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Application of quantum algorithms to direct measurement of concurrence of a two-qubit pure state*

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This paper proposes a method to measure directly the concurrence of an arbitrary two-qubit pure state based on a generalized Grover quantum iteration algorithm and a phase estimation algorithm. The concurrence can be calculated by applying quantum algorithms to two available copies of the bipartite system, and a final measurement on the auxiliary working qubits gives a better estimation of the concurrence. This method opens new prospects of entanglement measure by the application of quantum algorithms. The implementation of the protocol would be an important step toward quantum information processing and more complex entanglement measure of the finite-dimensional quantum system with an arbitrary number of qubits.

Keywords: concurrence, quantum algorithm, entanglement measure
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1. Introduction

Quantum entanglement, which is one of the main ingredients of nonintuitive quantum phenomena, has been identified as a nonlocal resource for quantum information processing[1] and is widely used in quantum communication.[2−11] Many protocols have been proposed to prepare entangled states using different physical systems, such as the ion trap system,[12] the cavity quantum electrodynamics (QED) system,[13,14] the nuclear magnetic resonance system,[15] quantum dot,[16] and so on. In practice, however, the coupling between physical systems and the external environment is unavoidable, which results in various decoherence processes. Thus the entanglement between qubits will be reduced consequently and we are often able to generate only a non-maximally entangled state due to the noise and imperfections in real experiments and distribution via a lossy quantum channel. Therefore, how to establish efficient methods to detect entanglement, not only to examine whether a given state (for both pure states and mixed states) is entangled or not, but also to quantify the degree of entanglement, is a very crucial task for quantum computing and quantum information processing. The quantification of entanglement of quantum states is of the utmost importance. Until now the quantification of the degree of entanglement for an arbitrary number of qubits is still an open problem in quantum information.

In recent years, several methods have been proposed to detect and measure entanglement.[17−30] One simple method to estimate the entanglement degree is quantum state tomography,[21−26] which is performed on one of the entangled pairs. In quantum state tomography, to evaluate the entanglement in the experiment, one must reconstruct the density matrix by a complete set of observables, which results in a rapidly growing experimental overhead as the system size increases, either in subsystem dimensions or in subsystem numbers. Therefore, quantum state tomography is an unscalable method. Entanglement witness,[27−30] which provides much more direct experimental insights to the entanglement property of a quantum state, is another important method for entanglement detection. However, this method depends on the detected states, which implies that a priori knowledge of the state is required. Thus entanglement witness cannot be freely applied for arbitrary unknown quantum states. Very recently, Walborn *et al*[31] reported a linear optics experiment for directly measuring the concurrence of a two-photon pure entangled state. The measurement set-up in the experiment in-

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cluded two copies of a quantum state that is prepared in the polarization and a momentum degree of freedom of two photons, and the concurrence was measured with a single, local measurement on just one of the photons. Romero *et al*[32] proposed a protocol to measure the concurrence of an arbitrary atomic two-qubit pure state in three-dimensional (3D) microwave cavities and trapped ion systems. The protocol was based on the fact that the concurrence can be encoded in the probability of finding all qubits in a certain state. Cai *et al*[33] proposed an entanglement measurement protocol based on two-particle interference, which could be easily realized with polarized entangled photons. Lee *et al*[34] also proposed an experimental protocol that allowed direct measurement of the concurrence of a two-qubit cavity system using cavity QED. In their protocol the atoms were used as flying qubits and the concurrence can be determined through the identity of the two-particle visibility of the probability of the detected atomic state.

