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Theoretical and experimental investigations of flexural wave propagation in periodic grid structures designed with the idea of phononic crystals*

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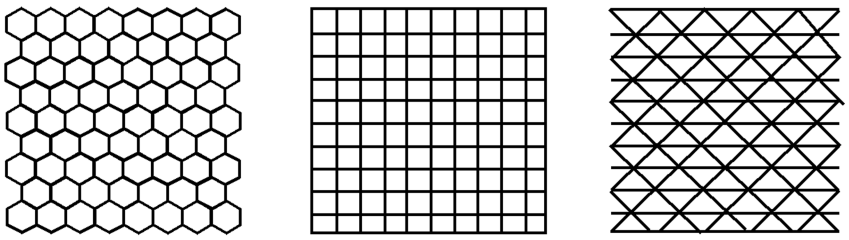
The propagation characteristics of flexural waves in periodic grid structures designed with the idea of phononic crystals are investigated by combining the Bloch theorem with the finite element method. This combined analysis yields phase constant surfaces, which predict the location and the extension of band gaps, as well as the directions and the regions of wave propagation at assigned frequencies. The predictions are validated by computation and experimental analysis of the harmonic responses of a finite structure with 11×11 unit cells. The flexural wave is localized at the point of excitation in band gaps, while the directional behaviour occurs at particular frequencies in pass bands. These studies provide guidelines to designing periodic structures for vibration attenuation.

Keywords: phononic crystal, phase constant surface, grid structure
PACC: 4340, 6320

1. Introduction

Structural vibrations are of great concern in the fields of aerospace, marine and mechanical engineering. Extensive efforts have been made to investigate vibration control of engineering structures. Generally, structural vibrations are related, not only to the location and the mode of vibration source, but also to the dynamic properties of the structure itself. The structural vibration can be reduced by suppressing or decreasing the excitation of the vibration source via techniques such as eliminating the source or appending a vibration isolation system, though, in most cases this is not possible. Therefore, there is a need to design and control the vibration performance of the structure itself.[1]

Periodic grid structures, each of which is composed of a number of identical unit cells connected together in a regular pattern, are widely used in aerospace engineering. The unit cell of grid structures can be any of various geometric shapes. Figure 1 shows hexagonal, rectangular and triangular lattices.[2] In the present paper, we investigate grids



(a) (b) (c)

Fig.1. Grid structures with different unit cell shapes: (a) hexagonal, (b) rectangular and (c) triangular.

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with rectangular unit cells. Grid structures are often used in design due to the high flexibility of manufacture and assembly, as well as high strength to weight performance.[2,3] Until now, investigations on grid structure have been focused on the static mechanical properties and technological design. However, very little work has been performed on the wave propagation characteristics for the purpose of structural vibration control.

In recent years, the propagation of elastic waves in phononic crystals (PCs) has received much attention. By introducing the philosophy of PCs into the design and the study of periodic structures, new phenomena appear which provide a new approach to vibration and noise control.[4−11] PCs have periodic composites with binary or multiple materials containing the so-called band gaps.[12] At the frequencies in the band gaps, elastic waves are all forbidden,[12−14] while the propagation of elastic waves may be directional and regional within the pass bands. It is desirable to design grid structures with internally tailored band gap properties and wave propagation characteristics based on the idea of PCs. Jeong[6] presented theoretical and experimental investigations of the flexural vibration of grid structures with periodic geometry parameters. The band gaps were predicted and confirmed, but the range of band gap that had been obtained was very narrow. In addition, the directional behaviour of wave propagation has not been validated experimentally.

The present work deals with grid structures each with periodic material parameters modeled after PCs. The vibrational band gaps and the phase constant surfaces of the grid structure are calculated by applying Bloch's theorem in conjunction with the finite element (FE) method. Analysis of the phase constant surfaces determines the directions and the regions of flexural wave propagation at assigned frequencies in the pass band. Then, the harmonic responses of the corresponding finite grid structures are calculated by using an FE software, which confirms the predictions about the band gaps and the phase constant surfaces. Finally, we prepare an experimental sample with this structure and test its dynamic behaviour. The experimental observations are shown to be in good agreement with the numerical predictions. This work provides new prospects for designing periodic structures for the technology of vibration reduction.

