sigma_{{\rm s}(E)}$) is from the spin-orbit coupling induced by the external electric field. From Eq.(19) one can see that the contribution from the spin-orbit coupling induced by the external electric field itself is dissipationless, i.e., it is independent of disorder scattering effects, and, as will be shown more clearly below, all states below the Fermi level can make contribution to it with equal weights, similar to the dissipationless spin-Hall current due to intrinsic spin-Hall effect.[6] The spin-Hall conductivity given by Eq.(20) is the usual extrinsic contribution from spin-orbit dependent impurity scatterings, which contains both the side-jump and skew scattering contributions due to spin-orbit dependent impurity scatterings. These two types of extrinsic spin-Hall contributions are usually described in a separative way in some previous theoretical treatments. Comparing with these theoretical treatments, we find that a merit of our approach is that these two types of extrinsic spin-Hall contributions can be described in a unified way. From Eq.(20) one can see that, unlike the spin-Hall conductivity contribution from the spin-orbit coupling induced by the external electric field, for the usual extrinsic spin-Hall conductivity contribution due to spin-orbit dependent impurity scatterings, only the states in the vicinity of the Fermi level can make substantial contribution to it, similar to the cases encountered with usual dissipative charge and/or spin transports. Mathematically, this is due to the fact that the extrinsic spin-Hall conductivity given by Eq.(20) contains a factor of $\partial f_0/\partial\epsilon_{\rm F}$, which is nonvanishing only for the states in the vicinity of the Fermi level.

An interesting question regarding the results given by Eqs.(18)–(20) is why the contribution from the spin-orbit coupling induced by the external electric field itself could be independent of disorder scattering effects. From the theoretical point of view, this is due to the fact that the spin-orientation dependent part of the velocities of charge carriers due to this type of spin-orbit interaction is independent of their wave vectors. To see this fact clearly, let us neglect the extrinsic spin-orbit coupling induced by impurity scatterings (i.e., setting the coupling constant $\alpha_{\rm im} = 0$

in the Hamiltonian (1)). Then in the direction perpendicular to the external electric field, the velocities of charge carriers with spin-up and spin-down shall be given, respectively, by

$$v_y^{(+)}(\boldsymbol{k}) = \frac{\hbar k_y}{m} + \frac{e\alpha_E E_x}{\hbar}, \tag{21}$$

$$v_y^{(-)}(\boldsymbol{k}) = \frac{\hbar k_y}{m} - \frac{e\alpha_E E_x}{\hbar}, \tag{22}$$

where $v_y^{(+)}(\boldsymbol{k})$ is the velocity of charge carriers with spin-up and $v_y^{(-)}(\boldsymbol{k})$ is the velocity of charge carriers with spin-down. At zero temperature each state below the Fermi level is occupied by two charge carriers with opposite spins. Then from Eqs.(21) and (22) one can see that all states below the Fermi level shall contribute to the spin-Hall current by the same amount, and the spin-Hall conductivity due to all states below the Fermi level shall be given by

$$\begin{aligned}\sigma_{\mathrm{s}(E)} &= E_x^{-1}\sum_{\lambda,\boldsymbol{k}}\frac{\hbar}{2}[v_y^{(+)}(\boldsymbol{k})\theta(\epsilon_{\mathrm{F}}-\varepsilon_{+,k}) \\ &\quad - v_y^{(-)}(\boldsymbol{k})\theta(\epsilon_{\mathrm{F}}-\varepsilon_{-,k})] \\ &= \frac{1}{2}N_{\mathrm{c}}e\alpha_E,\end{aligned} \tag{23}$$

where $\theta(\epsilon_{\mathrm{F}}-\varepsilon_{\lambda\boldsymbol{k}})=1$ for $\varepsilon_{\lambda\boldsymbol{k}}<\epsilon_{\mathrm{F}}$ and $\theta(\epsilon_{\mathrm{F}}-\varepsilon_{\lambda\boldsymbol{k}})=0$ for $\varepsilon_{\lambda\boldsymbol{k}}>\epsilon_{\mathrm{F}}$. This result is still valid if the temperatures are not zero. At nonzero temperatures, one has

