Vol 18 No 7, July 2009
1674-1056/2009/18(07)/2878-07

Chinese Physics B

© 2009 Chin. Phys. Soc.
and IOP Publishing Ltd

Lattice Boltzmann simulation of behaviour of particles moving in blood vessels under the rolling massage*

Yi Hou-Hui(伊厚会)[a)†], Yang Xiao-Feng(杨小锋)[b)],
Wang Cai-Feng(王彩凤)[a)], and Li Hua-Bing(李华兵)[c)]

[a)] *Department of Physics and Electronics Science, Binzhou University, Binzhou 256600, China*
[b)] *Department of Computer Engineering, Yiwu Industrial and Commercial College, Yiwu 322000, China*
[c)] *Department of Information Material Science and Engineering, Guilin University of Electronic Technology, Guilin 541004, China*

(Received 8 September 2008; revised manuscript received 12 February 2009)

The rolling massage is one of the most important manipulations in Chinese massage, which is expected to eliminate many diseases. Here, the effect of the rolling massage on a pair of particles moving in blood vessels under rolling massage manipulation is studied by the lattice Boltzmann simulation. The simulated results show that the motion of each particle is considerably modified by the rolling massage, and it depends on the relative rolling velocity, the rolling depth, and the distance between particle position and rolling position. Both particles' translational average velocities increase almost linearly as the rolling velocity increases, and obey the same law. The increment of the average relative angular velocity for the leading particle is smaller than that of the trailing one. The result is helpful for understanding the mechanism of the massage and to further develop the rolling techniques.

Keywords: lattice Boltzmann method, rolling massage, particles behaviour
PACC: 4760, 4755

1. Introduction

Blood flows through blood vessels are of great significance in medical science and are of great practical interest.[1,2] The blood flows are suspensions of formed elements in plasma and the formed elements are the red cells, white cells, and platelets. Red cells, white cells and platelets perform the most important duty of the blood, and a reduction in these elements poses grave health risks and is a major cause of mortality and morbidity in the industrialized world. Promoting blood circulation and blood cell transportation has become a significant topic, especially for stasis blood.

Massage is a method to aid circulation or relax the muscles by the rubbing or kneading of parts of the body. The Chinese traditional medical massage (CTMM) can even be a natural therapy, which has been applied to eliminate various diseases.[3−6] CTMM is also called the 'tui na' technique in China. The performance of 'tui na' is to push and to grasp the muscles of the sufferer, although the technique is not limited to these two actions. Although this technique has been used in the Orient for centuries,[3,4] the mechanism is far from being understood. Rolling massage is one of the most important manipulations in CTMM. Under this action, the superficial blood vessels will form a local moving stenosis, which changes the blood flows and blood cell transportation. In this paper, we focus on the behaviour of particles moving in blood vessels under rolling massage manipulation.

Since the proposal of the lattice Boltzmann method,[7−10] it has been recognized as an alternative method in computational fluid dynamics.[11,12] This method has been successfully extended to simulate fluid flows in various physical and engineering systems,[13,14] especially in the areas of complex fluids such as particle suspension flows,[15−22] binary mixtures,[23−27] flows in porous media[28−32] and blood flows.[33−42]

Recently, we have proposed a rolling massage model with a moving stenosis on blood vessels to explain the enhancement of the blood flow by assuming the blood as Newtonian fluid.[36] The blood flow was significantly influenced by the features of the rolling

*Project supported by the National Natural Science Foundation of China (Grant No 10747004), the National Key Basic Research Programme of China (Grant No 2006CB708612), and the Science Foundation of Guangxi, China (Grant No 0640064).
†To whom correspondence should be addressed. E-mail: houhuily@163.com

massage such as the rolling velocity and rolling depth. It was found that proper manipulation could aid the blood circulation. The effect of the rolling massage on a single particle's behaviour was also investigated by this model.[37] The particular interest in this field is modelling blood flows with swarms of blood cells. Studying the property of a pair of particles in fluid flows is the first step towards understanding of suspensions with swarms of particles, such as blood suspensions.

In this paper, we concentrate on the effect of rolling massage on a pair of particles' motions in blood vessels. In order to make comparisons, the behaviour of a pair of particles in blood vessels in the case of no rolling manipulation is studied first. Then the moving properties of a pair of particles in a long thin tube with a moving stenosis on one side are investigated by the lattice Boltzmann method. The particles' time-dependent positions, x-directional distance between them, translational velocities and angular velocities are computed. The average values, including translational velocities and angular velocities, for different rolling velocities and rolling depths are analysed numerically.

