Vol 18 No 7, July 2009
1674-1056/2009/18(07)/2667-07

Chinese Physics B

© 2009 Chin. Phys. Soc.
and IOP Publishing Ltd

Experiment and application of parameter-induced stochastic resonance in an over-damped random linear system

Jiang Shi-Qi(蒋世奇)[a)b], Hou Min-Jie(侯敏杰)[a)], Jia Chun-Hua(贾春华)[b)], He Ji-Rong(何吉荣)[a)], and Gu Tian-Xiang(古天祥)[b)]

[a)] *China Gas Turbine Establishment, Chengdu* 610500, *China*

[b)] *School of Automation Engineering, University of Electronic Science and Technology of China, Chengdu* 610054, *China*

(Received 19 November 2008; revised manuscript received 29 December 2008)

This paper investigates the parameter-induced stochastic resonance using experimental methods in an over-damped random linear system with asymmetric dichotomous noise. Non-monotonic dependence of signal-to-noise ratio on the system parameter is observed. Several potential applications of parameter-induced stochastic resonance are given in circuits.

Keywords: parameter-induced stochastic resonance, asymmetric dichotomous noise, over-damped linear system, signal-to-noise ratio

PACC: 0540, 0545

1. Introduction

Stochastic resonance (SR) is a cooperative effect between the internal mechanism and the external periodic forcing for stochastic dynamical systems.[1] The conventional SR, the SR in the broad sense, and the bona fide SR respectively denote non-monotonic dependence of the output characteristics, such as amplitude, moment, autocorrelation function, power spectrum, signal-to-noise ratio (SNR) or dynamic parameters, on some noise characteristics and the external field frequency.[2] The SNR is one of the focuses of attention in various physics systems.

Earlier, most authors who researched into SR concentrated on non-linear systems with periodic and random forces. There are many papers in which SR is studied by experimental methods in a non-linear system. In recent years, SR in a linear system has gradually become a research hotspot of SR.[3−12] But few articles involve SR using experimental methods in a linear system.

The dichotomous noise, i.e., random telegraph noise, is a realization of a two-state Poisson process.[13−17] The dichotomous noise includes symmetric noise and asymmetric noise. When the asymmetry of the asymmetric dichotomous noise is zero, the asymmetric dichotomous noise is degraded to be symmetric. The dichotomous noise widely exists in the various materials and devices, for example, metals,[18,19] tunnel junctions,[20−23] bipolar transistors,[19,24] junction field effect transistors,[25] metal–oxide–semiconductor transistors,[26] superconducting thin films,[27] nano-devices,[28,29] etc. It is exhibited in the form of resistance,[18,22,23,26,30] conductance,[21,31] voltage[20,29] or current.[25,28,31] The observed dichotomous noise has reached more than 0.3 V in 45-nm flash memory.[32]

Xu *et al* proposed the concept of parameter-induced stochastic resonance (PSR),[33] which characterized that SR is realized by tuning system parameters with fixed noise.[34,35] When the SR of SNR was investigated in an over-damped motion of a particle in a mono-stable potential driven simultaneously by an additive noise as well as by a multiplicative noise modulated by a bias periodic signal, we found that SNR non-monotonically depend on undisturbed system parameters.[10] In a practical first-order linear physical system, varying system parameters can be easier than varying noise parameters. In this case, the advantage of PSR will stand out.

It is verified experimentally in this paper that the PSR phenomena exist in the simple first-order random model with asymmetric dichotomous noise. Then, we give methods for using PSR to obtain maximum SNR in a few random circuit models. The result can be applied to circuit theory.

2. Realization of asymmetric dichotomous noise

Because of the need to use asymmetric dichotomous noise in investigations of experiments and applications, firstly, we study the realization method of this noise. The goal is, according to

$$\langle \xi(t) \rangle = 0, \ \langle \xi(t)\, \xi(t') \rangle = \sigma \exp\left(-s\left|t-t'\right|\right), \ \Delta, \quad (1)$$

to find a method in order to generate this noise $\xi(t) \in \{\Delta_+, -\Delta_-\}\,(\Delta_+ > 0, \Delta_- > 0)$, where $\sigma = \Delta_+\Delta_-$ is intensity, s is correlation rate, $\tau = 1/s$ is correlation time, and $\Delta = \Delta_+ - \Delta_-$ is asymmetry of noise.

