Vol 18 No 7, July 2009
1674-1056/2009/18(07)/2774-05

Chinese Physics B

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Study on the design and Zernike aberrations of a segmented mirror telescope*

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(Received 19 October 2008; revised manuscript received 13 November 2008)

The segmented mirror telescope is widely used. The aberrations of segmented mirror systems are different from single mirror systems. This paper uses the Fourier optics theory to analyse the Zernike aberrations of segmented mirror systems. It concludes that the Zernike aberrations of segmented mirror systems obey the linearity theorem. The design of a segmented space telescope and segmented schemes are discussed, and its optical model is constructed. The computer simulation experiment is performed with this optical model to verify the suppositions. The experimental results confirm the correctness of the model.

Keywords: Fourier optics, segmented mirror, Zernike, optical design
PACC: 4215E, 4230K

1. Introduction

The segmented primary mirror telescope is better than the traditional single primary mirror telescope in many aspects. For example, the primary mirror of the astronomical telescope can be made larger, and then the telescope can get higher angular resolution. The space-based remote sensing and detecting telescopes often use a segmented primary mirror to get high resolution and to compress the volume during the launching phase. Now, the diameter of payload in a carrier rocket must be less than 4.5 meters. The segmented mirrors can be adjusted to modulate the wave front of a system to compensate the influence of gravity and heat. So the image quality of the system can be improved. The segmented primary mirror telescope is already used in some telescopes.

In 1993 the KECK[1−6] telescope was built on the summit of Mauna Kea Hawaii astronomical observatory. It has a 10 meter diameter primary mirror which is divided into a mosaic of 36 hexagonal segments. Each segment is a part of an off-axis hyperbola with a focal ratio of $f/1.75$. Every segment mirror is 1.8-m from corner to corner and 75-mm thick, just as shown in the following figure. Figure 1 shows the KECK telescope in Wizinowich's paper.[1]

The James Webb Space Telescope (JWST) [7−11] is NASA's next great astronomical mission which they plan to launch in 2013. This telescope will be equipped with a large 18-segment, 6.5-meter primary mirror and a suite of infrared-sensing cameras and spectrometers. It is designed for seeing tiny objects in space. Its working wavelength is from 1.0 μm to 29 μm. The JWST has a large aperture (about 25 m^2) and will provide high angular resolution imaging and spectroscopy. Its diffraction-limited imaging can reach 2 μm. The JWST has been designed as a deployable payload of rocket. Figure 2 shows the James Webb telescope in Clampin's paper.[7]

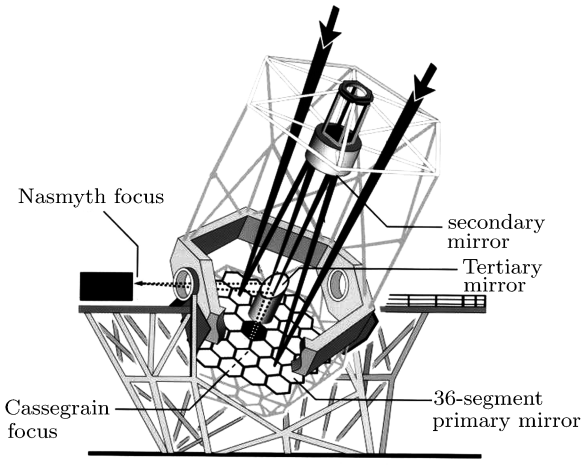


Fig.1. KECK telescope diagram.[1]

*Project supported by the National High Technology Research and Development Program of China (Grant No 2006AA01Z339).
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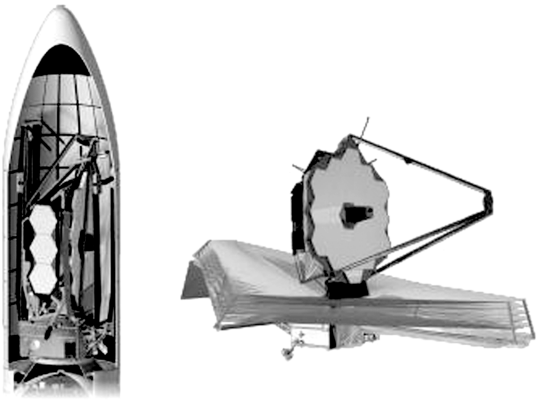


Fig.2. Deployment of James Webb.[7]

2. Imaging property of a segmented optical system

As compared with the traditional monolithic mirror telescope, the segmented mirror system's aperture is divided into parts. So the wave front is cut into many parts, thereby, the diffraction phenomenon appears. The diffraction pattern of the segmented aperture has been calculated in Ref.[12].