2. Finite element expression of the eigenvalue problem

The most widely used method to calculate band structures of PCs is based on the plane wave expansion (PWE) method,[15−17] for which the wave equations of the structure are needed. When a square grid structure is considered, it is usually not so easy to obtain the flexural wave equations unless the thickness of the structure is thin enough,[18] so there is a need to introduce new methods. In the present paper, we use the FE method along with the theory of periodic structures to obtain the dispersion relations. The basic idea of this method is to implement a discretization of unit cell of the structure by using the standard FE technique. Then, the cell FE model is replicated and coupled to the first original model through a set of boundary conditions that reflect the wave propagation condition as expressed by Bloch's theorem. This results in the formulation of an eigenvalue problem, which allows us to obtain the band structure and the phase constant surfaces by estimating the eigenvalues for a specific set of propagation constants.

Figure 2(a) illustrates the grid structure of square lattice with periodic material parameters. Figure 2(b) shows the unit cell and its interactions with adjacent cells. In this case, the grids consist of Al and PMMA glass, with lattice constant $a = 110$ mm, $l = 47.5$ mm and $d = 7.5$ mm. The thicknesses of Al and PMMA glass are $h_{\mathrm{Al}} = 15$ mm and $h_{\mathrm{Lu}} = 5$ mm, respectively. The unit cell is actually composed of 4 beams. Each beam can be approximated as a Euler-Bernoulli beam. The material parameters of Al are $\rho_{\mathrm{Al}} = 2600$ kg/m^3, $E_{\mathrm{Al}} = 8 \times 10^{10}$ Pa and $\sigma_{\mathrm{Al}} = 0.34$, and the material parameters of PMMA glass are $\rho_{\mathrm{Lu}} = 1200$ kg/m^3, $E_{\mathrm{Lu}} = 4 \times 10^9$ Pa and $\sigma_{\mathrm{Lu}} = 0.37$.

A general formulation for the unit cell equations of motion resulting from the FE discretization of its components can be expressed as follows:

$$(\boldsymbol{K} - \omega^2 \boldsymbol{M})\boldsymbol{q} = \boldsymbol{F}, \tag{1}$$

where ω is the frequency of wave propagation, $\boldsymbol{q}$ and $\boldsymbol{F}$ are the vectors containing generalized nodal displacements and forces of the cell, respectively, and can be written as $\boldsymbol{q} = \left\{ \boldsymbol{q}_{\mathrm{L}}\ \boldsymbol{q}_{\mathrm{R}}\ \boldsymbol{q}_{\mathrm{B}}\ \boldsymbol{q}_{\mathrm{T}}\ \boldsymbol{q}_{\mathrm{I}} \right\}^{\mathrm{T}}$ and $\boldsymbol{F} = \left\{ \boldsymbol{F}_{\mathrm{L}}\ \boldsymbol{F}_{\mathrm{R}}\ \boldsymbol{F}_{\mathrm{B}}\ \boldsymbol{F}_{\mathrm{T}}\ \boldsymbol{F}_{\mathrm{I}} \right\}^{\mathrm{T}}$. The notations L, R, B, T and I denote left, right, bottom, top and internal nodes of the unit cell, respectively. $\boldsymbol{K}$ and $\boldsymbol{M}$ are the

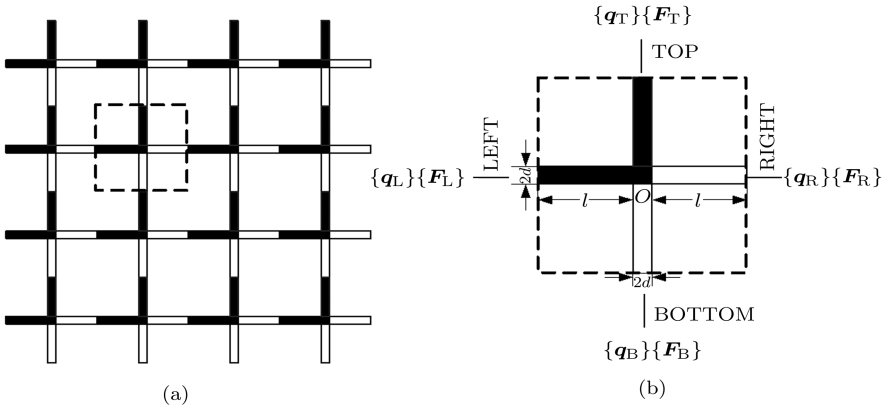


Fig.2. (a) Schematic of periodic grid structure and (b) a unit cell and its interactions with adjacent cells.

