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Evolutionary prisoner's dilemma game on highly clustered community networks*

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This paper studies the evolutionary prisoner's dilemma game on a highly clustered community network in which the clustering coefficient and the community size can be tuned. It finds that the clustering coefficient in such a degree-homogeneous network inhibits the emergence of cooperation for the entire range of the payoff parameter. Moreover, it finds that the community size can also have a marked influence on the evolution of cooperation, with a larger community size leading to not only a lower cooperation level but also a smaller threshold of the payoff parameter above which cooperators become extinct.

Keywords: prisoner's dilemma, cooperation, clustering coefficient, community structure
PACC: 0250, 0175, 0270

1. Introduction

Cooperative behaviours are found to be ubiquitous in many social and economic systems which consist of unrelated and selfish individuals, and such behaviours are essential in human society. Understanding the emergence and persistence of cooperation has thus become one of the fundamental problems. Evolutionary game theory has provided a powerful and convenient framework to investigate this problem.[1−5] The prisoner's dilemma game (PDG), as a general metaphor for characterizing the conflict between social welfare and individual selfishness, has been widely uesd (for a recent review, see Ref.[6]). In the traditional PDG, two players simultaneously decide whether to cooperate (C) or to defect (D) to obtain some payoffs. They both receive R under mutual cooperation, and only P for mutual defection; under the condition of unilateral cooperation or defection, the cooperator and defector receive S and T, respectively. Moreover, the four payoff values satisfy the following two inequalities: $T > R > P > S$ and $T+S < 2R$. It is easy to see that it is best to defect for any rational player irrespective of what the opponent does in a single round of the PDG; however, mutual cooperation leads to a higher collective income shared equally by the two participating players. Thus, the so-called dilemma is created by the above situation.

In past decades, considerable effort has been given to the exploration of mechanisms which support the emergence and maintenance of cooperation to explain the abundant and robust cooperative behaviours in real systems. The following mechanisms, e.g., kin selection,[7] direct[3] and indirect[8] reciprocity, network reciprocity,[9,10] and group selection,[11] are found to be favourable to the evolution of cooperation. Surprisingly, the spatial game, introduced by Nowak and May,[9] can lead to the emergence and maintenance of cooperative behaviour in the absence of strategic complexity. Much of the subsequent work has adopted this idea.[12,13,15,16] Axelrod *et al* [12] took the first step toward the structures of social networks by giving up a strictly local geographical structure. It has now become clear that many actual networks, social or technological, are neither regular nor random, but rather of the small-world type, that is, they

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are of short average path lengths and high clustering coefficients. Moreover, recent studies on complex networks revealed that many real-life networks are scale-free, i.e., their degree distributions follow a power-law as $P(k) \sim k^{-r}$, with r being typically constrained to the interval $2 < r < 3$. Santos and Pacheco have studied the PDG on Barabási–Albert (BA) scale-free networks, and discovered that scale-free networks can be a potent promoter for the emergence of cooperation.[13,16] However, BA scale-free networks can only be considered as approximations of real-world networks, since it has been observed that social networks usually have a higher clustering coefficient than the typical values reached in BA scale-free networks. More recently, Assenza *et al* [15] studied the effect of the clustering coefficient on cooperation in the PDG and found that a high clustering coefficient homogeneizes the invasion process of degree classes by defectors. Besides, in the context of social systems there exist other important topological features, e.g., degree–degree correlation (also known as mixing pattern).[17] References [18] and [19] gave detailed discussions on the influences of degree correlations on the emergence of cooperation. As another typical topological feature, community structure, i.e. sets of densely connected vertices with sparse connections between them, plays an important role in the way of information propagation throughout the network, and it is reasonable to expect that it will influence the evolution of cooperation profoundly. Regarding community structure, Chen *et al* [20] have studied the evolutionary PDG on a model network with three communities, and they found that reducing inter-community links can promote cooperation when the total number of links in the network is kept invariant. The work of Luthi *et al* [21] focused on the PDG on a more socially relevant model network and a real social network, i.e., the coauthorship network among researchers in the genetic programming community, which both possess community structures, and revealed that such structures can influence the evolution of cooperative behaviours.