In their paper,[12] Lightsey and Chrisp draw the conclusion that the diffraction from the small gaps ($< 0.001D$, where D is the diameter of the aperture) is quite small, and the point spread function (PSF) is dominated by diffraction from the serrated perimeter of the primary mirror. Most of the errors in a segmented mirror system are redistributed from low frequency aberrations to mid-frequency errors. Segmented mirror errors can be characterized by decomposition into Zernike polynomials.

The segmented primary mirror is divided into different sub mirrors. That is why the whole primary mirror loses its integrity. Any displacements of sub mirrors bring surface form error.

Goodman's book[13] describes the Fourier optics theory. In his book, the imaging system's linearity property is introduced.

According to the linearity theorem:[13]

$$F\{\alpha g + \beta h\} = \alpha F\{g\} + \beta F\{h\}. \tag{1}$$

Also the linear imaging system is space-invariant, (isoplanatic), and can be described with transfer function. Linear systems obey the superposition integral property as follows:

$$\begin{aligned} &S\{ap(x_1, y_1) + bq(x_1, y_1)\} \\ &= aS\{p(x_1, y_1)\} + bS\{q(x_1, y_1)\}. \end{aligned} \tag{2}$$

Goodman's book also introduced the scalar theory.[13] It yields very accurate results if two conditions are fulfilled. First, the diffracting aperture must be large in comparison with the wavelength. Second, the diffracted field observed must not be too close to the aperture.

The segmented mirror system is still an imaging system. The gaps between the sub mirrors bring some diffraction errors. If the gap size is small[12] (i.e. $< 0.01D$), its influence is small. The imaging system's aberration can be modeled with a Zernike polynomial.[14] The aberrations of sub mirrors can be described with Zernike polynomials too. So, a segmented mirror system meets the condition of invariant linearity as well as the scalar diffraction theorem.

In the alignment process of a segmented mirror system, Zernike polynomial aberrations are usually used. The relationship among the Zernike polynomials of sub mirrors is a useful tool for the alignment. As mentioned earlier, a segmented mirror system obeys the condition of invariant linearity and the superposition integral. According to Fourier optics,[13,15] the advantage of linearity is to express the response of the system to an arbitrary input in terms of the response to certain 'elementary' functions into which the input has been decomposed. In order to find a simple and convenient means of decomposing the input, the input error should be decomposed for every sub mirror. The whole system's global response due to each sub mirror's error can be expressed with the integral response of every single sub mirror's error. Because the Zernike[16] polynomial has orthogonal property, every term of the Zernike polynomial is independent and irrelevant. Thus a summation algorithm should be used instead of an integration algorithm.

To verify this supposition, a simulation test is presented in the section below.

3. Simulation and designing example

A space-based telescope is always used in satellite systems. Normally, a three-mirror anastigmatic (TMA) configuration[17] is adopted in such systems. This configuration is developed from the well known Cassegrain system with an additional third mirror. The third mirror in TMA provides more variables for the optimization process. Also TMA can provide a large field of view, and good image quality. Such tele-

scopes are made up of three conic mirrors.

Two main segmentation schemes of primary are adopted by Lake *et al*[]18 and Fender.[19] In the first scheme, hexagonal shaped sub mirrors are arranged in the tile floor in a pattern. While in the second scheme, the central larger sub mirror is surrounded by smaller fan-like sub mirrors. In the European Wide Aperture telescope and the US James Webb telescope, a hexagonal shaped scheme is employed. Both these two schemes have some merits and demerits. For the consideration of fold and deployment, it is recommended to adopt the fan-like sub mirror scheme.

We design a three mirror space telescope. Figure 3 shows the layout of this telescope.

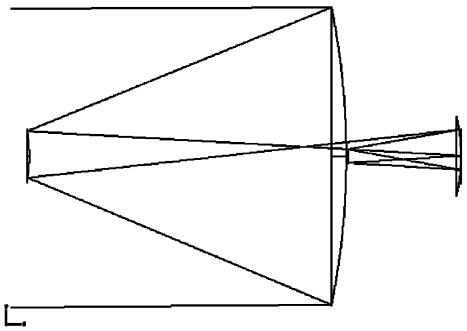


Fig.3. Layout of the space telescope.

The fan-like sub mirror scheme is adopted in this telescope's primary mirror. The primary mirror is divided into nine parts. The central part is an octagon shaped mirror with eight fan-like shaped mirrors surrounding it. The gap between sub mirrors is less than 4 mm.

The layout and the segmentation scheme are shown in Fig.4.

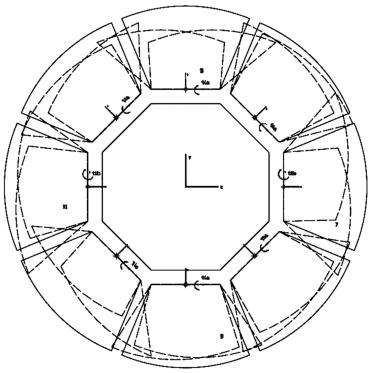


Fig.4. Segmentation scheme.