$$\begin{aligned}\sigma_{\mathrm{s}(E)} &= E_x^{-1}\sum_{\boldsymbol{k},\lambda}\frac{\hbar}{2}[v_y^{(+)}(\boldsymbol{k})f_+(\boldsymbol{k}) - v_y^{(-)}(\boldsymbol{k})f_-(\boldsymbol{k})] \\ &= \frac{1}{2}e\alpha_E\sum_{\boldsymbol{k}}[f_+(\boldsymbol{k})+f_-(\boldsymbol{k})] \\ &= \frac{1}{2}N_{\mathrm{c}}e\alpha_E,\end{aligned} \tag{24}$$

from which one can see clearly that, though the distribution function $f_\lambda(\boldsymbol{k})$ shall depend on disorder scattering effects and also depend on the temperatures, the spin-Hall conductivity contribution due to the spin-orbit coupling induced by the external electric field itself is independent of disorder scattering effects and all states below the Fermi level shall contribute to it with equal weights.

Since only the states in the vicinity of the Fermi level can make substantial contribution to the extrinsic spin-Hall conductivity $\sigma_{\mathrm{s(im)}}$ but all states below the Fermi level can contribute to the spin-Hall conductivity $\sigma_{\mathrm{s}(E)}$ with an equal weight, the two types of spin-Hall conductivity contributions shall be of the same orders of magnitude for an impure sample, though the spin-orbit coupling induced by impurity scatterings may be substantially stronger than that induced by the external electric field. To illustrate this fact explicitly, let us consider a simple case where the impurity scatterings are isotropic (i.e., the impurity scattering rates $w_{\lambda\boldsymbol{k},\lambda'\boldsymbol{k}'}$ are independent of the angle between $\boldsymbol{k}$ and $\boldsymbol{k}'$). If the impurity scatterings are isotropic, from Eq.(17) one shall get that $1/\tau'_{\lambda\boldsymbol{k}}=0$, then the extrinsic spin-Hall conductivity given by Eq.(20) shall reduce to the following form:

$$\sigma_{\mathrm{s(im)}} = \frac{e\hbar^2\alpha_{\mathrm{im}}}{2m}\sum_{\boldsymbol{k}\lambda}k_x^2(\partial f_0/\partial\epsilon_{\mathrm{F}}). \tag{25}$$

At low temperatures, the factor of $\partial f_0/\partial\epsilon_{\mathrm{F}}$ can be replaced approximately by a δ function, $\partial f_0/\partial\epsilon_{\mathrm{F}} \approx \delta\left(\frac{\hbar^2k^2}{2m}-\epsilon_{\mathrm{F}}\right)$. By making this approximation, the extrinsic spin-Hall conductivity given by Eq.(25) can be further reduced to a similar form as that given by Eq.(24),

$$\begin{aligned}\sigma_{\mathrm{s(im)}} &= \frac{e\hbar^2\alpha_{\mathrm{im}}}{2m}\sum_{\boldsymbol{k},\lambda}k_x^2(\partial f_0/\partial\epsilon_{\mathrm{F}}) \\ &\approx \frac{1}{2}N_{\mathrm{c}}e\alpha_{\mathrm{im}}.\end{aligned} \tag{26}$$

In obtaining this result the following identity has been used: $\sum_{\boldsymbol{k},\lambda}k_x^2\delta\left(\frac{\hbar^2k^2}{2m}-\epsilon_{\mathrm{F}}\right)=(m/\hbar^2)N_{\mathrm{c}}$, which can be verified by a few simple algebras. From Eqs.(25) and (26) one can see that, for δ-function shaped short-ranged isotropic impurity scatterings, the extrinsic spin-Hall conductivity due to spin-orbit dependent impurity scatterings shall be independent of the scattering times but depends only on the charge carrier density and the spin-orbit coupling constant. This result is in agreement with that obtained in some previous publications by different theoretical approaches (see, e.g., the corresponding results obtained in Refs.[3]–[5] and [26]). A similar result was also obtained in the study of the anomalous Hall effect due to spin-orbit dependent impurity scatterings in spin-polarized semiconductors, where it was also found that for δ-function shaped short-ranged isotropic impurity scatterings, the anomalous Hall conductivity shall be independent of the scattering times but depend only on the charge carrier density and the spin-orbit coupling constant.[43]