We assume that the transition rate is λ from Δ_+ to Δ_-, and the inverse rate is μ. Then, there are $s = \lambda + \mu$ and master equations

$$\begin{aligned} \partial_t P(\Delta_+, t|x, t_0) &= -\lambda P(\Delta_+, t|x, t_0) \\ &\quad + \mu P(\Delta_-, t|x, t_0), \\ \partial_t P(\Delta_-, t|x, t_0) &= \lambda P(\Delta_+, t|x, t_0) \\ &\quad - \mu P(\Delta_-, t|x, t_0). \end{aligned} \quad (2)$$

Using that the mean is zero from Eq.(1), we obtain

$$\begin{aligned} \sigma &= \Delta_+\Delta_-, \\ \Delta &= \Delta_+ - \Delta_-, \\ s &= \lambda + \mu, \\ \Delta_+\mu - \Delta_-\lambda &= 0. \end{aligned} \quad (3)$$

Solving these equations (3), we can describe Δ_+, Δ_-, λ and μ by σ, Δ and s, i.e.,

$$\begin{aligned} \Delta_+ &= \frac{1}{2}\left(\Delta + \sqrt{\Delta^2 + 4\sigma}\right), \\ \Delta_- &= \frac{1}{2}\left(-\Delta + \sqrt{\Delta^2 + 4\sigma}\right), \\ \lambda &= \frac{1}{2}\left(1 + \frac{\Delta}{\sqrt{\Delta^2 + 4\sigma}}\right) s, \\ \mu &= \frac{1}{2}\left(1 - \frac{\Delta}{\sqrt{\Delta^2 + 4\sigma}}\right) s. \end{aligned} \quad (4)$$

On the basis of different initial conditions, we solve the main equations Eq.(2), and obtain

$$\begin{aligned} P(\Delta_+, t|\Delta_+, 0) &= \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu}\exp\left[-(\lambda+\mu)\,t\right], \\ P(\Delta_+, t|\Delta_-, 0) &= \frac{\mu}{\lambda+\mu} - \frac{\mu}{\lambda+\mu}\exp\left[-(\lambda+\mu)\,t\right], \\ P(\Delta_-, t|\Delta_+, 0) &= \frac{\lambda}{\lambda+\mu} - \frac{\lambda}{\lambda+\mu}\exp\left[-(\lambda+\mu)\,t\right], \\ P(\Delta_-, t|\Delta_-, 0) &= \frac{\lambda}{\lambda+\mu} + \frac{\mu}{\lambda+\mu}\exp\left[-(\lambda+\mu)\,t\right]. \end{aligned} \quad (5)$$

Substituting Eq.(4) into Eq.(5), one obtains the following equations:

$$\begin{aligned} P(\Delta_+, t|\Delta_+, 0) &= \frac{1}{2}\left(1 - \frac{\Delta}{\sqrt{\Delta^2 + 4\sigma}}\right) + \frac{1}{2}\left(1 + \frac{\Delta}{\sqrt{\Delta^2 + 4\sigma}}\right)\exp\left[-st\right], \\ P(\Delta_-, t|\Delta_+, 0) &= \frac{1}{2}\left(1 + \frac{\Delta}{\sqrt{\Delta^2 + 4\sigma}}\right) - \frac{1}{2}\left(1 + \frac{\Delta}{\sqrt{\Delta^2 + 4\sigma}}\right)\exp\left[-st\right], \\ P(\Delta_+, t|\Delta_-, 0) &= \frac{1}{2}\left(1 - \frac{\Delta}{\sqrt{\Delta^2 + 4\sigma}}\right) - \frac{1}{2}\left(1 - \frac{\Delta}{\sqrt{\Delta^2 + 4\sigma}}\right)\exp\left[-st\right], \\ P(\Delta_-, t|\Delta_-, 0) &= \frac{1}{2}\left(1 + \frac{\Delta}{\sqrt{\Delta^2 + 4\sigma}}\right) + \frac{1}{2}\left(1 - \frac{\Delta}{\sqrt{\Delta^2 + 4\sigma}}\right)\exp\left[-st\right]. \end{aligned} \quad (6)$$

According to Eq.(6), We can generate a time series[36] of asymmetric dichotomous noise using the inverse transform method.[37] Then, the series is passed through digital-to-analog conversion. The noise source is obtained which satisfies asymmetric dichotomous characteristics.

3. Model

In the following, one can experiment on PSR by employing the noise source.