Different from the methods for measuring the concurrence mentioned above, Biham *et al*[35] defined a new entanglement measure for pure quantum states of n qubits based on the Grover quantum search algorithm,[36] namely, the Groverian entanglement measure. The Groverian measure of a state $|\psi\rangle$ is given by $G(\psi) = \sqrt{1 - P_{\max}}$. Here $P_{\max}$ is the maximal success probability of the algorithm for finding a marked element. Biham *et al*'s work provides an operational interpretation for a multi-party entanglement measure, explicitly connecting that measure to the success probability of a quantum algorithm. Subsequently, Shapira *et al*[37] generalized Biham *et al*'s method to the case of mixed states by a purification procedure into a pure state. Therefore, the Groverian measure, which is expressed in terms of the maximal success probability of the algorithm, provides an operational measure of entanglement for both pure and mixed quantum states of multiple qubits. In this paper, we propose a method to measure the concurrence of an arbitrary two-qubit pure state directly by using a generalized Grover quantum iteration algorithm and a phase estimation algorithm. We explicitly construct the generalized Grover iteration operator and show that the concurrence can be calculated by applying quantum algorithms to two available copies of the bipartite system. A final measurement on the auxiliary working qubits will give a good approximate estimation of the concurrence. The implementation of the protocol would be an important step toward quantum information processing and more complex entanglement measurement of the finite-dimensional quantum system with an arbitrary number of qubits, serving to show the power of quantum computation.

The structure of this paper is organized as follows. In Section 2, we briefly review the Grover quantum search algorithm. In Section 3, we introduce the generalized Grover iteration operator and discuss some of its useful properties. In Section 4, we show how to determine the concurrence of an arbitrary two-qubit pure state by using quantum algorithms. The discussion and conclusion are given in Section 5.

2. Grover quantum search algorithm

The goal of the Grover quantum search is to find one item in an unsorted database of size N ($N = 2^n$). Further, the search problem can be conveniently represented by a Boolean function f, which takes as inputting an integer x and satisfies $f : D \to \{0, 1\}$, where D is a set of N domain values x (or states). If $f(x) = 1$, then x is a solution to the search problem. While for $f(x) = 0$, x is not a solution to the search problem. The most efficient classical search, which is to examine items one by one, needs on average $N/2$ queries, while the Grover quantum search needs only $O(\sqrt{N})$ queries to obtain the marked item, providing a quadratic speed up over classical search algorithms. The process of Grover quantum search can be briefly described as below.

I. Reset all the qubits in the register to 0 and then perform the Hadamard transform (H transform) on each of the qubits so that the system is initially in the equal superposition state $|\phi\rangle$.

II. Repeat the following operations (named Grover quantum iteration) $R = \frac{\pi\sqrt{N}}{4}$ times:

(a) perform the f-controlled phase shift U_f on the state $|\phi\rangle$, giving $|x\rangle \xrightarrow{U_f} (-1)^{\delta_{m,x}}|x\rangle$, where $|m\rangle$ denotes the searched item;

(b) perform the H transform on each of the qubits in the register and obtain state $|\phi'\rangle$;

(c) perform the 0-controlled phase shift U_0 on the state $|\phi'\rangle$, giving $|x\rangle \xrightarrow{U_0} (-1)^{\delta_{0,x}}|x\rangle$.

(d) perform the H transform again.

III. Measure the resulting state.

The combined effect of steps (a), (b), (c), and (d) can be written by the sequence of operators:

$$G = -HU_0HU_f. \tag{1}$$

Here the operator G is called the Grover iteration operator.

3. The generalized Grover iteration operator and its properties

In this section we introduce the generalized Grover iteration operator by rewriting the unitary operations in Eq.(1) and discuss some of its useful properties which are previously revealed in Refs.[38] and [39].

The generalized Grover iteration operator, in the form of Eq.(1), can be written as

$$\mathcal{G} = \mathcal{G}(\mathcal{S}, f, \phi, \varphi) = -\mathcal{S}\mathcal{U}_0^{\phi}\mathcal{S}^{-1}\mathcal{U}_f^{\varphi}. \tag{2}$$

Here $\mathcal{S}$ represents any quantum algorithm and uses no measurements, and $\mathcal{U}_f^{\varphi}$ and $\mathcal{U}_0^{\phi}$ act on the state $|x\rangle$ having the following forms, respectively,

$$\mathcal{U}_f^{\varphi}|x\rangle = \begin{cases} \varphi|x\rangle, & \text{if } f(x)=1, \\ |x\rangle, & \text{if } f(x)=0; \end{cases}$$

$$\mathcal{U}_0^{\phi}|x\rangle = \begin{cases} \phi|x\rangle, & \text{if } x=0, \\ |x\rangle, & \text{if } x\neq 0. \end{cases} \tag{3}$$