This paper is organized as follows. In Section 2, we briefly describe the lattice Boltzmann method. We present our simulation model and results in Section 3. Conclusions are given in Section 4.

2. The lattice Boltzmann model

We choose to work on a two-dimensional square lattice with nine velocities. Let $f_i(\boldsymbol{x},t)$ be a non-negative real number describing the distribution function of the fluid density at site $\boldsymbol{x}$ at time t moving in direction $\boldsymbol{e}_i$. Here $\boldsymbol{e}_0=(0,0)$, $\boldsymbol{e}_i=(\cos(\pi(i-1)/2), \sin(\pi(i-1)/2)), i=1,2,3,4$, and $\boldsymbol{e}_i=\sqrt{2}(\cos(\pi(2i-1)/4), \sin(\pi(2i-1)/4))$, for $i=5$, 6, 7, 8 are the nine possible velocity vectors. The distribution functions evolve according to a Boltzmann equation that is discrete in both space and time,[8,9]

$$f_i(\boldsymbol{x}+\boldsymbol{e}_i,t+1)-f_i(\boldsymbol{x},t)=-\frac{1}{\tau}(f_i-f_i^{\mathrm{eq}}). \qquad (1)$$

The density ρ and macroscopic velocity $\boldsymbol{u}$ are defined by

$$\rho=\sum_i f_i, \quad \rho\boldsymbol{u}=\sum_i f_i\boldsymbol{e}_i, \qquad (2)$$

and the equilibrium distribution functions f_i^{eq} are usually supposed to be dependent only on the local density ρ and flow velocity $\boldsymbol{u}$. A suitable choice reads[8,9]

$$f_i^{\mathrm{eq}}=\rho\alpha_i\left[1+3\boldsymbol{e}_i\cdot\boldsymbol{u}+\frac{9}{2}(\boldsymbol{e}_i\cdot\boldsymbol{u})^2-\frac{3}{2}u^2\right], \qquad (3)$$

where $\alpha_0=4/9$, $\alpha_1=\alpha_2=\alpha_3=\alpha_4=1/9$, and $\alpha_5=\alpha_6=\alpha_7=\alpha_8=1/36$. The pressure and the kinematic viscosity are defined by the equations $p=c_{\mathrm{s}}^2\rho$ with $c_{\mathrm{s}}^2=1/3$ and $\nu=(2\tau-1)/6$.

3. Results and discussion

The two-dimensional model of the simulation system is shown in Fig.1. A fraction of a blood vessel is modelled by a planar channel with length L of 5200 and diameter (width) D of 30 with lattice unit. A stenosis, arc-shaped with a constant width $L_{\mathrm{s}}=800$ lattice unit, moves with a velocity u from $x_{\mathrm{c}}/L=0.115$ to $x_{\mathrm{c}}/L=0.885$ on the upper boundary due to rolling massage manipulation and the rolling time in this process is denoted by T. The thickness of the stenosis is h, which is the rolling massage depth. The radius of the arc is $r=h/2+L_{\mathrm{s}}^2/8h$. Here h is assumed to vary with respect to time t as

$$h=\begin{cases} h_{\mathrm{m}}t/T, & 0\le t<\dfrac{T}{2};\\ h_{\mathrm{m}}(1-t/T) & \dfrac{T}{2}\le t<T, \end{cases} \qquad (4)$$

where h_{m} is the maximal rolling massage depth. The maximal relative rolling massage depth is defined as h_{m}/D. The rolling velocity to the blood velocity (relative rolling velocity) is u/u_0, where u_0 is the blood velocity in the case without rolling manipulation.

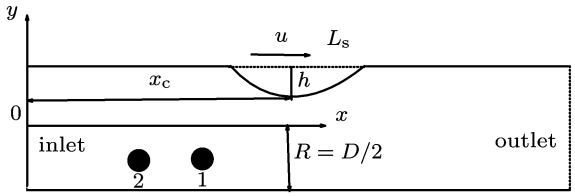


Fig.1. Schematic diagram of a pair of particles moving in a planar channel under rolling massage manipulation on the upper boundary.