assemblies of element mass and stiffness matrices of the cell[19]

$$\boldsymbol{K}=\sum_e \boldsymbol{K}^e \quad \text{and} \quad \boldsymbol{M}=\sum_e \boldsymbol{M}^e \tag{2}$$

with

$$\boldsymbol{K}^e=\boldsymbol{T}^{\mathrm{T}}\boldsymbol{K}_{\mathrm{l}}^e\boldsymbol{T} \quad \text{and} \quad \boldsymbol{M}^e=\boldsymbol{T}^{\mathrm{T}}\boldsymbol{M}_{\mathrm{l}}^e\boldsymbol{T}, \tag{3}$$

where $\boldsymbol{K}^e$ and $\boldsymbol{M}^e$ denote respectively the element stiffness and mass matrices in the global reference system, respectively, while $\boldsymbol{K}_{\mathrm{l}}^e$ and $\boldsymbol{M}_{\mathrm{l}}^e$ are the matrices in the local reference system, and $\boldsymbol{T}$ is the rotation matrix relating the local reference system to the global reference system.

According to Bloch's theorem, the generalized displacements and forces in the cell interfaces are related to each other by[6]

$$\boldsymbol{q}_{\mathrm{R}}=\mathrm{e}^{\mathrm{i}q_x a}\boldsymbol{q}_{\mathrm{L}}, \quad \boldsymbol{q}_{\mathrm{T}}=\mathrm{e}^{\mathrm{i}q_y a}\boldsymbol{q}_{\mathrm{B}} \tag{4}$$

and

$$\boldsymbol{F}_{\mathrm{R}}=-\mathrm{e}^{\mathrm{i}q_x a}\boldsymbol{F}_{\mathrm{L}}, \quad \boldsymbol{F}_{\mathrm{T}}=-\mathrm{e}^{\mathrm{i}q_y a}\boldsymbol{F}_{\mathrm{B}}, \tag{5}$$

where q_k $(k=x,\, y)$ is the wave vector, and $q_k a$ is the phase constant.

Equations (4) and (5) may be combined into matrix forms as follows:

$$\boldsymbol{q}=\boldsymbol{A}q_r \quad \text{and} \quad \boldsymbol{F}=\boldsymbol{B}F_r, \tag{6}$$

where $\boldsymbol{q}_r=\{\,\boldsymbol{q}_{\mathrm{L}}\ \boldsymbol{q}_{\mathrm{B}}\ \boldsymbol{q}_{\mathrm{I}}\,\}^{\mathrm{T}}$, $\boldsymbol{F}_r=\{\,\boldsymbol{F}_{\mathrm{L}}\ \boldsymbol{F}_{\mathrm{B}}\ \boldsymbol{F}_{\mathrm{I}}\,\}^{\mathrm{T}}$, and $\boldsymbol{A}$ and $\boldsymbol{B}$ are defined as

$$\boldsymbol{A}=\begin{bmatrix} \boldsymbol{I}_{\mathrm{L}} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{I}_{\mathrm{L}}\mathrm{e}^{\mathrm{i}q_x a} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{\mathrm{B}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{\mathrm{B}}\mathrm{e}^{\mathrm{i}q_y a} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_{\mathrm{I}} \end{bmatrix} \quad \text{and} \quad \boldsymbol{B}=\begin{bmatrix} \boldsymbol{I}_{\mathrm{L}} & \boldsymbol{0} & \boldsymbol{0} \\ -\boldsymbol{I}_{\mathrm{L}}\mathrm{e}^{\mathrm{i}q_x a} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{\mathrm{B}} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{I}_{\mathrm{B}}\mathrm{e}^{\mathrm{i}q_y a} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_{\mathrm{I}} \end{bmatrix}. \tag{7}$$

Substituting Eq.(6) into Eq.(1) yields

$$(\boldsymbol{K}-\omega^2\boldsymbol{M})\boldsymbol{A}q_r=\boldsymbol{B}F_r. \tag{8}$$

Pre-multiplying Eq.(8) by $\boldsymbol{A}^{\mathrm{H}}$, with H denoting a complex conjugate transpose, and assuming $\boldsymbol{F}_{\mathrm{I}}=\boldsymbol{0}$, give

$$(\boldsymbol{K}_r-\omega^2\boldsymbol{M}_r)\boldsymbol{q}_r=\boldsymbol{0}, \tag{9}$$

where $\boldsymbol{K}_r=\boldsymbol{A}^{\mathrm{H}}\boldsymbol{K}\boldsymbol{A}$, and $\boldsymbol{M}_r=\boldsymbol{A}^{\mathrm{H}}\boldsymbol{M}\boldsymbol{A}$.

