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Analytical solution of the Boltzmann–Poisson equation and its application to MIS tunneling junctions*

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In order to consider quantum transport under the influence of an electron–electron (e–e) interaction in a mesoscopic conductor, the Boltzmann equation and Poisson equation are investigated jointly. The analytical expressions of the distribution function for the Boltzmann equation and the self-consistent average potential concerned with e–e interaction are obtained, and the dielectric function appearing in the self-consistent average potential is naturally generalized beyond the Thomas–Fermi approximation. Then we apply these results to the tunneling junctions of a metal–insulator–semiconductor (MIS) in which the electrons are accumulated near the interface of the semiconductor, and we find that the e–e interaction plays an important role in the transport procedure of this system. The electronic density, electric current as well as screening Coulombic potential in this case are studied, and we reveal the time and position dependence of these physical quantities explicitly affected by the e–e interaction.

Keywords: Boltzmann–Poisson equation, quantum transport, self-consistent average potential, MIS tunneling junction

PACC: 7210B, 7300, 7340N

1. Introduction

In recent years, quantum transport in mesoscopic systems has attracted increasing attention,[1−5] and many methods, e.g. the non-equilibrium Green function and the Boltzmann equation etc., are used to investigate this problem. In particular, the Boltzmann equation is a very useful tool to study the electron transport in a mesoscopic system.[6−12] One can use it to conveniently describe the time and position dependence of a dissipative system. Since the Coulombic interaction between electrons is long-ranged, how to consider the electron–electron (e–e) interaction in the framework of the Boltzmann equation is an interesting problem. Vlasov used the Boltzmann equation and the Poisson equation to study the e–e interaction in plasma.[13] The so-called Vlasov equation was also utilized by Manfredi and Hervieux to describe the electronic transport in the thin metal film.[14] Here we will make use of the Boltzmann–Poisson equations to analytically investigate the influence of the e–e interaction on the transport in a mesoscopic system.

Since analytical analysis of the quantum transport involving the e–e interaction is a difficult problem, we make use of the Schrödinger equation together with the Poisson equation to treat this transport problem analytically in a mesoscopic conductor,[15,16] but we only obtain a one-dimensional analytical solution of the Schrödinger equation and the Poisson equation. In this paper, we try to study this problem by means of the Boltzmann equation and the Poisson equation. We can solve them analytically in the three-dimensional case, although they are usually solved numerically. The analytical expressions of the distribution function for the Boltzmann equation and the self-consistent average potential for the Poisson equation is helpful for intuitively analyzing quantum transport in a mesoscpic system.

In some multi-layer systems, such as metal–insulator–semiconductor (MIS) tunneling junctions, sometimes electrons accumulate near the interface between layers, and the e–e interaction will play an important role in the quantum transport of the whole system. In the MIS tunneling junction, if we want to know the transport properties, e.g. electric current, we must know the distribution of the self-consistent Coulombic average potential originating from the e–e interaction in this structure; however, it is related to the electronic distribution,[17] so we have to solve the Boltzmann and Poisson equations satisfied by

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the Coulombic average potential and the distribution function self-consistently, and this is widely done numerically.[18] It is worthwhile to obtain the solution analytically, then we can further obtain the analytical electronic density, electric current and other physical quantities in this structure, which can be compared with the previous results.[17]

2. Analytical solution of the Boltzmann–Poisson equation

Consider the electronic transport in a mesoscopic conductor, the distribution function of electrons in this conductor satisfies the following Boltzmann equation,

$$\frac{\partial f}{\partial t}+\boldsymbol{v}\cdot\frac{\partial f}{\partial \boldsymbol{r}}-\frac{e}{m}\boldsymbol{E}\cdot\frac{\partial f}{\partial \boldsymbol{v}}=-\left(\frac{\partial f}{\partial t}\right)_{\text{collision}}, \quad (1)$$