The primary mirror is first simulated by considering it as one single element and then considering it as segmented nine sub mirrors. Wave front peak valley (PV)-values of the system are presented in Table 1.

Table 1. Comparison of wave front data.

	single primary	segmented primary
PV value OPD	0.0204415200653	0.0215188635516
RMS OPD	0.0059724474096	0.006580291998
strehl ratio	0.9985416502126	0.9981534549215

The difference in the wave front optical path difference (OPD) between the single and segmented primary mirrors is very small according to the experimental data.

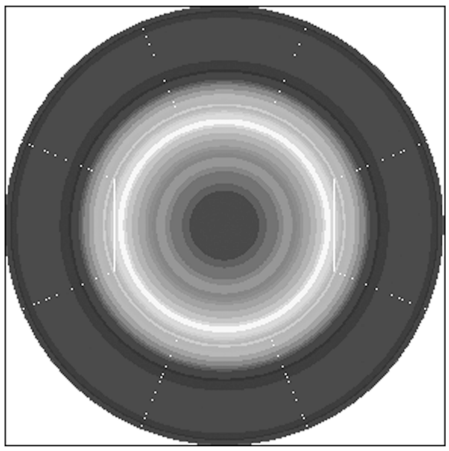


Fig.5. Wavefront map of the segmented system.

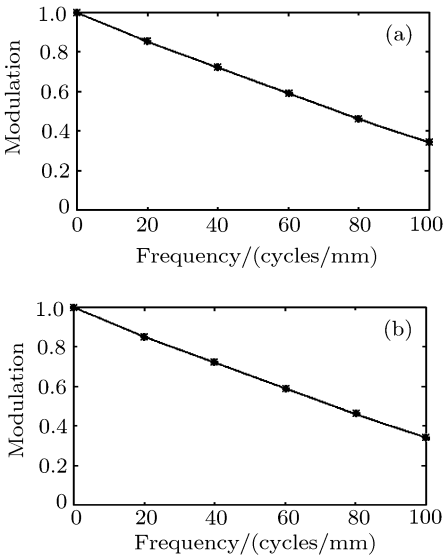


Fig.6. Comparison of MTF between un-segmented(a) and segmented(b) telescopes.

The wave front map and modulation transfer function (MTF) curves of the segmented system are shown in Figs.5 and 6.

The image quality of the single system is almost the same as that of the segmented system.

This result confirms the conclusions made by Lightsey.[11] It is concluded that the influence on the diffraction-limited MTF caused by the segmented sub mirrors is very little.

Every sub mirror can be folded about its edge close to the central mirror. Little displacements of sub mirrors cause position errors. In our design, Oslo optical software is used to simulate the proposed telescopic system.

Figure 7 shows four wavefront maps in four conditions.

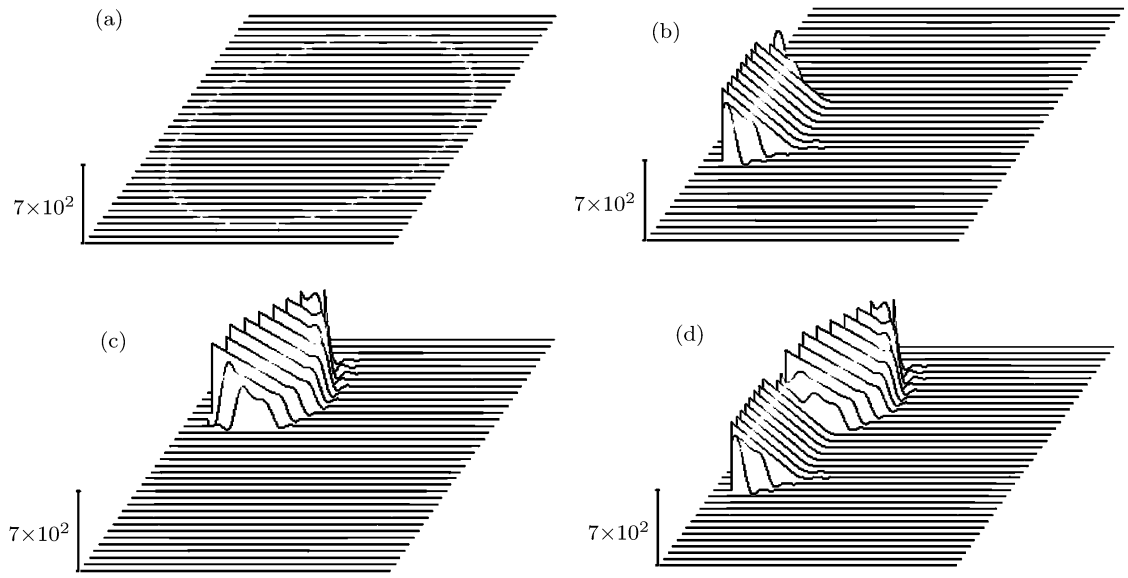


Fig.7. Wavefront map of four conditions. (a): no error; (b): one sub mirror has a 0.01 degree tilt error; (c): the 2nd one has a 0.01 degree tilt error; (d): both have 0.01 degree tilt errors