In the simulation, the particles are assumed to be identical cylinders with diameter d of 10 lattice units. The leading particle and the trailing particle are denoted by number 1 and number 2 respectively. In the study, the blood flow and the particles are assumed to have a mass density of 1 lattice unit; $\tau=0.517$ in each simulation. The pressure drop between inlet and outlet is set to be 1.998×10^{-3}. Initially the distribution functions at all the fluid nodes are set to be the equilibrium distribution functions

with zero velocity except for those at inlet and outlet. The rolling massage manipulation is exerted and the particles are released simultaneously after the first 100000 time steps ($10\Delta t$). The boundary conditions for the moving particles are treated by the method depicted in Refs.[20,35]. Periodical boundary conditions with a pressure difference are implemented at inlet and outlet.[43] The hydrodynamic forces acting on the particles are evaluated based on the method of stress-integration.[20] The translation and rotation of the particles are updated at each Newtonian dynamics time step by using a so-called half-step 'leap-frog' scheme.[44]

3.1. The behaviour of two particles in no rolling vessels

In this section, we first analyse the particle-particle interaction in vessels without rolling massage. In Fig.2(a), we display the x-directional distance between the centroids of the two particles, denoted by δx, with respect to time t. In the numerical simulations, initially, a pair of particles of the same size are both restrained at $y/R = 0.368$, which is the equilibrium radial position for a single particle with the blockage ratio $d/D = 0.33$ at the flow Reynolds number Re=26.6, where Re is defined as Du_0/ν.[45,46] The particles are released from their restrained radial positions with the particles x-directional distance $\delta x = 2d$. Figure 2(a) shows that the distance δx first increases rapidly and then decreases gradually and finally reaches an equilibrium distance with the value of $3.322d$. The simulation result indicates that the equilibrium state is the result of compromising of the attractive force and the repulsive force between the two particles.[47,48] In Figs.2(b)–2(d), we also show the dimensionless y-directional positions, translational velocities and angular velocities of both particles with respect to time t. The simulation results show that the leading particle's equilibrium position y_{10}/R is the same as that of the case with only one particle, i.e., $y_{10}/R = 0.368$, while the trailing particle's equilibrium position $y_{20}/R = 0.381$ is larger than that in the one-particle case. The particles' equilibrium translational velocities are $v_{10} = v_{20} = 5.835 \times 10^{-3}$, and angular velocities are $\omega_{10} = 1.898 \times 10^{-4}$ and $\omega_{20} = 1.634 \times 10^{-4}$ respectively. Due to the symmetry of the system under no rolling condition, there also exist the equilibrium states for both particles at the lower side of the vessel, i.e., $y_{10}/R = -0.368$, $y_{20}/R = -0.381$.

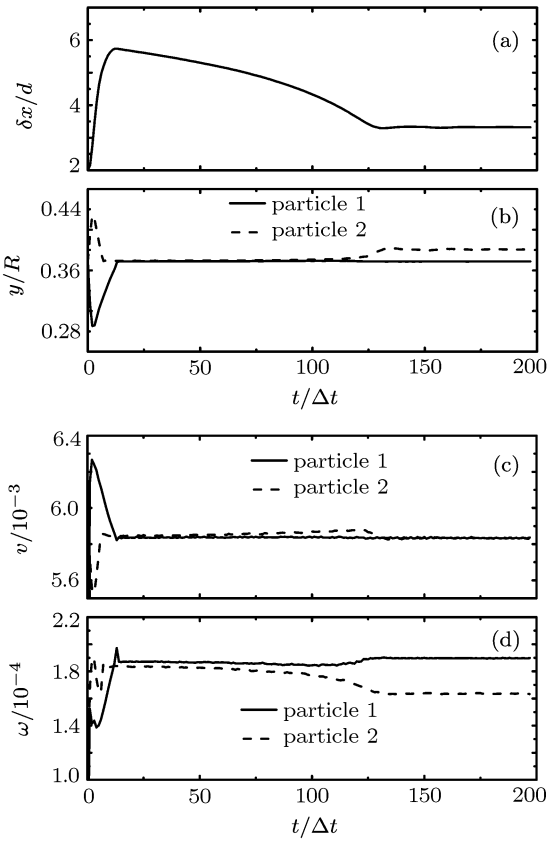


Fig.2. Time-dependent x-directional distance δx between two particles (a), dimensionless y-directional positions (b), translational velocities (c) and angular velocities (d) of both particles in the case without rolling vessels.