where $f(\boldsymbol{r},\boldsymbol{v},t)$ is the electronic distribution function that depends on the position $\boldsymbol{r}$, velocity $\boldsymbol{v}$ (or momentum $\boldsymbol{p}=m\boldsymbol{v}$), and time t; $\boldsymbol{E}$ is the external electric field applied on the conductor. The collision term $-\left(\frac{\partial f}{\partial t}\right)_{\text{collision}}$ on the right-hand side of the Boltzmann equation comes from the scattering of electron–impurity; if we adopt a relaxation time approximation, then it can be simplified as

$$-\left(\frac{\partial f}{\partial t}\right)_{\text{collision}}=-\frac{f-\langle f\rangle}{\tau_0}, \quad (2)$$

where τ_0 is the relaxation time.

In some mesoscopic systems, the influence of the e–e interaction on the transport of the system is obvious. In order to study the effect induced by this interaction analytically, we treat the e–e interaction in the conductor at the level of Hartree approximation, so each electron can be regarded as a particle moving in an average field provided by other electrons. The self-consistent average potential concerning this average field satisfies the Poisson equation,

$$\bigtriangledown^2\varphi=\frac{e}{\varepsilon_0}\int f_1 d\boldsymbol{v}, \quad (3)$$

where $\varphi(\boldsymbol{r},t)$ is the self-consistent average potential involving the e–e interaction, ε_0 is the dielectric function of vacuum, f_1 is the deviation from equilibrium distribution $f_1=f-f_0$, and f_0 is the Fermi equilibrium distribution. When we consider the e–e interaction in the process of quantum transport, the Boltzmann equation (1) becomes

$$\frac{\partial f}{\partial t}+\boldsymbol{v}\cdot\frac{\partial f}{\partial \boldsymbol{r}}-\frac{e}{m}\boldsymbol{E}\cdot\frac{\partial f}{\partial \boldsymbol{v}}+\frac{e}{m}\bigtriangledown\varphi\cdot\frac{\partial f}{\partial \boldsymbol{v}}=-\frac{f-\langle f\rangle}{\tau_0}, \quad (4)$$

where the term $\frac{e}{m}\bigtriangledown\varphi\cdot\frac{\partial f}{\partial \boldsymbol{v}}$ comes from the contribution of the self-consistent average potential. Equations (3) and (4) can be coupled together, and are named as the Boltzmann–Poisson equations. One usually solves them numerically by the iteration method, but these numerical solutions cannot give an intuitive structure of the physical quantities; it is better to give their analytical solutions in some theoretical analysis.

As we know, the electrons usually move near the Fermi surface in the mesoscopic transport process, so the velocity dependence (or the momentum dependence) in the term $\frac{e}{m}\boldsymbol{E}\cdot\frac{\partial f}{\partial \boldsymbol{v}}$ in Eq.(4) can be approximated as $\frac{e}{m}\boldsymbol{E}\cdot\frac{\partial f_0[1+h(\boldsymbol{r})]}{\partial \boldsymbol{v}}$, where f_0 is the Fermi equilibrium distribution function, and $h(\boldsymbol{r})$ is a small function chosen arbitrarily. Similarly, the term $\frac{e}{m}\bigtriangledown\varphi\cdot\frac{\partial f}{\partial \boldsymbol{v}}$ due to the e–e interaction can be approximated as $\frac{e}{m}\bigtriangledown\varphi\cdot\frac{\partial f_0}{\partial \boldsymbol{v}}$, and usually $\langle f\rangle=f_0$ in relaxation time approximation; these approximations are reasonable when the system is not far away from the equilibrium. After the above approximations, the electronic distribution under the influence of the e–e interaction satisfies the following equation,

$$\frac{\partial f_1}{\partial t}+\boldsymbol{v}\cdot\frac{\partial f_1}{\partial \boldsymbol{r}}-\frac{e}{m}\boldsymbol{E}\cdot\frac{\partial f_0[1+h(\boldsymbol{r})]}{\partial \boldsymbol{v}}+\frac{e}{m}\bigtriangledown\varphi\cdot\frac{\partial f_0}{\partial \boldsymbol{v}}=-\frac{f_1}{\tau_0}. \quad (5)$$