System's Zernike coefficients can be calculated in these four states. Table 2 shows four group Zernike coefficients.(only shows first 3 terms)

Table 2. First 3 terms of Zernike coefficients with two tilted sub mirrors.

no tilt	one tilt	other tilt	both tilt
$1.5135593620\times10^{-2}$	2.6039510674×10^{1}	4.0159042790×10^{1}	6.6256519199×10^{1}
$2.9336294000\times10^{-6}$	$2.9373485000\times10^{-6}$	9.0060837889×10^{1}	9.0196180449×10^{1}
$2.6427467000\times10^{-6}$	8.8416919439×10^{1}	9.0060843467×10^{1}	1.7860468595×10^{2}

Every tilt data value must be subtracted from no tilt data value in the above table to obtain the net value of tilt influence. Table 3 shows the net values (first 10 terms).

Table 3. Net Zernike coefficients of first 10 terms.

one(net)	other(net)	sum	both tilt (net)
2.6054646268×10^{1}	4.0174178383×10^{1}	6.6228824651×10^{1}	6.6271654792×10^{1}
$3.7191000000\times10^{-9}$	9.0060840823×10^{1}	9.0060840819×10^{1}	9.0196183382×10^{1}
8.8416916796×10^{1}	9.0060840824×10^{1}	1.7847775762×10^{2}	1.7860468331×10^{2}
3.5904051191×10^{1}	5.2611375230×10^{1}	8.8515426421×10^{1}	8.8525671045×10^{1}
1.0649657466×10^{2}	$-8.7400000000\times10^{-11}$	1.0649657466×10^{2}	1.0648217441×10^{2}
$5.8838000000\times10^{-9}$	1.5229976673×10^{2}	1.5229976674×10^{2}	1.5253543127×10^{2}
$0.0000000000\times10^{-13}$	3.9119780848×10^{1}	3.9119780848×10^{1}	3.9176408141×10^{1}
4.8159994297×10^{1}	3.9119782348×10^{1}	8.7279776645×10^{1}	8.7315219275×10^{1}
$2.3991841382\times10^{-1}$	3.3083328467×10^{0}	3.0684144329×10^{0}	2.9791790921×10^{0}
$1.8144000000\times10^{-9}$	1.0583144667×10^{2}	1.0583144667×10^{2}	1.0583119707×10^{2}

In the end, the difference between the sum value and the tilt (net) value in Table 3 table are listed in Table 4 (first 20 terms).

Table 4. Difference values.

$4.2830141000\times10^{-2}$	$1.3534256291\times10^{-1}$	$1.2692569111\times10^{-1}$	$1.0244624520\times10^{-2}$
$1.4400250040\times10^{-2}$	$2.3566453265\times10^{-1}$	$5.6627292980\times10^{-2}$	$3.5442630490\times10^{-2}$
$8.9235340870\times10^{-2}$	$2.4959730000\times10^{-4}$	$4.6402074050\times10^{-2}$	$4.1608208760\times10^{-2}$
$3.7628960020\times10^{-2}$	$5.1129038120\times10^{-2}$	$2.2001647310\times10^{-2}$	$1.1767681561\times10^{-1}$
$5.6726541690\times10^{-2}$	$7.9275131310\times10^{-2}$	$2.4622254817\times10^{-1}$	$1.1019697575\times10^{-1}$

The calculated data reveals that the sub mirror's Zernike coefficients of the segmented system obey the superposition integral property. It is evident that every single sub mirror's Zernike coefficients are independent. Also, the influence of the tilts of 2 or more sub mirrors can be expressed as the sum of tilts of each sub mirror of the system.

4. Conclusion

According to the simulation test, a conclusion can be drawn as follows:

The influence of the gap between the segmented sub mirrors is very little under the condition that the gap size is less than $0.001D$, where D is the diameter of the apertue. The Zernike coefficients of each sub mirror are independent. The Zernike coeficients of two tilted sub mirrors can be expressed as the sum of the coefficients of each independent tilted sub mirror.

This conclusion is useful and can instruct the alignment of the segmented mirror system. According to this conclusion, the sub mirrors can be aligned one by one. The influence between the two sub mirrors can be ignored. This conclusion will simplify the aberration relationship among sub mirrors.

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