Equation (9) forms a standard eigenvalue problem, whose solution constructs the band structure of wave frequency ω for an assigned set of wave vectors $(q_x,\, q_y)$. Varying q_x and q_y from $-\pi/a$ to π/a generates the phase constant surfaces in the first Brillouin zone. Figure 3 shows the band structure (a) and the

phase constant surfaces (b) for the given structure. There are two complete band gaps within 3500 Hz, indicated by the shaded area in Fig.3(a). The frequency ranges are 270–360 Hz and 1200–3000 Hz.

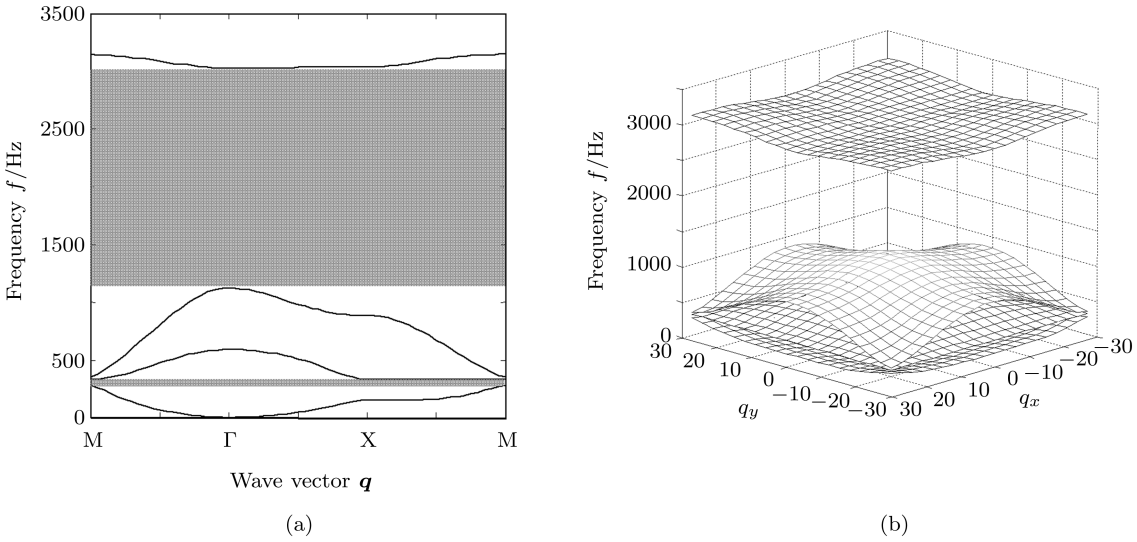


Fig.3. Band structure (a) and phase surfaces (b) of a periodic grid structure.

3. Directions of wave propagation

The group speed of wave motion in a two-dimensional periodic structure is given by[6]

$$c_{gx} = a_x \frac{\partial \omega}{\partial q_x} \quad \text{and} \quad c_{gy} = a_y \frac{\partial \omega}{\partial q_y}, \tag{10}$$

where c_{gx} and c_{gy} denote the group speed components along the x and the y directions, respectively, and a_x and a_y refer to the lattice constants along the x and the y directions, respectively. For undamped structures, the group speed is equal to the speed of propagation of the vibrational energy. Based on Eq.(10), one can find that the direction of wave propagation in a two-dimensional periodic grid structure can be estimated by taking the normal to the iso-frequency contour lines of the phase constant surfaces. Then, the direction of flexural wave propagation at a given frequency can be determined by evaluating the corresponding normal angle.

The first phase constant surface and its contour plot are illustrated in Figs.4(a) and 4(b), respectively.