Next, we try to obtain the analytical solutions of Eqs.(3) and (5) by the integral transformation method.[13] Firstly, we carry out a Fourier transformation for Eqs.(3) and (5),

$$\begin{cases}\dfrac{\partial F_1}{\partial t}+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}F_1-\dfrac{e}{m}\boldsymbol{E}\cdot\dfrac{\partial f_0}{\partial \boldsymbol{v}}[\delta(\boldsymbol{k})+H(\boldsymbol{k})]\\ +\dfrac{e}{m}\mathrm{i}\boldsymbol{k}\cdot\dfrac{\partial f_0}{\partial \boldsymbol{v}}\Psi=-\dfrac{F_1}{\tau_0},\\ -k^2\,\Psi=\dfrac{e}{\varepsilon_0}\displaystyle\int F_1\mathrm{d}\boldsymbol{v},\end{cases} \quad (6)$$

where $F_1(\boldsymbol{k},\boldsymbol{v},t)$ is the Fourier transformation of distribution function $f_1(\boldsymbol{r},\boldsymbol{v},t)$ with respect to $\boldsymbol{r}$, $F_1(\boldsymbol{k},\boldsymbol{v},t)=\int f_1(\boldsymbol{r},\boldsymbol{v},t)\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\mathrm{d}\boldsymbol{r}$; $H(\boldsymbol{k})$ is the Fourier transformation of function $h(\boldsymbol{r})$, $H(\boldsymbol{k})=\int h(\boldsymbol{r})\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\mathrm{d}\boldsymbol{r}$, and $\Psi(\boldsymbol{k},t)$ is the Fourier transformation of the self-consistent average potential $\varphi(\boldsymbol{r},t)$, $\Psi(\boldsymbol{k},t)=\int\varphi(\boldsymbol{r},t)\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\mathrm{d}\boldsymbol{r}$. The time variable t in

Eq.(6) can be further transformed by the Laplace transformation,

$$\begin{cases} s\mathcal{F}_1 - G + \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}\mathcal{F}_1 - \dfrac{e}{m}\boldsymbol{E}\cdot\dfrac{\partial f_0}{\partial \boldsymbol{v}}[\delta(\boldsymbol{k}) + H(\boldsymbol{k})]\dfrac{1}{s} \\ +\dfrac{e}{m}\mathrm{i}\boldsymbol{k}\cdot\dfrac{\partial f_0}{\partial \boldsymbol{v}}\Phi = -\dfrac{\mathcal{F}_1}{\tau_0}, \\ -k^2\Phi = \dfrac{e}{\varepsilon_0}\displaystyle\int \boldsymbol{F}_1 \mathrm{d}\boldsymbol{v}, \end{cases} \tag{7}$$

where $\mathcal{F}_1(\boldsymbol{k},\boldsymbol{v},s)$ is the Laplace transformation of function $F_1(\boldsymbol{k},\boldsymbol{v},t)$, $\mathcal{F}_1(\boldsymbol{k},\boldsymbol{v},s) = \int F_1(\boldsymbol{k},\boldsymbol{v},t)\mathrm{e}^{-st}\mathrm{d}t$; $\Phi(\boldsymbol{k},s)$ is the Laplace transformation of function $\Psi(\boldsymbol{k},t)$, $\Phi(\boldsymbol{k},s) = \int \Psi(\boldsymbol{k},t)\mathrm{e}^{-st}\mathrm{d}t$; $G_0(\boldsymbol{k}) = \int(f^0 - f_0)\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\mathrm{d}\boldsymbol{r}$, $f^0 = f_0 g(\boldsymbol{r})$ is the initial distribution of the system, f_0 is the equilibrium distribution function, and $g(\boldsymbol{r})$ is a known function we used.