3.2. The behaviour of two particles under rolling manipulation

3.2.1. The rolling effect for different particle positions

3.2.1.1. The particles' trajectories

In order to investigate the effect of the rolling massage on the particles' motions, we define the time-dependent translational relative velocities v_1/v_{10}, v_2/v_{20}, time-dependent relative angular velocities ω_1/ω_{10}, ω_2/ω_{20} of the two particles, where v_1, v_1 and ω_1, ω_2 are the particles' translational velocities and angular velocities as functions of time t. Before rolling massage manipulation, two particles are assumed to be situated at their equilibrium states as mentioned in Section 3.1. In the following, according to the relations of both particles in their equilibria, only the position of the trailing particle (number 2) is used when we depict the particles' initial positions under rolling manipulation.

Figures 3(a)–3(d) display the numerical results of the particles' trajectories for different initial positions under rolling manipulation. It is clear that the rolling massage can considerably change the behaviour of the moving particle, and the equilibria of the two particles are disturbed. The particles' behaviour is affected by the rolling massage manipulation in two aspects:

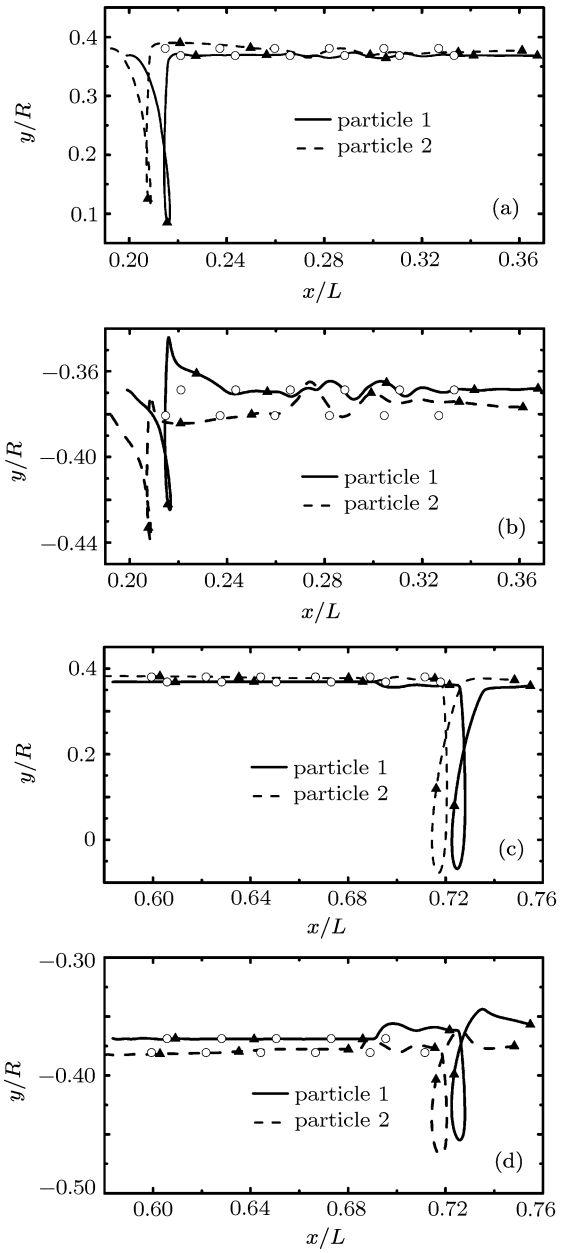


Fig.3. Particle trajectories for the trailing particle initial positions $x_2^0/L = 0.192$, $y_2^0/R = 0.381$ (a), $x_2^0/L = 0.192$, $y_2^0/R = -0.381$ (b), $x_2^0/L = 0.576$, $y_2^0/R = 0.381$ (c) $x_2^0/L = 0.576$, $y_2^0/R = -0.381$ (d) under rolling manipulation. The circles and the triangles are the snapshots of the particle positions with interval $T/6$, for the case without and with rolling massage respectively. The ratio of rolling massage velocity to the blood velocity is 6.6 and the maximal relative rolling depth h_m/D is 0.6.

the rightward motion of the stenosis and the change of depth h. The rightward motion of the stenosis leads to the particles moving downward if it moves closer to them, while it causes the particles to move upward if it moves away from them. The particles' x-directional motions are also modified by the rightward motion of the stenosis and the moving behaviour is affected by the ratio of rolling velocity to blood velocity. An increase of depth h hinders the motion of particles behind it and pushes the particles forward before it. In contrast, a decrease of depth h results in pushing the particles before the stenosis backward. In the vertical direction, an increase of depth h favors the particles downward while a decrease results in the particles moving upward. As the stenosis moves forward, the abnormality of the particles' motions is clearly found in Fig.3. The effects of rolling massage on the particles' motions are very similar but the trajectory of the leading particle lags that of the trailing one.