Let $|\Psi\rangle$ be any superposition of pure state. In two orthogonal subspaces we can write $|\,\Psi\rangle$ as

$$\begin{aligned} |\,\Psi\rangle &= |\,\Psi^a\rangle + |\,\Psi^b\rangle \\ &= \sum_{i\in f^{-1}(1)} a_i|x_i\rangle + \sum_{j\in f^{-1}(0)} a_j|x_j\rangle, \end{aligned} \tag{4}$$

where $|\,\Psi^a\rangle = \sum_{i\in f^{-1}(1)} a_i|x_i\rangle$ represents the searched states corresponding to $f(x_i)=1$; $|\,\Psi^b\rangle = \sum_{j\in f^{-1}(0)} a_j|x_j\rangle$ represents the unsearched states corresponding to $f(x_j)=0$; and $\sum_{i\in f^{-1}(1)} a_i^2 + \sum_{j\in f^{-1}(0)} a_j^2 = 1$. It can be easily obtained that the probability of finding a searched state is $a = \langle\,\Psi^a|\,\Psi^a\rangle$, and the probability of finding an unsearched state is $b = \langle\,\Psi^b|\,\Psi^b\rangle$, and a and b satisfy $a+b=1$.

Applying the generalized Grover iteration operator $\mathcal{G}$ to $|\,\Psi^a\rangle$ and $|\,\Psi^b\rangle$, respectively, we obtain[38]

$$\begin{aligned} \mathcal{G}|\,\Psi^a\rangle &= -\varphi|\,\Psi^a\rangle + \varphi(1-\phi)\langle\,\Psi^a|\mathcal{S}|\mathbf{0}\rangle^*\mathcal{S}|\mathbf{0}\rangle, \\ \mathcal{G}|\,\Psi^b\rangle &= -|\,\Psi^b\rangle \quad + \quad (1-\phi)\langle\,\Psi^b|\mathcal{S}|\mathbf{0}\rangle^*\mathcal{S}|\mathbf{0}\rangle. \end{aligned} \tag{5}$$

If let $\mathcal{S}|\mathbf{0}\rangle = |\,\Psi\rangle = |\,\Psi^a\rangle + |\,\Psi^b\rangle$, we have

$$\begin{aligned} \mathcal{G}|\,\Psi^a\rangle &= \varphi\left[(1-\phi)a - 1\right]|\,\Psi^a\rangle \\ &\quad + \varphi(1-\phi)a|\,\Psi^b\rangle, \\ \mathcal{G}|\,\Psi^b\rangle &= -\left[(1-\phi)a + \phi\right]|\,\Psi^b\rangle \\ &\quad + (1-\phi)(1-a)|\,\Psi^a\rangle. \end{aligned} \tag{6}$$

Let $\phi = \varphi = -1$, that is, $\mathcal{G} = \mathcal{G}(\mathcal{S}, f, -1, -1)$. Then

$$\begin{aligned} \mathcal{G}|\,\Psi^a\rangle &= (1-2a)|\,\Psi^a\rangle - 2a|\,\Psi^b\rangle, \\ \mathcal{G}|\,\Psi^b\rangle &= (1-2a)|\,\Psi^b\rangle + 2b|\,\Psi^a\rangle. \end{aligned} \tag{7}$$

Define the new normalized basis vectors

$$\begin{aligned} |\alpha\rangle &= \frac{1}{\sqrt{a}}|\,\Psi^a\rangle, \\ |\beta\rangle &= \frac{1}{\sqrt{b}}|\,\Psi^b\rangle, \\ |\,\Psi^+\rangle &= \frac{1}{\sqrt{2}}|\alpha\rangle + \frac{\mathrm{i}}{\sqrt{2}}|\beta\rangle, \\ |\,\Psi^-\rangle &= \frac{1}{\sqrt{2}}|\alpha\rangle - \frac{\mathrm{i}}{\sqrt{2}}|\beta\rangle. \end{aligned} \tag{8}$$