Then from Eq.(7), we have

$$\Phi = -\frac{e}{\varepsilon_0\varepsilon_l k^2}\int \frac{\dfrac{e}{m}\boldsymbol{E}\cdot\dfrac{\partial f_0}{\partial \boldsymbol{v}}[\delta(\boldsymbol{k}) + H(\boldsymbol{k})]\dfrac{1}{s}}{s + \dfrac{1}{\tau_0} + \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}}\mathrm{d}\boldsymbol{v} - \frac{e}{\varepsilon_0\varepsilon_l k^2}\int \frac{G}{s + \dfrac{1}{\tau_0} + \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}}\mathrm{d}\boldsymbol{v}, \tag{8}$$

where ε_l is the dielectric function of the system.

$$\varepsilon_l = 1 - \frac{\mathrm{e}^2}{m\varepsilon_0 k^2}\int \frac{\boldsymbol{k}\cdot\frac{\partial f_0}{\partial \boldsymbol{v}}}{\boldsymbol{k}\cdot\boldsymbol{v} - \mathrm{i}s - \dfrac{\mathrm{i}}{\tau_0}}\mathrm{d}\boldsymbol{v} = 1 + \frac{\mathrm{e}^2 n}{\varepsilon_0 k^2 k_\mathrm{B} T} - \frac{4\pi^2\mathrm{e}^2\left(s + \dfrac{1}{\tau_0}\right) f_0\left(\dfrac{\mathrm{i}s + \dfrac{\mathrm{i}}{\tau_0}}{k}\right)}{m\varepsilon_0 k^3} \tag{9}$$

with n being the electron concentration in the system. The first two terms in the dielectric function are just the well-known Thomas–Fermi dielectric function, while the third term is the correction we obtained, which indicates that our result is better than the Thomas–Fermi approximation. The integrations in Eq.(8) can be carried out by using the residual theorem; and the first term in Eq.(8) (denoted by Φ_1) is given by

$$\Phi_1 = -\mathrm{i}\frac{e}{\varepsilon_0\varepsilon_l k^3}\frac{eE_z}{m}H(\boldsymbol{k})\frac{mn}{k_\mathrm{B}T}\frac{1}{s}, \tag{10}$$

where we consider that the external electric field is in the z-axis direction. After the inverse Laplace transformation of Φ_1, we can express $\Psi_1 = \dfrac{1}{2\pi\mathrm{i}}\displaystyle\int_{\sigma-\mathrm{i}\infty}^{\sigma+\mathrm{i}\infty}\Phi_1\mathrm{e}^{st}\mathrm{d}s$ as

$$\Psi_1 = -\mathrm{i}\frac{e}{\varepsilon_0\varepsilon_l k^3}\frac{eE_z}{m}H(\boldsymbol{k})\frac{mn}{k_\mathrm{B}T}. \tag{11}$$

Then the self-consistent average potential φ_1 corresponding to the first term in Eq.(8) can be finally obtained by further inverse Fourier transformation of Ψ_1,

$$\varphi_1 = \left(\frac{1}{2\pi}\right)^3\int_{-\infty}^{\infty}\Psi_1\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\mathrm{d}\boldsymbol{k} = A_1\frac{\mathrm{e}^{-\lambda_\mathrm{D}r}}{r}, \tag{12}$$

where $\lambda_\mathrm{D} = \sqrt{\dfrac{n\mathrm{e}^2}{\varepsilon_0 k_\mathrm{B}T}}$ is the Debye wavelength, and $A_1 = -\dfrac{E_z}{4\pi}H(\mathrm{i}\lambda_\mathrm{D})\lambda_\mathrm{D}$ is the coefficient which is related to the external electric field.