Experimental model is

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\left[a + \xi(t)\right]x + A\cos(\omega t), \quad (7)$$

where, a is the damping coefficient, $A\cos(\omega t)$ is the periodic signal, A and ω are the amplitude and frequency of the periodic signal respectively, $\xi(t)$ is an asymmetric dichotomous noise defined by Eq.(1).

There are many authors who have investigated SR phenomena theoretically in Eq.(7).[3−7] These phenomena include the conventional SR, the SR in a broad sense and the bona fide SR. But, we have seen few papers investigate those phenomena by experiment with regard to this model, and have never seen the case where $\xi(t)$ is asymmetric dichotomous noise in Eq.(7). When $\xi(t)$ degrades asymmetric dichotomous noise to symmetric, the Eq.(7) is in Ref.[4]. Moreover, the PSR in this model is not seen to be reported.

4. PSR theoretically

From Eq.(7) with Eq.(1), the value R of SNR is elicited by the method in the Ref.[10]

$$R = \frac{\int_0^\infty S_2(\omega)\,\delta(\Omega-\omega)\mathrm{d}\Omega}{S_1(\Omega=\omega,\omega)}, \tag{8}$$

where,

$$S_1(\Omega,\omega) = \frac{1}{N_1-N_2}\left\{\langle x^2\rangle_{\mathrm{st}}\left(\frac{2N_1b_2}{b_2^2+\Omega^2}-\frac{2N_2b_1}{b_1^2+\Omega^2}\right)+A\left(\frac{G_1b_1}{b_1^2+\Omega^2}-\frac{G_2b_2}{b_2^2+\Omega^2}\right)\right\},$$

$$S_2(\omega) = \frac{\pi A(G_2-G_1)}{N_1-N_2}, \quad \langle x^2\rangle_{\mathrm{st}} = \frac{A[(a+\Delta+\lambda/2)f_1-f_3]}{2a^2+(\lambda+2\Delta)a-2\sigma},$$

$$G_1 = f_1J_2b_1+f_2J_2\omega, \quad G_2 = f_1J_1b_2+f_2J_1\omega, \quad J_1 = \frac{N_1}{\omega^2+b_2^2},$$

$$J_2 = \frac{N_2}{\omega^2+b_1^2}, \quad b_{1,2} = a+N_{1,2}, \quad N_1 = \frac{\lambda+\Delta}{2}+\sqrt{\frac{(\lambda+\Delta)^2}{4}+\sigma}, \quad N_2 = \frac{\lambda+\Delta}{2}-\sqrt{\frac{(\lambda+\Delta)^2}{4}+\sigma},$$

$$f_1 = \frac{A\left[a^3+2(\Delta+\lambda)a^2+\left((\Delta+\lambda)^2+\omega^2-\sigma\right)a-\sigma(\Delta+\lambda)\right]}{(\omega^2+b_1^2)(\omega^2+b_2^2)},$$

$$f_2 = \frac{A\left[a^2+2(\Delta+\lambda)a+\sigma+\omega^2+(\Delta+\lambda)^2\right]\omega}{(\omega^2+b_1^2)(\omega^2+b_2^2)}, \quad f_3 = \frac{-A\sigma\left[a(a+\lambda+\Delta)-\sigma-\omega^2\right]}{(\omega^2+b_1^2)(\omega^2+b_2^2)}. \tag{9}$$

In terms of Eq.(8), the influences of the system parameter on the SNR can be discussed. The curves of R versus the system parameter a for different intensity σ of asymmetric dichotomous noise are shown in Fig.1. It is shown that the asymmetry Δ effects the curves of R vs a in Fig.2. These figures illustrate that SNR non-monotonically depends on the system parameter a.

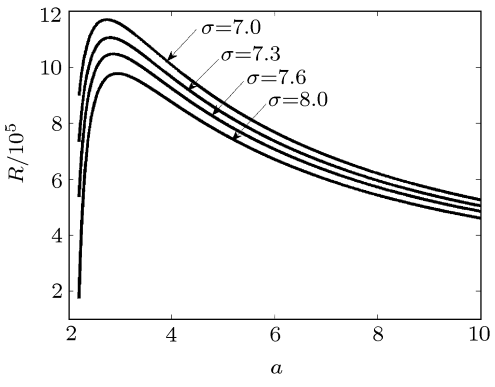


Fig.1. R as a function of the system parameter a with the intensity σ ($\Omega = 2\pi\times 11.25$, $A=1$, $\lambda=1$, $\Delta=1$).