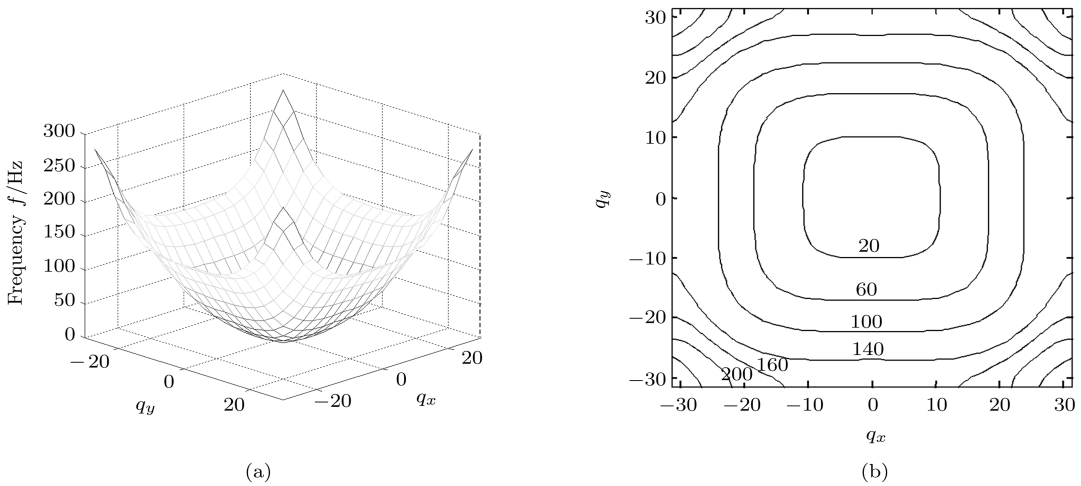


Fig.4. The first phase constant surface (a) and its contour plot (b).

The iso-frequency contour line for a frequency of 200 Hz is shown in Fig.5(a), where the arrows indicate normals to the contours. For a given point on the contour, let θ be the angle between the direction of the normal and the positive direction of x-axis, which means θ varies within a range of 0°-to-360°. N points that are evenly distributed on the contour are chosen, and N θ-values are obtained. The θ-values are interspaced with each other at intervals of 5° for the full range of 0°-to-360°, thereby yielding a total of 72 θ-values, which are plotted as a function of angle in a polar plot. The numbers 5, 10, 15 and 20 along the direction outward from the centre in the polar plot are the numbers of θ values each located within a specified 5° range. Figure 5(b) indicates that the main directions of the flexural wave propagation at 200 Hz are along the lines of ±45°.

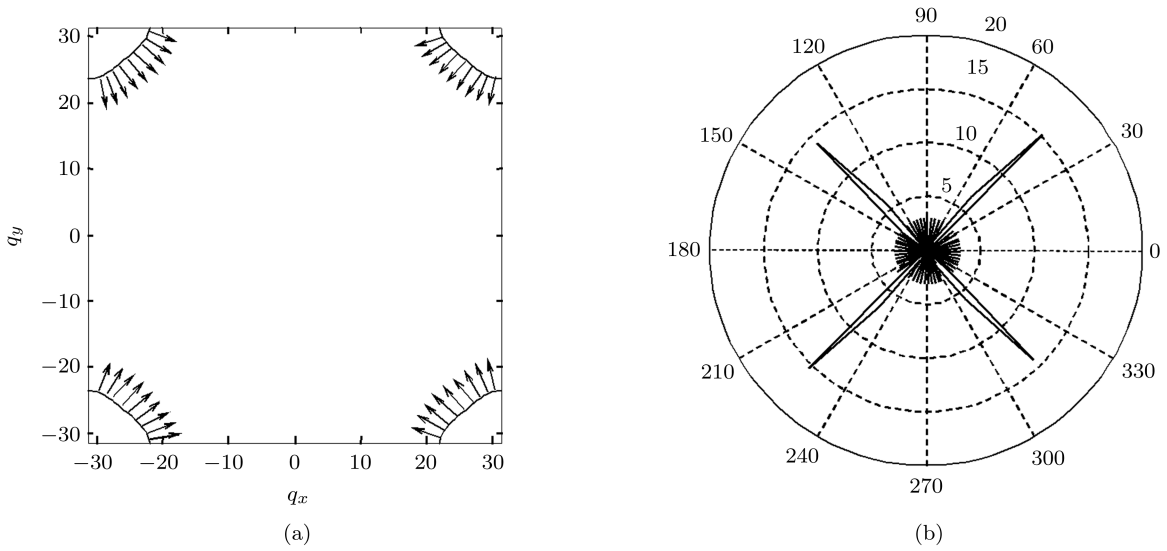


Fig.5. Plots of contour line and directional propagation at a frequency of 200 Hz, where panel (a) is for the contour line with directional vector and panel (b) is for the polar plot of the directional propagation.