Figure 4 displays the variation of x-directional distance δx between the two particles with time t for different initial positions under rolling massage. The simulation results show that there is a short period that δx is obviously larger than that under no rolling massage as the stenosis passes by the particles. The maximal x-directional distance δx_m for $x_2^0/L = 0.192$ is smaller than that for $x_2^0/L = 0.576$.

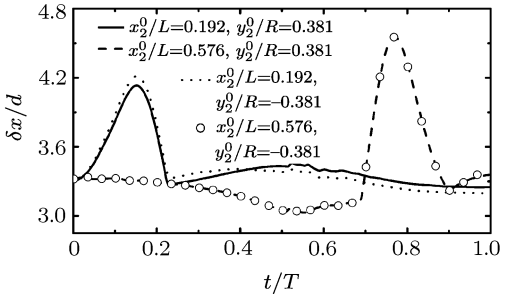


Fig.4. Time-dependent x-directional distance δx between the two particles for different initial positions under rolling manipulation. The ratio of rolling massage velocity to blood velocity is 6.6 and the maximal relative rolling depth h_m/D is 0.6.

3.2.1.2. Particle velocities and angular velocities

Figure 5(a) displays the translational relative velocities v_1/v_{10} and v_2/v_{20} versus time t for different initial positions of the particles. In most of the second half of the rolling time, both particles' velocities are larger than those in a vessel without rolling massage for $x_2^0/L = 0.192$. Similar results can be expected

in most of the first half of the rolling time for $x_2^0/L = 0.576$. The behaviour of the particles' translational relative velocities is similar to that of the blood flux at inlet (outlet) in Ref.[36] if the particles' position is nearer to the inlet (outlet). There exists a clear difference of translational velocity between the two particles for $x_2^0/L = 0.192$ when $0.02T < t < 0.23T$, so δx at this stage is obviously larger than that under normal conditions as shown in Fig.4. As for $x_2^0/L = 0.576$, the time that the clear difference of translational velocity between the two particles appears is from $0.69T$ to $0.90T$. The small difference of v_1/v_{10} (v_2/v_{20}) between $y_2^0/R = -0.381$ and $y_2^0/R = 0.381$ for the same x_2^0/L indicates that the effect of y-directional positions on the particles' translational velocities is negligible. Figure 5(b) displays the impact of rolling massage on the particles' relative angular velocities. The simulation result is similar to that for the translational velocities.

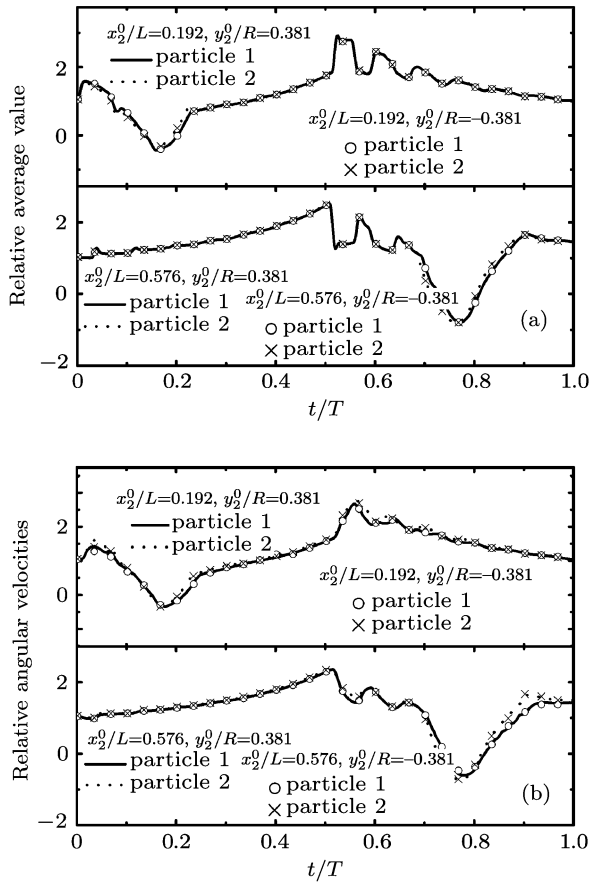


Fig.5. Time-dependent translational relative velocities v_1/v_{10} and v_2/v_{20} (a) and relative angular velocities ω_1/ω_{10} and ω_2/ω_{20} (b) for different initial positions of the particles. The maximal relative rolling massage depth h_m/D is 0.6 and the ratio of rolling velocity to blood velocity is 6.6.