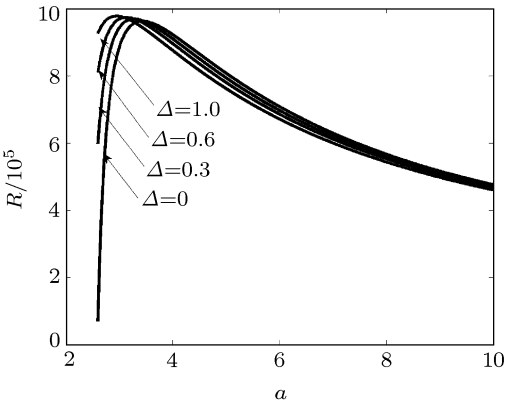


Fig.2. R as a function of the system parameter a with the asymmetry Δ($\Omega = 2\pi\times 11.25$, $A=1$, $\lambda=1$, $\sigma=8$).

5. Experimental Principle

From Eq.(7), there is an equation

$$x(t) = -\int ax(t)\,\mathrm{d}t - \int \xi(t)x(t)\,\mathrm{d}t + \int A\cos(\omega t)\mathrm{d}t. \tag{10}$$

In accordance with Eq.(10), we design an experimental circuit shown in Fig.3. The function of each part in the circuit is as follows: CON1, CON2, CON3 and CON4 are connectors of the input signal $A\cos(\omega t)$, the output signal $x(t)$, the circuit power and noise $\xi(t)$, respectively; A_1 and its peripheral circuit implements a function of reverse integral; A_2 and its peripheral circuit carries out amplifying reverse voltage, but its gain coefficient is 1; U_1 is a multiplier which multiplies $x(t)$ by $\xi(t)$; K_3 is a rheostat which is used to adjust the damping coefficient; r_{13} presents the resistance between pin 1 and pin 3 of K_3, r_{12} is the resistance between 1 and 2, and K is a ratio of r_{13} to r_{12}, that is $K = r_{13}/r_{12}$.

From Fig.3 we have

$$U_\mathrm{o} = -\frac{1}{93\times10^3\times C_1}\times K\int U_\mathrm{o}\mathrm{d}t - \frac{1}{93\times10^3\times C_1}\int \xi(t)\,U_\mathrm{o}\mathrm{d}t + \frac{1}{93\times10^3\times C_1}\int A\cos(\omega t)\,\mathrm{d}t. \tag{11}$$

The relation is consistent with Eq.(7).