In this work, we employ the network model proposed in Ref.[22] which can produce highly clustered networks with community structures. Importantly, both the clustering coefficient and the size of the community in the network can be tuned, by which we can perform thorough studies of the effects they have on the emergence of cooperation. The authors of Ref.[20] have elaborated on the role of community structure with the emphasis on the patterns of links inner and inter communities; however, they only dealt with the case of three communities, which gave no hint of the effect of the community size on the emergence of cooperation. Assenza *et al* [15] have studied the evolution of cooperation in the PDG on a class of highly clustered scale-free networks with highly heterogeneous degree distributions which have no apparent community structures. Therefore, we actually do not know what happens when highly clustered networks are degree-homogeneous or with community structure. In the present paper, we show that clustering in networks does not always enhance cooperation, especially for degree-homogeneous networks. Furthermore, it shows that larger communities turn out to be unfavourable to cooperative behaviour.

2. Network model

We use the model proposed by Newman.[22] Consider a network of N individuals divided into M communities. Individuals can belong to one or more communities. When determining neighbours of each individual, it is assumed that individuals are not necessarily acquainted with all other members within the same community. Thus, if two individuals belong to the same community, then they are acquainted with probability p and not with probability $q = 1 - p$; in contrast, if they have no common communities, then they are not acquainted.

Here we consider a simplified version of the model in which all communities have the same size ν. For each of the M communities, ν individuals are chosen independently and uniformly at random from the population of N individuals. The clustering coefficient, which represents the probability that two neighbours of a given node are themselves connected (see Ref.[23] for details), can be adjusted by varying the number of groups per node μ. Thus, the number of groups can be calculated as $M = N\mu/\nu$. Following the original work of Ref.[22], we find that the connection probability is given by

$$p = \frac{k}{\mu(\nu - 1)}, \tag{1}$$

and the estimated clustering coefficient is

$$C = \frac{p}{1 + \mu(\nu - 1)/(\nu - 2)}. \tag{2}$$

3. Evolutionary dynamics

We consider the evolutionary PDG with each player located on one node of the underlying interaction network. Initially, the strategies (C and D) are randomly distributed with equal probability among all individuals. Evolution is carried out for a population of N individuals analogue of replicator dynamics[9,24] for the PDG with payoffs $T = b$, $R = 1$, and $P = S = 0$, where $1 < b < 2$ represents the advantage of defectors over cooperators. At each generation, each individual, e.g., individual i, plays once with all its neighbours and accumulates the payoffs from the PDG with all its neighbours, stored as P_i. Then all players update their strategies synchronously according to the following rules. Whenever individual i updates its strategy, a neighbour j is drawn randomly among all k_i neighbours, and then compares its payoff P_i with P_j. If $P_i \geq P_j$, player i keeps the same strategy for the next round of game; on the contrary, if $P_i < P_j$, player i adopts the strategy of the neighbour j with probability given by

$$\Pi_{i\to j} = \beta(P_j - P_i), \tag{3}$$

where $\beta = (k_> b)^{-1}$, $k_>$ denotes the larger one between k_i and k_j.

4. Simulation results

In what follows, we will report the simulation results carried out for a population of $N = 5000$ individuals. The relevant quantity used here is the frequency of cooperation, f_c, which is defined as the fraction of cooperators in the whole population. Initially, cooperators and defectors are uniformly and randomly distributed in the population. In all simulations, f_c is obtained by averaging 5000 generations after a transient time of 10000 generations. Each data point results from 100 realizations corresponding to 10 network realizations and 10 initial conditions for each network realization.

First, let us plot the degree distributions corresponding to networks with clustering coefficients $C = 0.1, 0.3, 0.5$, respectively, as shown in Fig.1. One can find that the degree distributions follow an exponential form. It is noteworthy that the resulting networks have a giant connected component (GCC), with some nodes being isolated, and the size of the GCC decreases with the increase of the clustering coefficient (not shown here, and we have checked that our results reported below are robust with respect to the size of GCC by adopting a larger network size, e.g., $N = 10000$). In this paper, we focus our attention on the GCC and investigate the evolution of cooperation on the GCC. The inset of Fig.1 is the average degree variance of the nodes in the GCC as a descending function of the clustering coefficient, according to which one can find that as the clustering coefficient increases, the corresponding network becomes more and more homogeneous in terms of node degree.