3.2.2. The effect of rolling velocity on the particle motions

The simulation results for different relative rolling velocities for $x_2^0/L = 0.192$, $y_2^0/R = 0.381$ are displayed in Fig.6. The average relative velocity V_1/v_{10} (V_2/v_{20}) reflects the impact of the rolling massage. $V_1/v_{10} > 1$ ($V_2/v_{20} > 1$) indicates that the rolling massage increases the leading (trailing) particle motion while $V_1/v_{10} < 1$ ($V_2/v_{20} < 1$) denotes that the rolling massage decreases the leading (trailing) particle motion. Figure 6 shows that average relative velocities and the average relative angular velocities of the particles increase as the rolling velocity increases. The results are similar to that for the blood flux in vessels in Ref.[36] and those for the average relative translational velocity V/v_0 and average relative angular velocity ϖ/ω_0 for a single-particle case in Ref.[37]. In a single particle case, $\varpi/\omega_0 \approx V/v_0$,[37] which denotes that the rolling massage has almost the same effect both on the translational velocity and on the angular velocity at a fixed rolling velocity. In the two-particle case, the average relative angular velocity of the trailing particle is larger than that of the leading particle for the same rolling velocity. The results indicate that rolling massage has a better effect on the angular velocity of the trailing particle than that of the leading one. For the relative rolling velocity $u/u_0 = 6.66$, the increment of the average velocities is about 25%, which is a little larger than that in the single-particle case in Ref.[37].

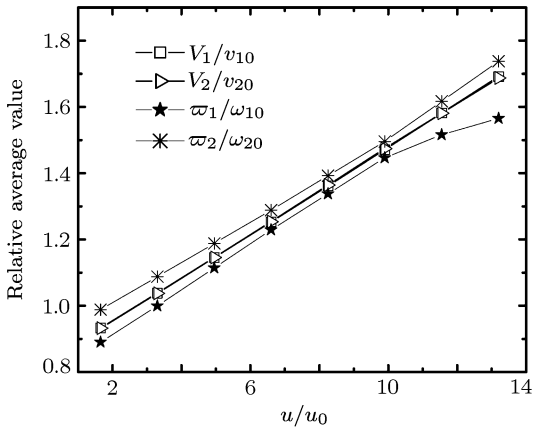


Fig.6. Average relative translational velocity and average relative angular velocity versus the relative rolling velocity. The maximal relative rolling massage depth h_m/D is 0.6.

Figure 7 shows the x-directional distance δx between the two particles for different relative rolling velocities as a function of time t. Smaller relative rolling velocity results in a longer rolling time on both

particles as the stenosis moves forward, so the larger maximal distance δx_{m} is found, as shown in Fig.7.

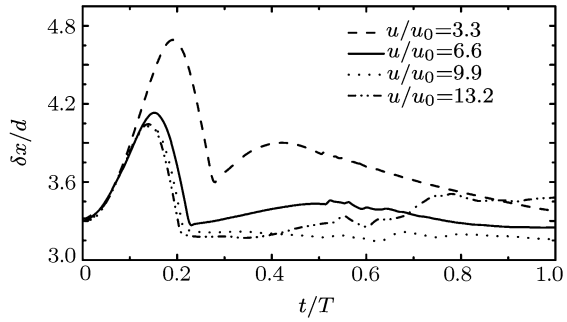


Fig.7. Time-dependent x-directional distance δx between two particles for different relative rolling velocities. The maximal relative rolling massage depth h_{m}/D is 0.6.

3.2.3. The effect of rolling depth on particle motions

The properties of both particles also depend on the maximal rolling depth h_{m}. The particles' average relative values, including average relative translational velocity and average relative angular velocity, are displayed in Fig.8. Both increase as h_{m}/D increases. Similar results are obtained for the average blood flux in blood vessels in Ref.[36]. The increment of the angular velocity of the trailing particle is larger than that of the leading one.