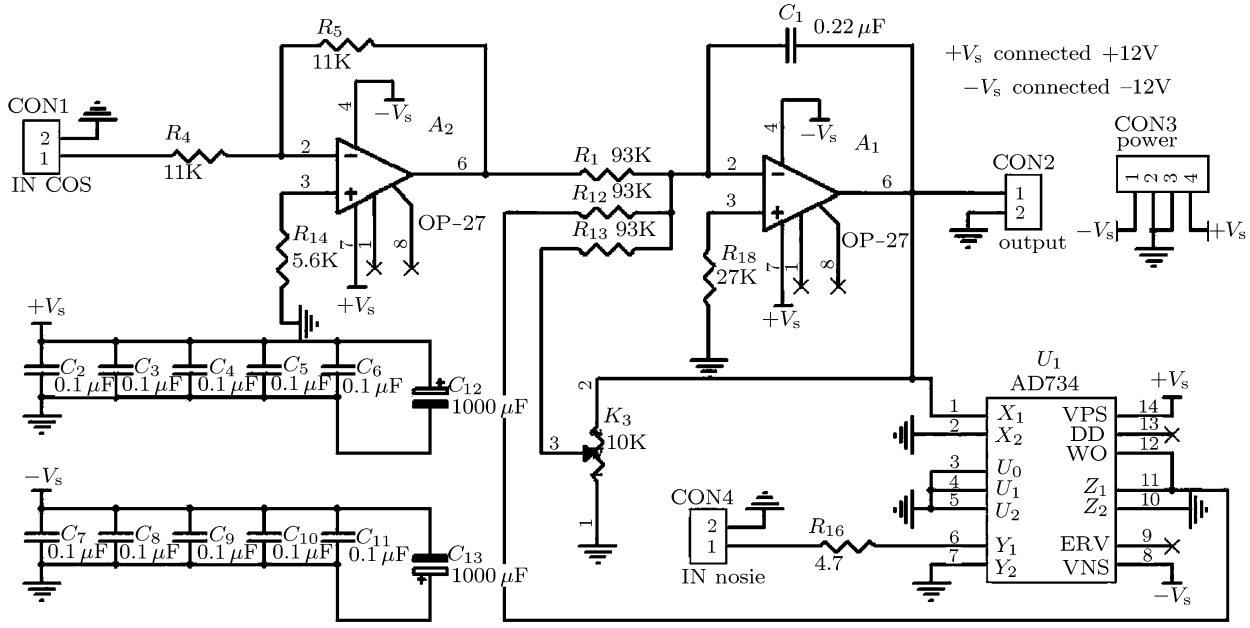


Fig.3. Parameter-induced stochastic resonance experimental circuit in an over-damped linear system.

6. Tuning system parameter to realize stochastic resonance

Comparing Eq.(7) with Eq.(11), we find that the system parameter a in Eq.(7) is directly proportional to the coefficient K in Eq.(11). One knows that K is directly proportional to r_{13} since r_{12} is a constant and the coefficient K is defined by $K = r_{13}/r_{12}$. So the system parameter a is directly proportional to r_{13}, too. And varying r_{13} is equal to tuning the system parameter a. Studying the relationship between SNR and a is equal to studying the dependence of SNR on r_{13}. In the latter, we will verify PSR by adjusting the resistance r_{13} of rheostat K_3 when the capacitor $C_1 = 0.22$ μF, the input sine signal amplitude 1.53 V (root mean square or RMS value), frequency 11.25 Hz, and noise $\sigma = 8$, $\Delta = 1$, $\tau = 1$. Figures 4 and 5 are the waveform and the spectrum of output at $r_{13} = 6.056$ kΩ, respectively. The figures are derived from the data acquisition analyser (from OROS Inc.) acquired and analysed. We employ a definition[17] that SNR is a ratio of the spectrum

at signal frequency to the mean of spectrums close to the signal frequency, i.e., the ratio of the spectrum at 11.25 Hz to the average spectrum at 10.1563 Hz and at 12.3438 Hz. Compared with the SNR in Figs.1 and 2, this SNR from the experiment is much smaller. It is the reason why the SNR in Figs.1 and 2 come from Eq.(8) which originates in response to the delta function, however, the analysis of bandwidth based on fast Fourier transform theory does not go to zero when we calculate the SNR from measurement of the spectrum.

For each confirmed r_{13}, we experiment 20 times, and average all the SNR, as shown in Table 1. We plot the curve about the dependence of SNR on r_{13} in Fig.6. It is observed that the SNR initially increases as the system parameter r_{13} increases, and the SNR reaches maximum in the case of $r_{13} = 6.056$ kΩ. Then, if the system parameter r_{13} increases again, the SNR decreases. It is indicated that the optimal SNR can be obtained by tuning the system parameter in an over-damped random linear system, that is, parameter-induced stochastic resonance exists in the linear system. The same to the non-monotonic phenomena in Figs.1 and 2 is observed.

Table 1. The relation between system parameter r_{13} and SNR when $\sigma = 8$, $\Delta = 1$, $\tau = 1$, $C_1 = 0.22$ μF.

system parameter r_{13}/kΩ	SNR	system parameter r_{13}/kΩ	SNR
2.664	285.80	6.056	535.77
3.429	306.13	7.424	338.63
4.254	372.56	9.011	316.23
5.720	430.25		

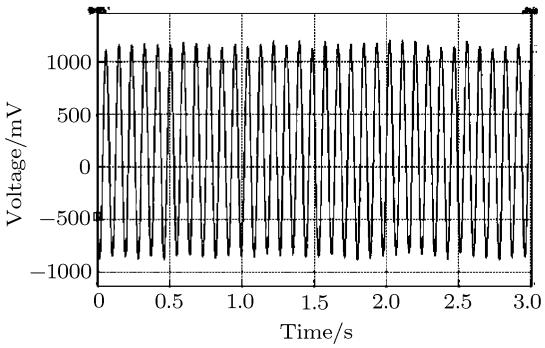


Fig.4. Waveform of output at $r_{13} = 6.056$ kΩ.