It then follows from Eqs.(7) and (8) that

$$\begin{aligned} \mathcal{G}|\,\Psi^+\rangle &= \frac{1}{\sqrt{2}}\left[(1-2a) + \mathrm{i}2\sqrt{ab}\right]|\,\Psi^a\rangle \\ &\quad + \frac{\mathrm{i}}{\sqrt{2}}\left[(1-2a) + \mathrm{i}2\sqrt{ab}\right]|\,\Psi^b\rangle \\ &= \mathrm{e}^{2\pi\mathrm{i}\theta_a}|\,\Psi^+\rangle, \end{aligned} \tag{9}$$

where $\cos(2\pi\theta_a) = 1-2a$, $\sin(2\pi\theta_a) = 2\sqrt{ab} = 2\sqrt{a(1-a)}$, with $0 \le \theta_a \le \frac{1}{2}$. Similarly, we can obtain $\mathcal{G}|\,\Psi^-\rangle = \mathrm{e}^{-2\pi\mathrm{i}\theta_a}|\,\Psi^-\rangle$. From the above we can see that $|\,\Psi^+\rangle$ and $|\,\Psi^-\rangle$ are eigenvectors of $\mathcal{G}$ with eigenvalues $\mathrm{e}^{2\pi\mathrm{i}\theta_a}$ and $\mathrm{e}^{-2\pi\mathrm{i}\theta_a}$, respectively.

From Eq.(8), we rewrite state $|\,\Psi\rangle$ of Eq.(4) as

$$\begin{aligned} |\,\Psi\rangle &= \mathcal{S}|\mathbf{0}\rangle \\ &= -\frac{\mathrm{i}\mathrm{e}^{\mathrm{i}\pi\theta_a}}{\sqrt{2}}|\,\Psi^+\rangle + \frac{\mathrm{i}\mathrm{e}^{-\mathrm{i}\pi\theta_a}}{\sqrt{2}}|\,\Psi^-\rangle, \end{aligned} \tag{10}$$

where $\sin(\pi\theta_a) = \sqrt{a}$, $\cos(\pi\theta_a) = \sqrt{1-a}$.

4. Direct measurement of concurrence using quantum algorithms

Consider a general two-qubit pure state whose concurrence we want to measure has the following form

$$|\varphi\rangle = a_0|00\rangle + a_1|01\rangle + a_2|10\rangle + a_3|11\rangle, \tag{11}$$

where a_k are arbitrary complex numbers and satisfy $\sum_{k=0}^{3} a_k^2 = 1$. The concurrence of state $|\varphi\rangle$ is given by $C(\varphi) = 2|a_0a_3 - a_1a_2|$. Under the same interaction between the system and environment, using the same technology and method, we can produce two identical copies of $|\varphi\rangle$ of Eq.(11), that is, $|\varphi\rangle_{12} \otimes |\varphi\rangle_{34}$. Introducing unitary operator $\mathcal{A}$, we describe the process of generating the state $|\varphi\rangle$ as

$$|\varphi\rangle = \mathcal{A}|00\rangle = a_0|00\rangle + a_1|01\rangle + a_2|10\rangle + a_3|11\rangle, \quad (12)$$

here $\mathcal{A}$ denotes the combination of arbitrary unitary operations.

The state of the system consisting of two copies of $|\varphi\rangle$ is

$$\begin{aligned}|\psi\rangle &= |\varphi\rangle_{12} \otimes |\varphi\rangle_{34} \\ &= a_0^2|0000\rangle_{1234} + a_0a_1|0001\rangle_{1234} + a_0a_2|0010\rangle_{1234} + a_0a_3|0011\rangle_{1234} \\ &\quad + a_0a_1|0100\rangle_{1234} + a_1^2|0101\rangle_{1234} + a_1a_2|0110\rangle_{1234} + a_1a_3|0111\rangle_{1234} \\ &\quad + a_0a_2|1000\rangle_{1234} + a_1a_2|1001\rangle_{1234} + a_2^2|1010\rangle_{1234} + a_2a_3|1011\rangle_{1234} \\ &\quad + a_0a_3|1100\rangle_{1234} + a_1a_3|1101\rangle_{1234} + a_2a_3|1110\rangle_{1234} + a_3^2|1111\rangle_{1234}. \end{aligned} \quad (13)$$