Similarly, we can obtain the second integral in Eq.(8) (denoted by Φ_2),

$$\Phi_2 = -\frac{e}{\varepsilon_0\varepsilon_l k^2}\int \frac{G}{s + \dfrac{1}{\tau_0} + \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\mathrm{d}\boldsymbol{v}. \tag{13}$$

After inverse Laplace and Fourier transformations, we have the self-consistent average potential φ_2 corresponding to the second term in Eq.(8)

$$\varphi_2 = A_2\mathrm{e}^{-\frac{t}{\tau_0}}\frac{\mathrm{e}^{-\lambda_\mathrm{D}r}}{r}, \tag{14}$$

where the coefficient $A_2 = \dfrac{1}{6\sqrt{3\pi}}\dfrac{ne}{\varepsilon_0}G(\mathrm{i}\lambda_\mathrm{D})(1 - \mathrm{e}^{1-\sqrt{\frac{2ne^2t^2}{m\varepsilon_0}}})$ is related to the initial distribution of the system. Then the analytical expression of the self-consistent average potential in Eq.(3) is finally obtained as

$$\varphi = (A_1 + A_2\mathrm{e}^{-\frac{t}{\tau_0}})\frac{\mathrm{e}^{-\lambda_\mathrm{D}r}}{r}. \tag{15}$$

The first term in Eq.(15) is just the usual Coulombic screening potential, while the second term is a correction term we obtain, which depends on the initial distribution and is due to the time dependence of the Boltzmann equation. If the system starts with an initial distribution, then the self-consistent average potential will include time in an exponential form. It is meaningful to have the time-dependence of the self-consistent average potential.

Substituting Eq.(15) into Eq.(5), we can obtain the analytical form of the electronic distribution function as

$$f_1 = f^0 - f_0 + \frac{eE_z\tau_0}{m}\frac{\partial f_0}{\partial v_z}(1 - \mathrm{e}^{-\frac{t}{\tau_0}}) - \int_0^t \frac{e}{m}\mathrm{e}^{-\frac{t-t'}{\tau_0}} \bigtriangledown\varphi[\boldsymbol{r} - \boldsymbol{v}(t-t')] \times \frac{\partial f_0}{\partial \boldsymbol{v}}\mathrm{d}t'. \tag{16}$$

Using this expression, the electronic density, electric current as well as other physical quantities can also be expressed analytically. This is helpful for explicitly analysing the electronic transport in a system.

3. Application of the MIS tunneling junction

Here, we will apply the above results to the transport in the MIS tunneling junction whose structure is shown in Fig.1, where the semiconductor in the junction is chosen to be an n-type one. If a bias V is applied along the z-axis, which is perpendicular to

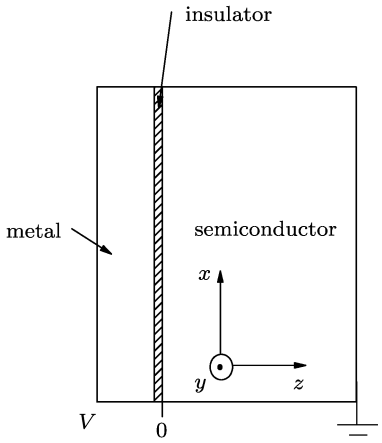


Fig.1. Schematic structure of the MIS tunneling junction.

the plane of the layers, we can only study the electronic transport along the z-axis, i.e. a quasi-one-dimensional problem. In this case, the term concerning the external electric field $\frac{e}{m}\boldsymbol{E}\cdot\frac{\partial f_0[1+h(\boldsymbol{r})]}{\partial \boldsymbol{v}}$ in Eq.(5) will reduce to $\frac{e}{m}E_z\frac{\partial f_0[1+h(z)]}{\partial v_z}$. We express $h(z)$ in Eq.(5) as $h(z)=-a\mathrm{e}^{-cz}$, where a and c are constants. After Fourier and Laplace transformations of the Boltzmann equation, the integral transformation Φ of the self-consistent average potential φ can be obtained. It has the following simple form

$$\Phi = -\frac{e}{\varepsilon_r\varepsilon_0\varepsilon_l k^2}\int\frac{\frac{e}{m}E_z\frac{\partial f_0}{\partial v_z}H(k_z)\delta(k_x)\delta(k_y)\frac{1}{s}}{s+\frac{1}{\tau_0}+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}}\mathrm{d}\boldsymbol{v} - \frac{e}{\varepsilon_r\varepsilon_0\varepsilon_l k^2}\int\frac{G}{s+\frac{1}{\tau_0}+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}}\mathrm{d}\boldsymbol{v}, \tag{17}$$

where ε_r is the dielectric constant of the system.