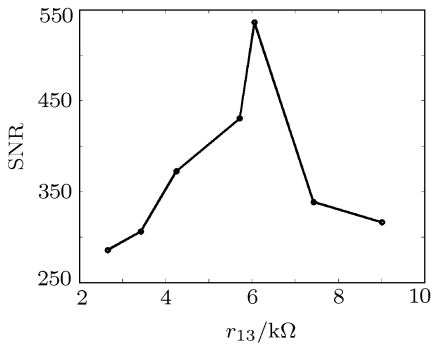


Fig.6. The relation between system parameter r_{13} and SNR in case of $\sigma = 8$, $\Delta = 1$, $\tau = 1$, $C_1 = 0.22$ μF.

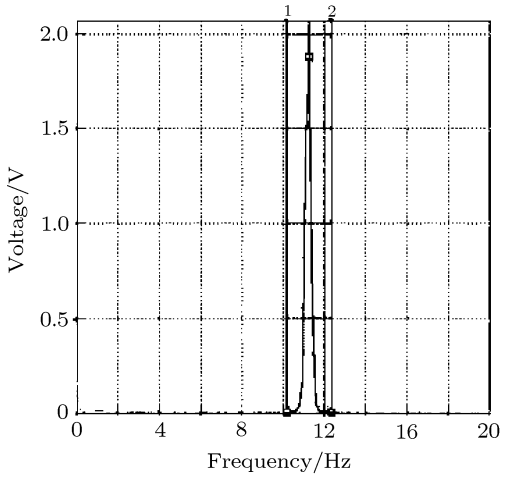


Fig.5. Spectrum of output at $r_{13} = 6.056$ kΩ.

7. Applications of PSR in circuits

Equation (7) has a strong background of application in first-order linear circuit. In the following, several examples are given, which shows that PSR appies to first-order linear circuits.

Example 1. Figure 7 is a RC differential circuit. Its relation between input and output is

$$\frac{\mathrm{d}V_\mathrm{o}}{\mathrm{d}t} = -\frac{1}{C}G'V_\mathrm{o} + \frac{\mathrm{d}V_\mathrm{i}}{\mathrm{d}t}, \tag{12}$$

where $G' = 1/R$ is a conductance. When $\mathrm{d}V_\mathrm{i}/\mathrm{d}t = A\cos(\omega t)$ and noise $\xi(t)$ occur in G', i.e., $G' = G + \xi(t)$, Eq.(12) becomes

$$\frac{\mathrm{d}V_\mathrm{o}}{\mathrm{d}t} = -\frac{1}{C}\left[G + \xi(t)\right]V_\mathrm{o} + A\cos(\omega t). \tag{13}$$

It is seen that Eq.(13) is similar to Eq.(7). The method of tuning parameter is seen in Fig.8, that is, a resistance r shunts with R. Here, the output voltage $V_{\rm o}$ has a formula

$$\frac{{\rm d}V_{\rm o}}{{\rm d}t} = -\frac{1}{C}\left[1/r + G + \xi\left(t\right)\right]V_{\rm o} + A\cos\left(\omega t\right). \quad (14)$$

The PSR denotes that tuning resistance r can obtain optimal SNR of output voltage $V_{\rm o}$ when PSR exists.

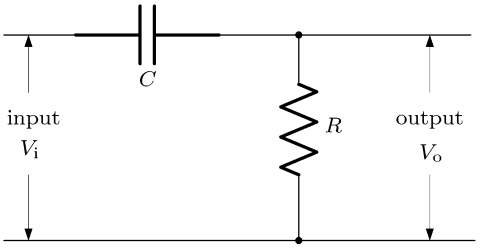


Fig.7. Output voltage in a differential circuit with conductance noise.

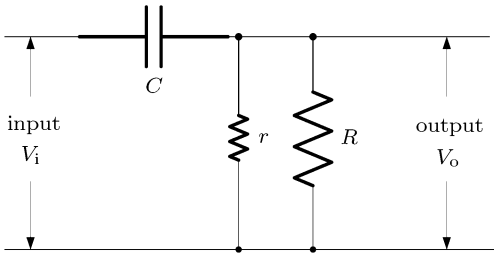


Fig.8. Application of PSR to improve SNR of output voltage in differential circuit with conductance noise.