4. Response to harmonic loading

The predictions from the analysis of the phase constant surface and its contour lines can be readily verified through the computation of harmonic response of a finite structure of a 2D periodic grid structure.

Figure 6(a) shows an FE model of a finite periodic grid structure with 11×11 unit cells. The point acceleration excitation perpendicular to the structure is applied to the lower right corner, and the acceleration responses at the upper left corner and the upper right corner can be calculated by using a finite element method (FEM) as shown in Fig.6(b). There occur two drops in the frequency response function, separately in frequency ranges of 230–500 Hz and 1145–3340 Hz, which correspond to the band gaps. It is obtained that the frequency response function is in good accord with the band structure.

Figure 7 shows the harmonic response of a periodic grid structure according to the FEM for various frequencies, with the point excitation applied to the lower right corner of the structure. Figure 7(a) shows how the flexural wave freely propagates in all directions at a frequency of 150 Hz (in pass band). Figure 7(b) exhibits that the flexural wave is located at the excitation point at a frequency of 350 Hz (in band gap). Figure 7(c) indicates that the wave propagates along –45° at a frequency of 211 Hz.

Due to boundary effects for a finite size sample, the characteristics of directional propagation of the flexural wave occur only in a very narrow frequency range, while for an infinite structure such characteristics can be found in a relatively wide frequency range.

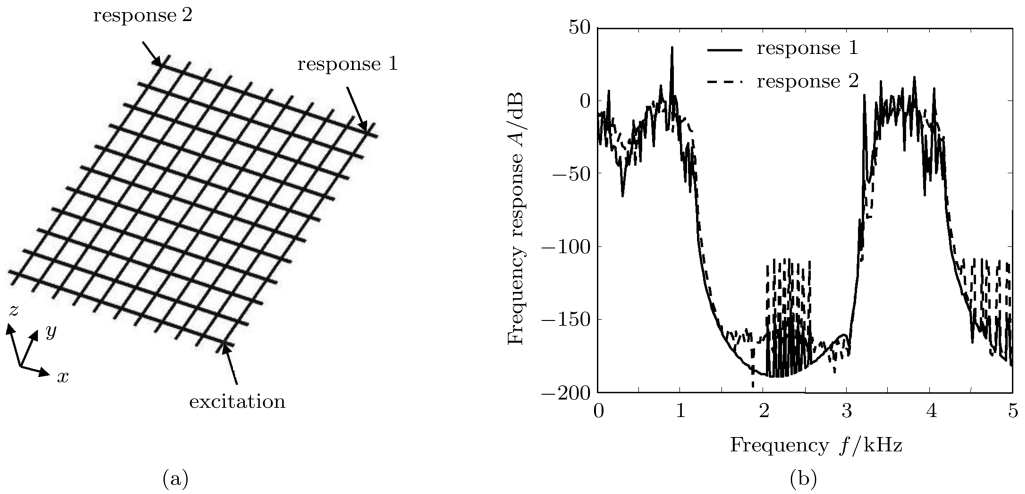


Fig.6. Finite element model (a) and frequency response function curves (b) of periodic grid structure.

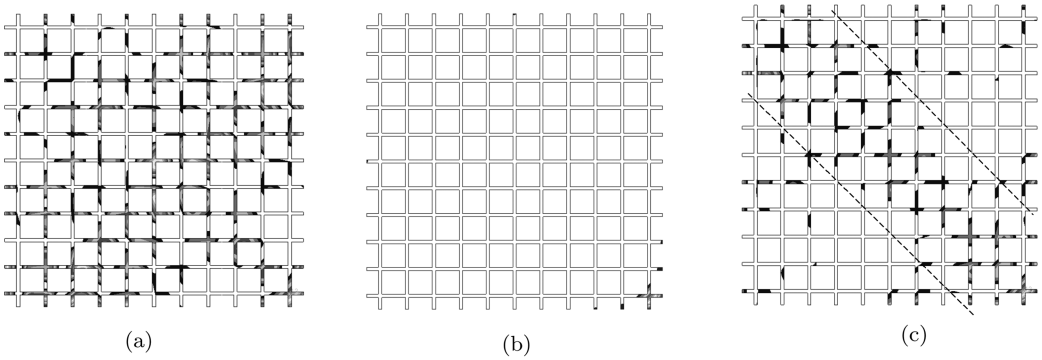


Fig.7. Harmonic responses of the periodic grid structure at frequencies of 150 (a), 350 (b) and 211 Hz (c).