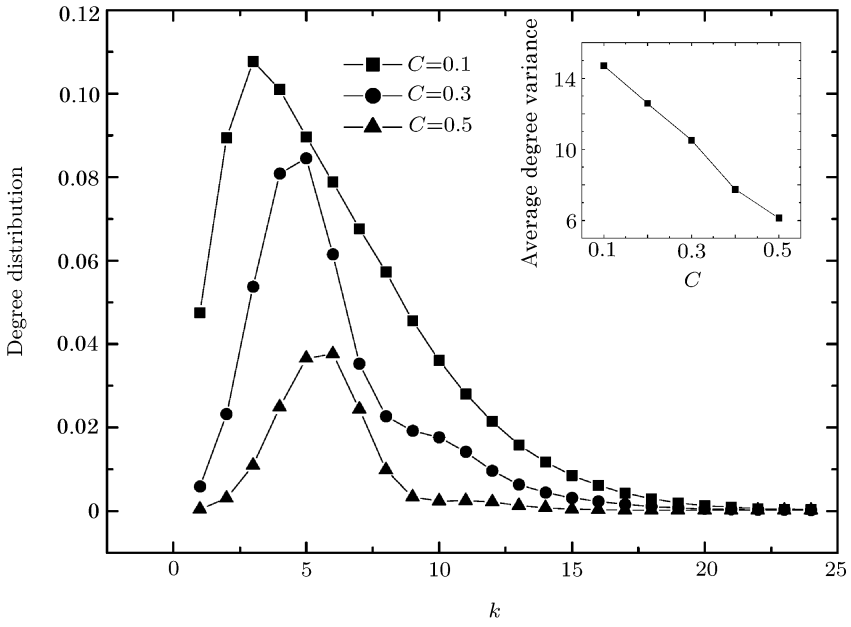


Fig.1. Degree distributions for networks with clustering coefficients $C = 0.1, 0.3, 0.5$. The inset is the degree variance vs C. The average degree of the nodes in the GCC is $\langle k \rangle = 6$. The results are obtained by averaging over 100 networks.

Figure 2 shows the cooperation level for the above clustered networks (filled symbols) as a function of the temptation to defect b with community size $\nu = 10$. As mentioned above, in this work, we study the PDG on the GCC of the networks. Here the average degree of the GCC corresponding to each clustered network is set to be $\overline{k} = 6$. Thus, the results shown in Fig.2 is only affected by the clustering coefficient and the degree distribution. It can be observed that all the curves descend monotonically as b increases. Moreover, we can find that cooperation for $C = 0.3$ is enhanced with respect to the case of $C = 0.1$ when b is small, however, when moving to high values of b, cooperation is inhibited and reflected by the fact that the curve for $C = 0.3$ descends more quickly. And for $C = 0.5$, the cooperation is the lowest for the entire range of b. This is in part due to the fact that with the increase of C, the degree distribution becomes more and more homogeneous.

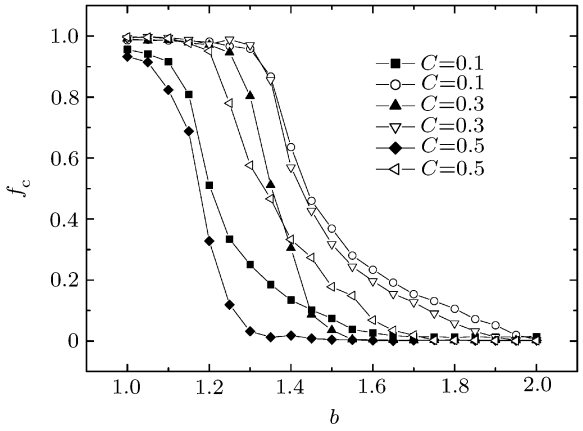


Fig.2. Frequency f_c of cooperation vs b for networks with different clustering coefficients as shown in the legend.