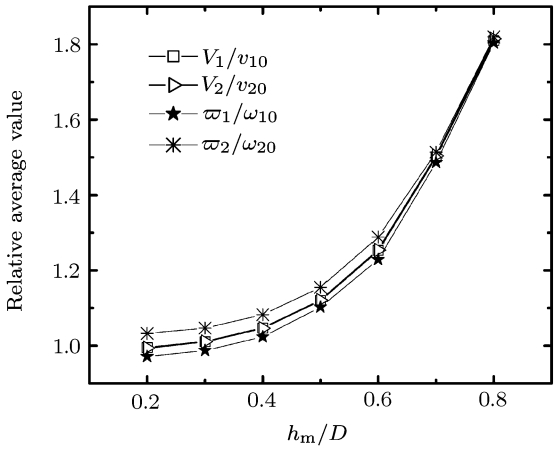


Fig.8. Average relative translational velocities and average relative angular velocities versus the maximal relative rolling depth h_{m}/D. The ratio of rolling velocity to the blood velocity is 6.6.

Figure 9 shows the x-directional distance δx between the two particles versus time t for $x_2^0/L = 0.192$, $y_2^0/R = 0.381$ for different maximal rolling depths h_{m}/D under rolling massage. The simulation results show that larger rolling depth results in the larger maximal distance δx_{m} when the stenosis moves forward.

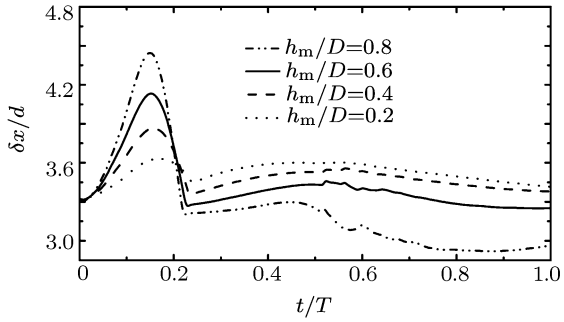


Fig.9. Time-dependent x-directional distance δx between the two particles for different maximal rolling depths h_{m}/D under rolling manipulation. The ratio of rolling massage velocity to the blood velocity is 6.6.

4. Conclusion

In summary, we have used a rolling massage model to study the properties of a pair of particles moving in blood vessels under rolling massage manipulation. It is found that the particles' behaviour will be greatly changed by the rolling massage. The average values, including those of particle translational velocity and angular velocity, depend on the rolling velocity and rolling depth. The increment of the angular velocity of the trailing particle is larger than that of the leading one. The simulation results show that the rolling massage does have the function of promoting blood circulation and removing the blood stasis if a proper rolling technique is exerted.

Our study may be helpful for the understanding of the mechanics of the rolling massage and provide a basis for massage therapy. However, the effects of the rolling massage could be more complex, i.e., the blood flow may get in bifurcations as well as the deformation of the red blood cells and blood vessels. More complex models are needed and we are working in this direction.

The authors thank professor Fang Haiping and Qian Yuehong for helpful comments and discussions, and Shanghai Supercomputer Center of China for the computational support.

References

[1] Fung Y C 1997 *Biomechanics Circulation* (Berlin: Springer-Verlag)

[2] Stround J S, Berger S A and Saloner D 2002 *J. Biomech. Eng.* **124** 9

[3] Maria Mercati 1997 *The Handbook of Chinese Massage: Tuina Techniques to Awaken Body and Mind* (Rochester Vermont: Healing Art Press)

[4] Sun C N 1990 *Chinese Massage Therapy* (Jinan: Shandong Science and Technology Press)

[5] Xu S X 1997 *Proceeding of 9th International Conference of Biomedical Engingeering* (Singapore: Singapore University Press) p306–308

[6] Xu S X, Ji L and Wang Q W 2005 *Appl. Math. Mech. Engl.* **26** 753

[7] McNamara G R and Zanetti G 1988 *Phys. Rev. Lett.* **61** 2332

[8] Qian Y H, d'Humiéres D and Lallemand P 1992 *Europhys. Lett.* **17** 479

[9] Chen S, Chen H, Martinez D O and Matthaeus W H 1991 *Phys. Rev. Lett.* **67** 3776

[10] Chen H, Chen S and Matthaeus W H 1992 *Phys. Rev.* A **45** 5339

[11] Chen H, Kandasamy S, Orszag S, Shock R, Succi S and Yakhot V 2003 *Science* **301** 633

[12] Succi S 2001 *The Lattice Boltzmann Equation for Fluid Dynamics and Beyond* (Oxford: Clarendon Press)