5. Experiment and results

To verify the theoretical analysis and simulation results, an experimental sample was designed, whose material and dimension were the same as those required by the finite element model described above. Figure 8 shows a schematic of the experimental setup. Here, the main experimental instrument is a Polytec PSV-400-M4 Scanning Vibrometer. An excitation signal produced by a data management system is input into the vibration shaker, which transmits vibrations into the lower right corner. A laser scanning head based on the Doppler effect is used to measure displacements and/or speeds of the sample, and it is able to offer the experimental frequency response function and the vibration characteristics of the periodic grid structure through data processing.

Figure 9 shows the experimental setup. The periodic grid structure is hung with an elastic string to simulate free boundary conditions.

Figure 10 shows the frequency response function curves. A point acceleration excitation perpendicular to the structure is applied to the lower right corner, and the acceleration response is measured at the upper left corner. As seen in Fig.10, there occur two drops in the frequency response function curve, separately in frequency ranges of 220–500 Hz and 850–3700 Hz, which correspond to the band gaps shown in Fig.3(a) and simulation results in Fig.6(b) respectively.

The Polytec PSV-400-M4 Scanning Vibrometer is able to record the vibration speed and phase information of each measured point on the grid structure, which can describe the propagation characteristics of the flexural wave shown in Figs.11–13. The black points in these figures represent the positive direction of velocity along the z-axis, while the white points denote the negative direction of velocity. The darker colour refers to the bigger magnitude of velocity at the measured point, while the gray represents the velocity magnitude that is close to 0.

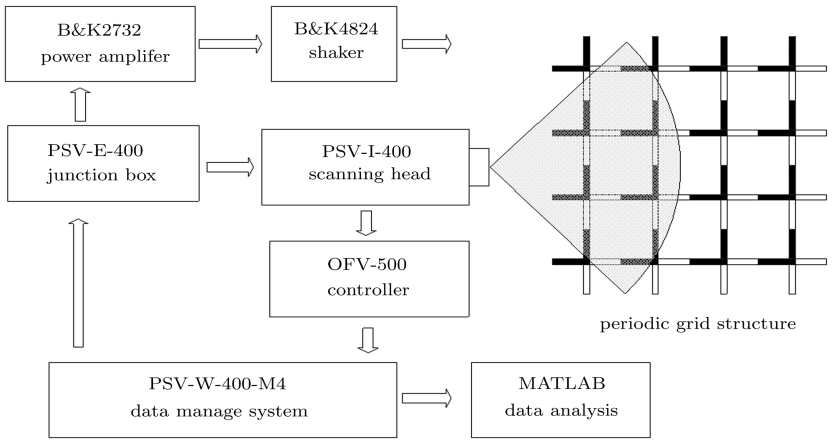


Fig.8. Schematic block diagram of the experimental setup.

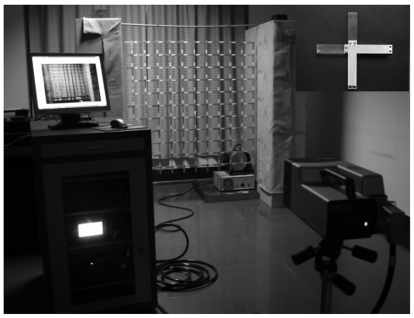


Fig.9. The experimental setup.

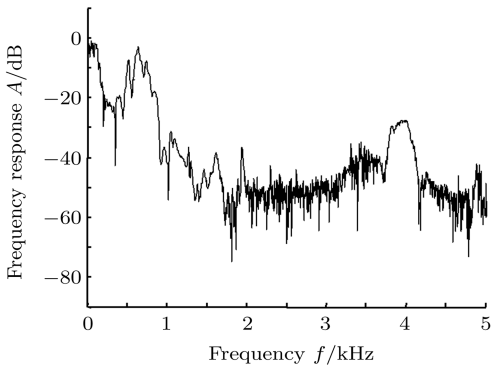


Fig.10. Measured frequency response function curves of a periodic grid structure.