In order to distinguish the roles of the clustering coefficient and the degree distribution, we construct configuration networks according to the approach proposed by Molloy and Reed.[25,26] The configuration model allows us to generate networks maximally random under the constraint of prescribed degree sequence, for which we take those produced with the original networks, that is, each clustered network has the same degree sequence as its corresponding configuration one. Furthermore, we have also examined that both of them have no degree correlations. (Thus, the main ingredient that can affect the cooperation levels between the original clustered networks and the configuration ones is the clustering coefficient.) In the same figure, we report the cooperation level for the configuration networks (open symbols). Looking at the results for configuration networks indicated by open symbols in Fig.2, one can find that, indeed, the cooperation level on configuration networks corresponding to the larger clustering coefficient, i.e., $C = 0.5$, is lower than that for smaller one, just because when increasing the clustering coefficient, the degree is distributed more homogeneously. It is well known that degree-heterogeneous networks instead of homogeneous ones are favourable to cooperation.[13]

Now, let us compare the cooperation level between the results for clustered networks and the corresponding configuration ones. In this respect, one can find that for all clustering coefficients, cooperation on each of the configuration networks is higher than that on the corresponding clustered network, revealing that on such homogeneous networks, the clustering coefficient can be a hindrance to promote cooperation, which is in contrast with the results reported previously for highly heterogeneous scale-free networks.[15] In the work, the authors found that the clustering coefficient produces an overall enhancement of cooperation.

In the following, we explore the effect of the community size on the evolution of cooperation. In Fig.3, we report the simulation results of cooperation for both the clustered networks and their corresponding configuration ones with community size $\nu = 10, 20, 30$, respectively. During the process of simulations, the clustering coefficient is set to be $C = 0.2$. In order to exclude the influence of average degree, the average degree of nodes in the corresponding GCC of each of the networks under consideration is fixed as $\overline{k} = 12$. Thus, the factors leading to the different cooperation levels on clustered networks in Fig.3 are the community size and the degree distribution, while in the configuration networks, the cooperation behaviours are only influenced by the degree distributions. It can be observed that larger community sizes lead to not only the lower cooperation level, but also the smaller threshold of the temptation to defect above which cooperators becomes extinct. We can also find that although the configuration network with larger community sizes has a slightly lower cooperation level, the difference caused by the degree distribution is actually very minute. Therefore, it is reasonable to conclude that the main ingredient which results in the big difference in cooperation on clustered networks is the community size. In other words, larger community sizes inhibit the emergence of cooperation in the

PDG.

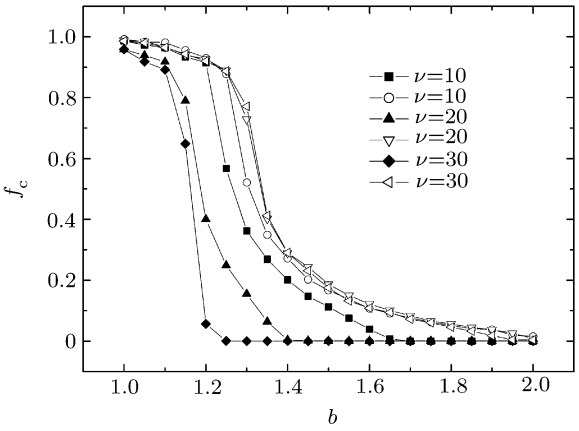


Fig.3. Frequency f_c of cooperation for clustered networks (filled symbols) and the corresponding configuration ones (open symbols) with community sizes $\nu = 10$, 20, 30.

To give a more complete description on the role of community size, we also study the cooperation level at the community level, that is, we calculate the distribution of cooperation for all communities, as shown in Fig.4. The calculation is performed in the following way. For each community, we calculate the cooperation level, i.e., the fraction of cooperators, and for each interval of cooperation, we calculate the total number of communities in which the frequency of cooperation falls into this interval. These results are obtained by averaging over 50 initial states, and for each initial condition, we average the time series of cooperation from 10000 to 15000 generations. It is worth pointing out that with the increase of the number of individuals in each community, the number of communities decreases, since the total number of individuals is fixed.