Example 2. Figure 9 is a RL circuit. When the R exists resistance noise, i.e., $R = R_0 + \xi\left(t\right)$, and $V_{\rm i} = A\cos\left(\omega t\right)$, the current i is satisfied by

$$\frac{{\rm d}i}{{\rm d}t} = -\frac{1}{L}\left[R_0 + \xi\left(t\right)\right]i + A\cos\left(\omega t\right). \quad (15)$$

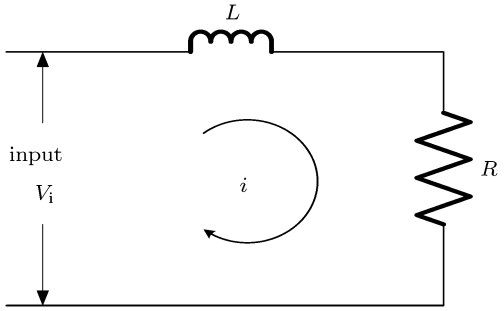


Fig.9. A current in RL circuit with resistance noise.

Obviously, Eq.(15) is similar to Eq.(7), too. So, we can obtain maximum SNR of current i by PSR, that is, r is in series with R in the circuit. Then, the SNR of output current achieves a peak by adjusting resistance r, as shown in Fig.10.

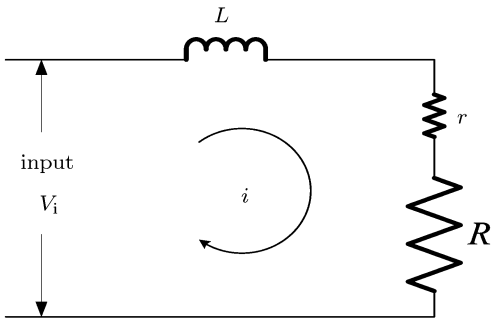


Fig.10. Application of PSR to improve SNR of current in RL circuit with resistance noise.

Example 3. Figure 11 is a RL differential circuit. The R is the same to example 2, and ${\rm d}V_i/{\rm d}t = A\cos\left(\omega t\right)$. So, $V_{\rm L}$ abides by

$$\frac{{\rm d}V_{\rm L}}{{\rm d}t} = -\frac{1}{L}\left[R_0 + \xi\left(t\right)\right]V_{\rm L} + A\cos\left(\omega t\right). \quad (16)$$

The form of Eq.(16) is similar to Eq.(7). So, in the same way, we apply PSR to get an optimal SNR of $V_{\rm L}$, and the method is still seriate with a resistance r in the sub-circuit R. Varying r, seeing Fig.12, we can get the maximum SNR of output voltage $V_{\rm L}$.

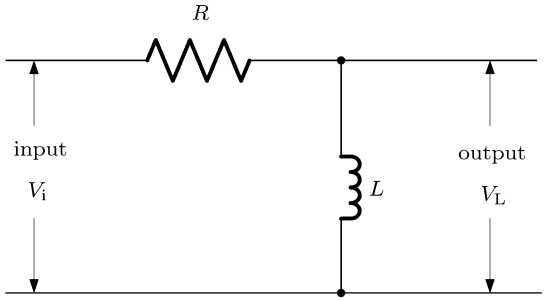


Fig.11. A voltage of inductance in RL circuit with resistance noise.

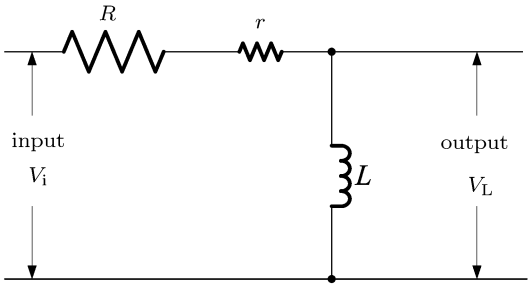


Fig.12. Application of PSR to improve SNR of voltage of inductance in RL circuit with resistance noise.

8. Conclusion

In this paper, using experimental methods, the PSR is investigated. It is observed that the maximum SNR can be obtained by tuning the system parameters in an over-damped linear system with a multiplicative asymmetric dichotomous noise. And then several application examples of PSR in circuit models are given.

Considering noise effects of different components or disturbed parameters, one can also get many models which are similar to Eq.(7) described. In the above examples, obviously, tuning the system parameter is easier than varying the noise parameters in some cases. So, PSR has probably more application values than the conventional SR and SR resonance in a broad sense.