[13] Guo W B, Wang N C, Shi B C and Guo Z L 2003 *Chin. Phys.* **12** 67

[14] Liu C F and Ni Y S 2008 *Chin. Phys.* B **17** 4554

[15] Ladd A J C 1994 *J. Fluid Mech.* **271** 285

[16] Wan R Z, Fang H P, Lin Z F and Chen S Y 2003 *Phys. Rev.* E **68** 011401

[17] Zhang C Y, Shi J, Tan H L, Liu M R and Kong L J 2004 *Chin. Phys. Lett.* **21** 1108

[18] Zhang C Y, Li H B, Tan H L, Liu M R and Kong L J 2005 *Acta Phys. Sin.* **54** 1982 (in Chinese)

[19] Zhang C Y, Tan H L, Liu M R, Kong L J and Shi J 2005 *Chin. Phys. Lett.* **22** 896

[20] Li H B, Lu X Y, Fang H P and Qian Y H 2004 *Phys. Rev.* E **70** 026701

[21] Fang H P and Chen S Y 2004 *Chin. Phys.* **13** 47

[22] Yi H H, Chen Y Y and Li H B 2007 *Chin. Phys.* **16** 2444

[23] Shan X and Chen H D 1993 *Phys. Rev.* E **47** 1815

[24] Guo Z L and Zhao T S 2003 *Phys. Rev.* E **68** 035302

[25] Guo Z L and Zhao T S 2005 *Phys. Rev.* E **71** 026701

[26] Swift M R, Orlandini S E, Osborn W R and Yeomans J M 1995 *Phys. Rev. Lett.* **75** 830

[27] Xu A G, Gonnella G and Lamura A 2004 *Physica* A **331** 10

[28] Xu Y S, Wu F M, Chen Y Y and Xu X Z 2003 *Chin. Phys.* **12** 621

[29] Xu Y S, Wu F M, Chen Y Y and Xu X Z 2003 *Mod. Phys. Lett.* B **19** 1351

[30] Xu Y S 2003 *Acta Phys. Sin.* **52** 626 (in Chinese)

[31] Xu Y S, Li H B, Fang H P and Huang G X 2004 *Acta Phys. Sin.* **53** 773 (in Chinese)

[32] Wu B Z, Xu Y S, Liu Y and Huang G X 2005 *Chin. Phys.* **14** 2046

[33] Fang H P, Lin Z F and Wang Z W 1998 *Phys. Rev.* E **57** **R**25

[34] Fang H P, Wang Z W, Lin Z F and Liu M R 2002 *Phys. Rev.* E **65** 051905

[35] Li H B, Fang H P, Lin Z F, Xu S X and Chen S Y 2004 *Phys. Rev.* E **69** 031919

[36] Yi H H, Xu S X, Qian Y H and Fang H P 2005 *Chin. Phys. Lett.* **22** 3210

[37] Yi H H, Fan L J, Yang Y F and Chen Y Y 2008 *Chin. Phys. Lett.* **25** 3496

[38] Kang X Y, Liu D H, Zhou J and Jin Y J 2005 *Chin. Phys. Lett.* **22** 2873

[39] Lu X Y, Yi H H, Chen J Y and Fang H P 2006 *Chin. Phys. Lett.* **23** 738

[40] Li H B, Yi H H, Shan X W and Fang H P 2008 *Europhys. Lett.* **81** 54002

[41] Migliorini C, Qian Y H, Chen H, Brown E B, Jain R K and Munn L L 2002 *Biophys. J.* **83** 1834

[42] Sun C H, Migliorini C and Munn L L 2003 *Biophys. J.* **85** 208

[43] Inamuro T, Maeba K and Ogino F 2000 *Int. J. Multiphase Flow* **26** 1981

[44] Allen M P and Tildesley D J 1987 *Computer Simulation of Liquid* (Oxford: Clarendon Press)

[45] Segré G and Silberberg A 1961 *Nature* **189** 209

[46] Segré G and Silberberg A 1962 *J. Fluid Mech.* **14** 115

[47] Segré G and Silberberg A 1965 *Proc. 4th. Int. Gong. Rheol.* **4** 103

[48] Zhu M Y 2000 *Direct Numerical Simulation of Particulate Flow of Newtonian and Viscoelastic Fluids.* (Ph. D Dissertation) (Philadelphia: University of Pennsylvania)