From Fig.4, we can find that cooperation is distributed within narrow ranges irrespective of the community sizes and the values of b. More importantly, with the increase of the community size, cooperation is inhibited, reflected by the fact that most communities for larger ν have a lower cooperation level. In Ref.[21], the authors found that in most communities of the model networks studied there, either cooperators or defectors dominate no matter what the cooperation level is; in other words, almost all communities are composed of players with the same strategies. In our case, such strategy clustering phenomenon is not very evident, except for the cases of very high or very low cooperation. Actually, due to the networks considered here are degree-homogeneous, cooperators and defectors can be regarded as uniformly and randomly distributed among all individuals. Consequently, there exist a largest number of communities having cooperation levels similar to the global one. One may ask the following question: what is the reason for the lower cooperation when the community size becomes larger? Note that the networks considered here are of community structures, moreover, within the same community, individuals are densely connected, while the links between different communities are sparse. In combination with the fact that the homogeneous networks studied here are unfavourable to cooperation, therefore, larger community sizes will facilitate the spread of defection, and thus result in the lower cooperation level, which is clearly demonstrated in Fig.4. The above conclusion is valid at least for our case of homogeneous networks.

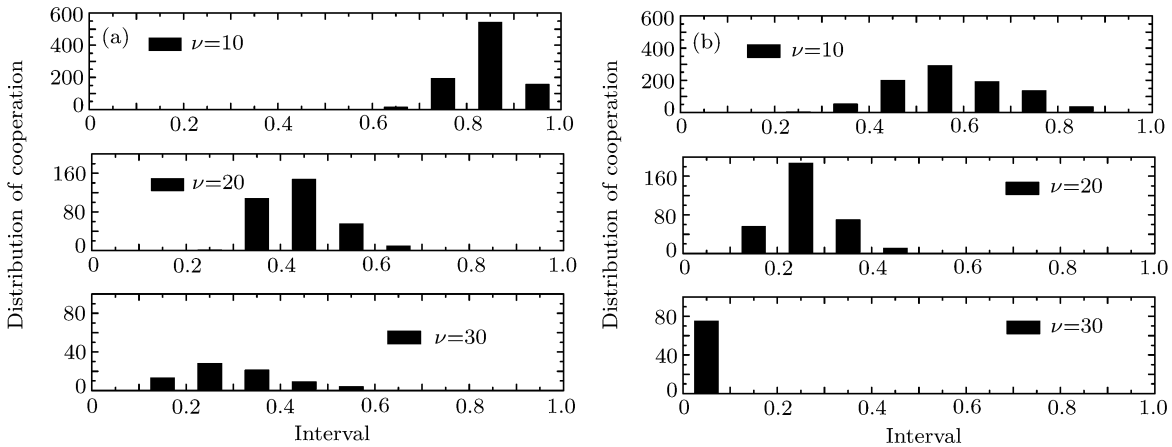


Fig.4. Distribution of cooperation for all communities for $b = 1.2$ (panel a) and $b = 1.25$ (panel b). Each bar denotes the number of communities in which the cooperation frequency falls into the corresponding interval.

5. Conclusions

In order to study the roles of the clustering coefficient and the community structure in the evolution of cooperation, we have used a network model which allows us to construct networks with both a tunable clustering coefficient and a tunable degree distribution. Our study indicates that the clustering coefficient in degree-homogeneous networks inhibits the emergence of cooperation. Furthermore, we found that the size of community can also affect the cooperative behaviours; in other words, as the community size increases, not only does the frequency of cooperation decrease, there also exists a smaller threshold of the payoff parameter above which the cooperators vanish for increased group size. Our study reveals that cooperative behaviours on homogeneous networks are indeed different from those on heterogeneous networks, especially for the highly clustered community networks considered here. Although the networks employed here cannot describe the topologies of real-world networks realistically, the obtained results can indeed give us some clues on the effects that the clustering coefficient and the community size have on the emergence of cooperation.

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