We first apply local unitary operators σ_y ($\sigma_y = i(|1\rangle\langle 0| - |0\rangle\langle 1|)$) to qubits 2 and 3, respectively. Then we perform a controlled-not gate operation U_{CNOT} on qubits 1 and 3, here qubit 1 is the control qubit and qubit 3 is the target qubit. The CNOT gate operation is defined such that if the control qubit is in state $|0\rangle$, the target qubit is flipped. Otherwise, the target qubit is not affected. Finally, we perform a rotation operation R on qubit 1. The R operation can transform each qubit as

$$\begin{aligned} R|0\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \\ R|1\rangle &= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle). \end{aligned} \quad (14)$$

After a sequence of unitary operations mentioned above (shown in Fig.1), the state $|\psi\rangle$ in Eq.(13) becomes

$$\begin{aligned}|\psi'\rangle = \frac{1}{\sqrt{2}}\big[&(a_0a_1 - a_2a_3)|0000\rangle_{1234} + (a_1^2 - a_3^2)|0001\rangle_{1234} + (a_0a_3 - a_1a_2)|0010\rangle_{1234} \\ &+ (a_2^2 - a_0^2)|0100\rangle_{1234} + (a_2a_3 - a_0a_1)|0101\rangle_{1234} + (a_0a_3 - a_1a_2)|0111\rangle_{1234} \\ &+ (a_0a_1 + a_2a_3)|1000\rangle_{1234} + (a_1^2 + a_3^2)|1001\rangle_{1234} - (a_0a_3 + a_1a_2)|1010\rangle_{1234} \\ &+ (a_0a_3 + a_1a_2)|1111\rangle_{1234} - (a_0^2 + a_2^2)|1100\rangle_{1234} - (a_0a_1 + a_2a_3)|1101\rangle_{1234} \\ &+ 2a_0a_2|1110\rangle_{1234} - 2a_1a_3|1011\rangle_{1234}\big]. \end{aligned} \quad (15)$$

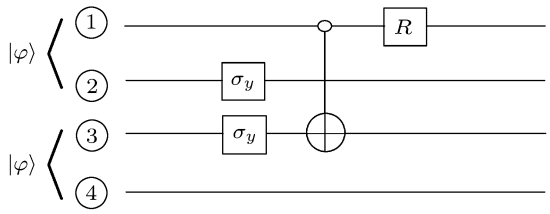


Fig.1. Schematic quantum circuit for performing a sequence of unitary operations on two copies of $|\varphi\rangle$.

Let $\mathcal{B} = R^1 \otimes U_{\mathrm{CNOT}}^{1,3} \otimes \hat{\sigma}_y^3 \otimes \hat{\sigma}_y^2$, where the superscripts 1, 2, and 3 represent the qubits which are operated, respectively. In this way the process of $|\psi\rangle \to |\psi'\rangle$, using the operators $\mathcal{A}$ and $\mathcal{B}$, can be described as

$$|\psi'\rangle = \mathcal{B}\mathcal{A}^2\mathcal{A}^1|00\rangle_{12}|00\rangle_{34}, \quad (16)$$

where the superscripts 1 and 2 indicate that the operator $\mathcal{A}$ acts on $|00\rangle_{12}$ and $|00\rangle_{34}$ states, respectively.

In Section 3, let $\mathcal{S} = \mathcal{B}\mathcal{A}^2\mathcal{A}^1$, and let

$$
\begin{aligned}
|\Psi\rangle &= |\psi'\rangle = \mathcal{S}|00\rangle_{12}|00\rangle_{34} = |\Psi^a\rangle + |\Psi^b\rangle,\\
|\Psi^a\rangle &= \frac{1}{\sqrt{2}}\big[(a_0a_3 - a_1a_2)|0010\rangle_{1234} + (a_0a_3 - a_1a_2)|0111\rangle_{1234}\big],\\
|\Psi^b\rangle &= \frac{1}{\sqrt{2}}\big[(a_0a_1 - a_2a_3)|0000\rangle_{1234} + (a_1^2 - a_3^2)|0001\rangle_{1234} + 2a_0a_2|1110\rangle_{1234}\\
&\quad + (a_2^2 - a_0^2)|0100\rangle_{1234} + (a_2a_3 - a_0a_1)|0101\rangle_{1234} - 2a_1a_3|1011\rangle_{1234}\\
&\quad + (a_0a_1 + a_2a_3)|1000\rangle_{1234} + (a_1^2 + a_3^2)|1001\rangle_{1234} - (a_0a_3 + a_1a_2)|1010\rangle_{1234}\\
&\quad + (a_0a_3 + a_1a_2)|1111\rangle_{1234} - (a_0^2 + a_2^2)|1100\rangle_{1234} - (a_0a_1 + a_2a_3)|1101\rangle_{1234}\big].
\end{aligned} \tag{17}
$$