After inverse Fourier and Laplace transformations, we can obtain the analytical expression of the self-consistent average potential in the system

$$\varphi = \varphi_1 + \varphi_2, \tag{18}$$

where

$$\varphi_1 \approx \frac{aE_z}{2\lambda_{\mathrm{D}1}}\mathrm{e}^{-\lambda_{\mathrm{D}1}z} \tag{19}$$

and

$$\varphi_2 \approx \frac{ne\sqrt{\pi}G(\mathrm{i}\lambda_{\mathrm{D}1})\mathrm{e}^{-\frac{t}{\tau_0}-\lambda_{\mathrm{D}1}z}}{3\sqrt{3}\varepsilon_0\varepsilon_r\lambda_{\mathrm{D}1}}\left(1-\mathrm{e}^{1-\sqrt{\frac{2ne^2t^2}{m\varepsilon_0\varepsilon_r}}}\right), \tag{20}$$

in which $\lambda_{\mathrm{D}1}=\sqrt{\frac{ne^2}{\varepsilon_r\varepsilon_0 k_{\mathrm{B}}T}}$ is the Debye length of the semiconductor.

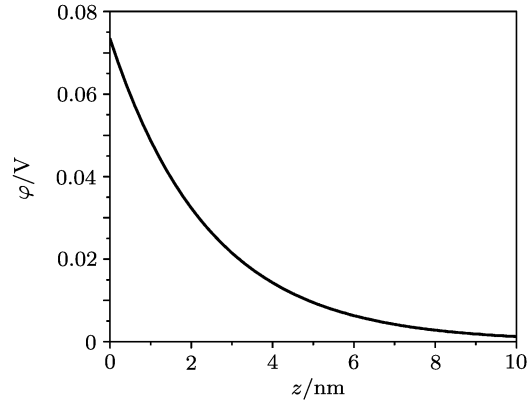


Fig.2. The self-consistent average potential in the semiconductor as a function of z.

In our calculation the electronic concentration in the semiconductor is chosen to be $n=1.5\times10^{22}$ m^{-3}, the external electric field $E_z=3\times10^7$ V/m, meanwhile the relaxation time of electron is $\tau_0=10^{-14}$ s^{-1}, the temperature $T=300$ K and the effective mass $m_{\mathrm{eff}}=0.19\ m$.[19] In this paper the effective Debye length near the surface of the semiconductor is $L_{\mathrm{D}1}\approx2.5$ nm, $|\varphi_1|\gg|\varphi_2|$, so we can omit φ_2 in the self-consistent average potential and only consider the first term φ_1 in Eq.(18). The self-consistent average potential as a function of position is shown in Fig.2, it decreases with position exponentially, indicating the screening effect caused by the e–e interaction.

Substituting Eq.(18) of the self-consistent average potential into Eq.(5), we can obtain the electronic distribution function in the MIS junction as

$$f_1 \approx f^0 - f_0 + \frac{eE_z\tau_0}{m}\frac{\partial f_0}{\partial v_z}(1-\mathrm{e}^{-\frac{t}{\tau_0}}) + \frac{aE_z}{2}\frac{e\tau_0}{m}\frac{\partial f_0}{\partial v_z}\frac{1-\mathrm{e}^{-\frac{t}{\tau_0}}\mathrm{e}^{\lambda_{\mathrm{D}1}v_z}}{1-\lambda_{\mathrm{D}1}v_z\tau_0}\mathrm{e}^{-\lambda_{\mathrm{D}1}z}, \tag{21}$$

which depends on time t, position z and velocity $\boldsymbol{v}$. After integrating over the velocity $\boldsymbol{v}$, the electronic