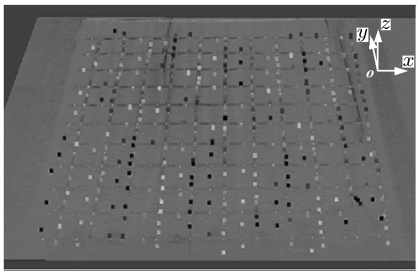


Fig.11. Flexural wave freely propagating in all directions at a frequency of 115 Hz.

Figure 11 shows the propagation characteristics of the periodic grid structure at a frequency of 115 Hz (in pass band). In the figure, there is a significant black-white colour conversion at the measured point, and this phenomenon still exists at different time moments in one vibration period. This indicates that the flexural wave freely propagates in all directions at such a frequency.

Figure 12 exhibits the propagation characteristics of the structure at a frequency of 350 Hz (in band gap). In the figure, significant black-white colour conversion occurs only near the excitation point, while the colour of measured point remains gray. This indicates that the flexural wave is localized at the excitation point.

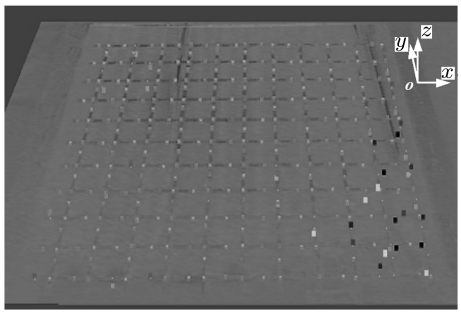


Fig.12. Flexural wave located at the excitation point at a frequency of 350 Hz.

Figure 13 shows the propagation characteristics of the structure at a frequency of 210 Hz (in pass gap). In the figure, significant black-white colour conversion occurs along about –45°, and the other measured points remain gray. This indicates that the flexural wave propagates along –45°.

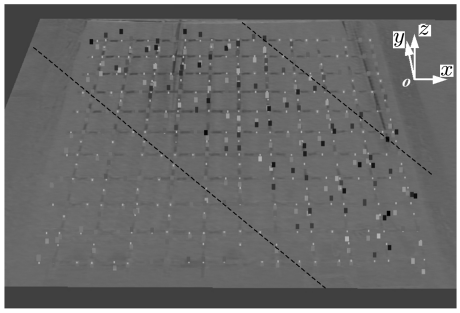


Fig.13. Flexural wave propagating along –45° at a frequency of 210 Hz (in pass gap).

The test results of the flexural wave propagation characteristics show that they are in good agreement with the FE simulation results. However, some discrepancies can be observed. This is because different materials are jointed by bolts in the experimental sample, while the structure is assumed to be continuous in the FE simulation model.

6. Conclusions

In the present work the investigations are focused on the characteristics of flexural wave propagation in grid structures with periodic material parameters. The analysis is performed by applying Bloch's theorem in conjunction with the FEM, which yields the band structure and phase constant surfaces for a grid structure with specific size and material parameters. The directions and the regions of wave propagation at particular frequencies within the pass band are studied through the analysis of the phase constant surfaces. The numerical predictions are confirmed by simulating the harmonic responses of the corresponding finite grid structure with using finite element software. In addition, the numerical analysis is also validated by testing an experimental sample.

The studies demonstrate the band gap properties of a grid structure with periodic materials. The frequency ranges of vibration attenuation of the finite grid structure agree with the calculated band gaps. Flexural waves are localized in the area around the point of excitation at the frequencies in the band gaps, while the flexural waves can freely propagate through the structure along all directions within the pass band, but they indicate strong directional behaviour at particular frequencies.

Grid structures are widely used in the field of aerospace engineering, for which vibration control is of great importance. These studies provide guidelines for the design of such periodic structures to attenuate vibration. Here, we have studied grids with rectangular unit cells; future investigations will deal with extending of the present analysis and the integration of the effects of cell shape, the material and the geometric parameters and the boundary conditions. Optimizing the design of such structures in order to take full advantage of their unique dynamic properties is highly desirable in engineering applications for vibration reduction.

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