References

[1] Benzi R, Sutera A and Vulpiani A 1981 *J. Phys.* A **14** L453

[2] Berdichevsky V and Gitterman M 1999 *Phys. Rev.* E **60** 1494

[3] Fulinski A 1995 *Phys. Rev.* E **52** 4523

[4] Berdichevsky V and Gitterman M 1996 *Europhys. Lett.* **36** 161

[5] Barzykin A V and Seki K 1997 *Europhys. Lett.* **40** 117

[6] Barzykin A V, Seki K and Shibata F 1998 *Phys. Rev.* E **57** 6555

[7] Bezák V 1998 *Czech. J. Phys.* **48** 529

[8] Fuliński A and Góra P F 2000 *J. Stat. Phys.* **101** 483

[9] Jin Y, Xu W, Xu M and Fang T 2005 *J. Phys.* A **38** 3733

[10] Jiang S, Guo F, Zhou Y and Gu T 2007 *Physica* A **375** 483

[11] Gitterman M 2005 *Physica* A **352** 309

[12] Cao L and Wu D J 2007 *Physica* A **376** 191

[13] Horsthemke W and Lefever R 1984 *Noise-Induced Transition: Theory and Applications in Physics, Chemistry, and Biology* (Berlin: Springer-Verlag)

[14] Broeck C V D 1983 *J. Stat. Phys.* **31** 467

[15] Gardiner C W 1983 *Handbook of Stochastic Methods for Physics, Chemistry and the Natural Sciences* (Berlin: Springer-Verlag)

[16] Broeck C V D and Hänggi P 1984 *Phys. Rev.* A **30** 2730

[17] Hu G 1994 *Stochastic Forces and Nonlinear System* (Shanghai: Shanghai Scientific and Technological Education Publishing House) (in Chinese)

[18] Ralls K S and Buhrman R A 1988 *Phys. Rev. Lett.* **60** 2434

[19] Grose J E, Pasupathy A N, Ralph D C, Ulgut B and Abruña H D 2005 *Phys. Rev.* B **71** 035306

[20] Ng S H, Surya C, Brown E R and Maki P A 1993 *Appl. Phys. Lett.* **62** 2262

[21] Rogers C T and Buhrman R A 1984 *Phys. Rev. Lett.* **53** 1272

[22] Rogers C T and Buhrman R A 1985 *Phys. Rev. Lett.* **55** 859

[23] Scola J, Polovy H, Fermon C, Pannetier-Lecoeur M, Feng G, Fahy K and Coey J M D 2007 *Appl. Phys. Lett.* **90** 252501

[24] Pogany D, Chroboczek J A and Ghibaudo G 2001 *J. Appl. Phys.* **89** 4049

[25] Kandiah K 1994 *IEEE Trans. on Electron Devices* **41** 2006

[26] Howard R E, Skocpol W J, Jackel L D, Mankiewich P M, Fetter L A, Tennant D M, Epworth R and Ralls K S 1985 *IEEE Trans. on Electron Devices* **32** 1669

[27] Yuzhelevski Y, Yuzhelevski M and Jung G 2000 *Rev. Sci. Instr.* **71** 1681

[28] Jhang S H, Lee S W, Lee D S, Yu H Y, Dettlaff U, Campbell E E B, Roth S and Park Y W 2006 *Cur. Appl. Phys.* **6** 987

[29] Xi H, Loven J, Netzer R, Guzman J I, Franzen S and Mao S 2006 *J. Phys.* D **39** 2024

[30] Ralls K S, Skocpol W J, Jackel L D, Howard R E, Fetter L A, Epworth R W and Tennant D M 1984 *Phys. Rev. Lett.* **52** 228

[31] He H X, Li X L, Tao N J, Nagahara L A, Amlani I and Tsui R 2003 *Phys. Rev.* B **68** 045302

[32] Kurata H, Otsuga K, Kotabe A, Kajiyama S, Osabe T, Sasago Y, Narumi S, Tokami K, Kamohara S and Tsuchiya O 2007 *IEEE Journal of Solid-State Circuits* **42** 1362

[33] Xu B, Li J and Zheng J 2003 *J. Phys.* A **36** 11969

[34] Xu B, Li J and Zheng J 2004 *Physica* A **343** 156

[35] Li J and Xu B 2006 *Physica* A **361** 11

[36] Barik D, Ghosh P K and Ray D S 2006 *J. Stat. Mech.* **3** P03010

[37] Law A M and Kelton W D 2000 *Simulation Modeling and Analysis* (Third Edition) (New York: McGraw-Hill)