density and electric current density can be further obtained analytically,

$$n_e = \int f \mathrm{d}\boldsymbol{v}, \tag{22}$$

and

$$j = -e \int f \boldsymbol{v} \mathrm{d}\boldsymbol{v}. \tag{23}$$

The electronic density as a function of position is shown in Fig.3, where different distributions of electronic density are shown at different time $t = \tau_0$, $t = 2\tau_0$, $t = 3\tau_0$, $t = 6\tau_0$. At time $t = 6\tau_0$, the electronic distribution is rather stable, because it nearly reaches the equilibrium state, while at $t = \tau_0$, the system is out of equilibrium, which leads to the rapid variation of electronic density. Because of the screening effect induced by the e–e interaction, the relaxation time in our system becomes $\tau_\mathrm{a} = \dfrac{\tau_0}{1 - \lambda_{\mathrm{D}1} v_\mathrm{F} \tau_0}$ (where v_F is the Fermi velocity) which is larger than τ_0, so the system reaches equilibrium after $6\tau_0$. From

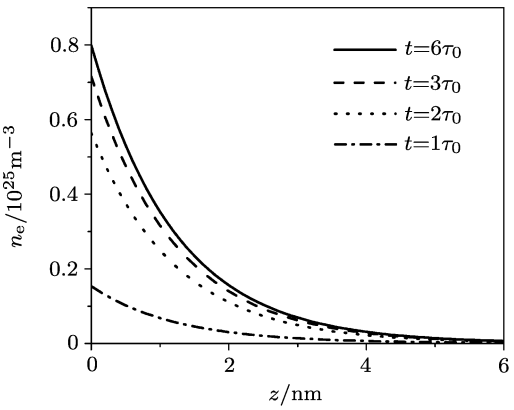


Fig.3. The electronic density in the semiconductor along the z-axis at different times.

Fig. 3, we can see the obvious electronic accumulation near the surface of the semiconductor, which is due to the positive voltage applied on the metal; this means that the e–e interaction is important in this region. Using the expression of electronic density, the quantum capacitance of this system can be easily obtained analytically,

$$C = \frac{\varepsilon_0 \varepsilon_\mathrm{r}}{d} \mathrm{e}^{-\lambda_{\mathrm{D}1} d}, \tag{24}$$

where d is the thickness of the semiconductor.

We draw the electric current density versus position in Fig.4, which shows that the electric current decreases as z increases in the semiconductor. This is caused by the external electric field which has the opposite direction to the electronic current. The external electric field and the self-consistent average potential will together affect the electron movement in the semiconductor; the time dependence of the electric current is similar to that of the electronic density shown in Fig.3.

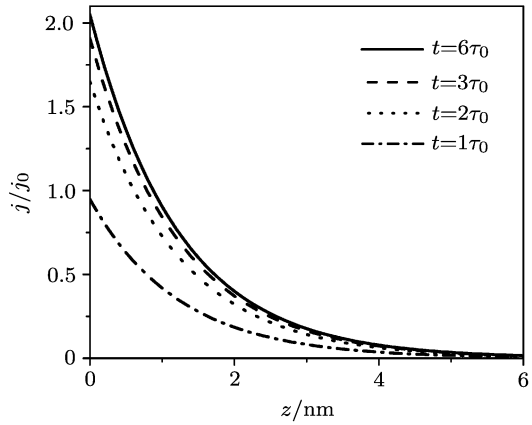


Fig.4. The electric current density versus position in the semiconductor at different times for $j_0 = \dfrac{ne^2 E_z \tau_\mathrm{a}}{m}$.

4. Summary

We have used the Boltzmann–Poisson model to investigate the electronic transport involving the e–e interaction in a mesoscopic conductor. The analytical solutions to the distribution function and the self-consistent average potential are obtained by means of the integral transformation method, also the corrections are obtained for the Thomas–Fermi dielectric function and the screening Coulombic potential; their applications to MIS tunneling junctions are further discussed. The self-consistent average potential, electronic density and electric current density as functions of position z are plotted in Figs.2, 3 and 4, respectively. The time dependence of the electronic density and electric current density are also shown in Figs.3 and 4, which is helpful for understanding the out-of-equilibrium transport properties in the system.

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