Therefore, we have $a = \langle\Psi^a|\Psi^a\rangle = |a_0a_3 - a_1a_2|^2$, and the concurrence of the state $|\varphi\rangle$ is $C = 2\sqrt{a}$. In the following we show how to determine the concurrence C by calculating the value of a.

Step 1 Initialize the system in the state

$$
\begin{aligned}
|\Phi\rangle &= \frac{1}{\sqrt{2^t}}\sum_{x=0}^{2^t-1}|y\rangle\mathcal{S}|00\rangle_{12}|00\rangle_{34}\\
&= -\frac{\mathrm{i}\mathrm{e}^{\mathrm{i}\pi\theta_a}}{\sqrt{2^{t+1}}}\sum_{x=0}^{2^t-1}|y\rangle|\Psi^+\rangle\\
&\quad + \frac{\mathrm{i}\mathrm{e}^{-\mathrm{i}\pi\theta_a}}{\sqrt{2^{t+1}}}\sum_{x=0}^{2^t-1}|y\rangle|\Psi^-\rangle,
\end{aligned} \tag{18}
$$

where $\mathcal{S} = \mathcal{B}\mathcal{A}^2\mathcal{A}^1$, t is the number of qubits in the first register, and $|\Psi^+\rangle$ and $|\Psi^-\rangle$ are defined by $|\Psi^a\rangle$ and $|\Psi^b\rangle$ revealed in Eqs.(8) and (17).

Step 2 Applying the generalized Grover iteration operator $\mathcal{G} = -\mathcal{S}\mathcal{U}_0^{-1}\mathcal{S}^{-1}\mathcal{U}_f^{-1}$ to the second register y times when the first register is in state $|y\rangle$, we obtain

$$
\begin{aligned}
|\Phi_1\rangle &= -\frac{\mathrm{i}\mathrm{e}^{\mathrm{i}\pi\theta_a}}{\sqrt{2^{t+1}}}\sum_{y=0}^{2^t-1}\mathrm{e}^{2\pi\mathrm{i}\theta_a y}|y\rangle|\Psi^+\rangle\\
&\quad + \frac{\mathrm{i}\mathrm{e}^{-\mathrm{i}\pi\theta_a}}{\sqrt{2^{t+1}}}\sum_{x=0}^{2^t-1}\mathrm{e}^{-2\pi\mathrm{i}\theta_a y}|y\rangle|\Psi^-\rangle.
\end{aligned} \tag{19}
$$

Step 3 Applying the inverse quantum Fourier transform F_{p}^-, which maps each state $|m\rangle$ into a superposition given by

$$
F_{\mathrm{p}}^-|m\rangle = \frac{1}{\sqrt{2^p}}\sum_{n=0}^{2^p-1}\mathrm{e}^{-2\pi\mathrm{i}mn/2^p}|n\rangle \tag{20}
$$