By making comparison between Eqs.(24)–(26) one gets $\sigma_{\mathrm{s}(E)}/\sigma_{\mathrm{s(im)}} \approx \alpha_E/\alpha_{\mathrm{im}}$. Since $\alpha_E \approx \alpha_{\mathrm{im}}$ in general, the spin-Hall conductivity contributions due to the two types of spin-orbit coupling shall be on the

same orders of magnitude in general. If impurity scatterings are anisotropic, the extrinsic spin-Hall conductivity given by Eq.(20) shall depend on the scattering times and may be somewhat smaller than that given by Eq.(26). In such cases the dissipationless spin-Hall conductivity contribution from the spin-orbit coupling induced by the external electric field itself may even be larger than that due to spin-orbit dependent impurity scatterings. But it should be stressed that the results obtained above is based on a perturbation treatment of both external electric fields and internal electric fields due to impurities, which is valid only in the cases where the density of impurities is in a modern range such that the total scattering potential due to all randomly distributed impurities is a smooth function of the space coordinate. Such a condition can be satisfied for heavily doped semiconductors but may fail in the clean limit. So, though the results obtained above is formally correct in the clean limit, the perturbation treatment applied above is actually valid only in a moderate range of impurity densities.

To give a quantitative estimate of the magnitudes of the spin-Hall conductivities obtained by the theory, we take n-doped GaAs for example. For n-doped GaAs, the magnitude of the spin-orbit coupling constants is around 5.3Å^2.[41] For the charge carrier density N_c, we take the parameters from the experiment reported in Ref.[8], where the electron density $N_c \approx 3 \times 10^{16}\text{cm}^{-3}$. To make comparisons with the relevant experimental results, the spin-Hall conductivities will be multiplied by a factor of $e/(\hbar/2)$ so that their dimensions become the same as usual charge conductivities.[8] Then from Eqs.(19) and (20) we get that $\sigma_{s(\text{im})} \approx \sigma_{s(E)} \approx 0.38\Omega^{-1}\cdot\text{m}^{-1}$ and the total spin-Hall conductivity $\sigma_s = \sigma_{s(\text{im})} + \sigma_{s(E)} \approx 0.76\Omega^{-1}\cdot\text{m}^{-1}$, this is in good agreement with the relevant experimental results ($\sigma_s \approx 0.7\Omega^{-1}\cdot\text{m}^{-1}$) reported in Ref.[8]. But it should be pointed out that, though the theoretical results obtained above are in good agreement with the relevant experimental results, a number of important factors were not considered in our calculation, e.g., the contribution of intrinsic spin-orbit coupling[21] and Coulomb interactions between charge carriers[32] were left out in our calculation. The total spin-Hall conductivity in real semiconductors will also depend on these factors. For example, as was shown in Ref.[32], the extrinsic spin-Hall conductivity shall be reduced by the Coulomb interactions for typical experimental mobilities. In order to get a clear understanding of the relevant experimental results, these factors also need to be considered. But this is beyond the scope of the approach applied in the present paper.

In summary, we have shown that in addition to the usual extrinsic contribution from spin-orbit dependent impurity scatterings, the spin-orbit coupling induced by the external electric field itself may also make a substantial contribution to the spin-Hall current in a nonmagnetic semiconductor (or metal). Assuming that the impurity density is in a moderate range such that the total scattering potential due to all randomly distributed impurities is a smooth function of the space coordinate, we find that the contribution due to the spin-orbit coupling induced by the external electric field itself shall be independent of disorder scattering effects and has the same orders of magnitude as the usual extrinsic contribution from spin-orbit dependent impurity scatterings, or may even be larger than the latter one. The results obtained are in good agreement with the recent relevant experimental results.

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