to the first register, we obtain

$$
\begin{aligned}
|\Phi_2\rangle &= -\frac{\mathrm{i}\mathrm{e}^{\mathrm{i}\pi\theta_a}}{2^t\sqrt{2}}\sum_{x=0}^{2^t-1}\sum_{y=0}^{2^t-1}\mathrm{e}^{2\pi\mathrm{i}y(\theta_a - x/2^t)}|x\rangle|\Psi^+\rangle\\
&\quad + \frac{\mathrm{i}\mathrm{e}^{-\mathrm{i}\pi\theta_a}}{2^t\sqrt{2}}\sum_{x=0}^{2^t-1}\sum_{y=0}^{2^t-1}\mathrm{e}^{-2\pi\mathrm{i}y(\theta_a + x/2^t)}|x\rangle|\Psi^-\rangle\\
&= -\frac{\mathrm{i}\mathrm{e}^{\mathrm{i}\pi\theta_a}}{2^t\sqrt{2}}|\widetilde{\theta_a}\rangle|\Psi^+\rangle + \frac{\mathrm{i}\mathrm{e}^{-\mathrm{i}\pi\theta_a}}{2^t\sqrt{2}}|\widetilde{-\theta_a}\rangle|\Psi^-\rangle.
\end{aligned} \tag{21}
$$

Here $\widetilde{\theta_a}$ is a close estimate of θ_a with high precision.

Step 4 Measuring the first register, we obtain

$$
\theta_a = \begin{cases} x/2^t, & \text{for } 0 \le x \le 2^t/2,\\ 1 - x/2^t, & \text{for } 2^t/2 < x \le 2^t. \end{cases} \tag{22}
$$

Then we substitute the value of θ_a into the equation $\sin(\pi\theta_a) = \sqrt{a}$ and obtain the exact value of a. Therefore, the concurrence of the state $|\varphi\rangle$ can be determined directly by using the above method.

5. Discussion and conclusion

In this protocol we require two copies of a quantum state $|\varphi\rangle$ and assume that state $|\varphi\rangle$ can be generated by using the same technology and method under the same environment during the process of performing generalized Grover quantum iteration, i.e., $|\varphi\rangle = \mathcal{A}|00\rangle$. Here the unitary operator $\mathcal{A}$ is unknown and represents the explicit process of generating the state $|\varphi\rangle$. Experimentally, Walborn *et al*[31] have demonstrated the direct observation of entanglement measure on two copies of a pure state by using linear optics means. The experiment showed that perfect copies of a quantum state could be experimentally generated with a certain precision and the entanglement properties could be well depicted by

the expectation value of a certain Hermitian operator corresponding to two copies of a quantum state. Romero *et al*[32] proposed a general protocol to measure the concurrence by using two available copies of the bipartite system. In the protocol Romero *et al* showed how to produce two identical copies in a 3D microwave cavity and trapped ion system based on the entangling technique that has already been experimentally realized in Refs.[40, 41]. Therefore, the process of producing identical copies of the initial pure quantum state (i.e., $|\varphi\rangle = \mathcal{A}|00\rangle$) in our protocol is feasible. Essentially, in addition, the process of determining the value of θ_a (steps (1–4)) is equivalent to a phase estimation process.[42] The accuracy of θ_a and the probability, which we wish the process of estimating the concurrence C to be successful, mainly depend on the number of qubits chosen in the first register (auxiliary working qubits), that is, the size of t, which is referred in Eq.(18). Suppose we wish to approximate θ_a to an accuracy 2^{-p} and to successfully obtain θ_a to p qubits with the success probability at least $1-\epsilon$, we should choose

$$t = p + \left[\log\left(2 + \frac{1}{2\epsilon}\right)\right].$$

Compared with the previous protocols mentioned above, our protocol has the following merits: (i) the method for estimating the concurrence of a pure state was straightforward with the success probability of unit in ideal case; (ii) the precision of calculating the value of concurrence was very high by choosing the number of qubits in the first register appropriately; (iii) the repeated rounds of experiments for determining the probability of finding all relevant qubits in the certain state were unnecessary, which simplified the experimental realization of measuring concurrence. So far many protocols for implementing a Grover quantum search algorithm and a phase estimation algorithm have been proposed both in theoretical and experimental aspects,[43−50] and protocols for realizing quantum Fourier transform have also been proposed experimentally.[51,52] Therefore, our protocol might be experimentally feasible.

In conclusion, we have proposed a method to measure the concurrence of an arbitrary two-qubit pure state directly by applying quantum algorithms to two available copies of the bipartite system and discussed the issues that we may face. The method of measuring the concurrence of a pure state in our protocol was straightforward and the value of concurrence could be estimated with high precision when a measurement was made. We hope that further work in this direction could prove to be useful.

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