Vol 18 No 6, June 2009
1674-1056/2009/18(06)/2398-06

Chinese Physics B

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Mode-field half-widths of Gaussian approximation for the fundamental mode of two kinds of optical waveguides*

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(Received 23 September 2008; revised manuscript received 15 January 2009)

This paper analyzes the characteristic of matching efficiency between the fundamental mode of two kinds of optical waveguides and its Gaussian approximate field.Then, it presents a new method where the mode-field half-width of Gaussian approximation for the fundamental mode should be defined according to the maximal matching efficiency method. The relationship between the mode-field half-width of the Gaussian approximate field obtained from the maximal matching efficiency and normalized frequency is studied; furthermore, two formulas of mode-field half-widths as a function of normalized frequency are proposed.

Keywords: mode-field half-width, Gaussian approximation, normalized frequency, optical waveguide

PACC: 4282, 4280L

1. Introduction

As one of the most important parameters for optical waveguides,[1,2] the mode-field half-width is closely related to various transmission characteristics, such as the splice loss,[3] micro-bending loss,[4,5] two waveguides and source-to-fiber coupling efficiency[6,7] and so on. Moreover, because the concept of the Gaussian beam and the computing mean of Fourier transform for the Gaussian function are classical scientific ideas, the Gaussian approximation[8−13] for the fundamental mode of optical waveguides applies widely. The key is how to obtain the mode-field half-width. Thus, it always attracts research interest.

There are two kinds of mode-field half-width based on two well-known definitions introduced by Petermann.[14,15] However, there are some differences. It is difficult for us to take one of them as the mode-field half-width of Gaussian approximation so that the Gaussian field can be more closed to its principal field. Furthermore, due to the characteristic of normalized frequency itself, the relation between the mode-field half-width and normalized frequency is widely investigated, but formulas for the two kinds of mode-field half-widths as a function of normalized frequency are not found. In view of this, Marcuse[6,16] has presented two well-known approximate formulas and the authors of this paper have presented an exponential model to describe their relation[17,18] . But further research indicates that they are not ideal, especially as the deviation in the formula provided by Marcuse for planar waveguides is large.

The research in this paper aims at analyzing the mode-field half-width of Gaussian approximation and presents more accurate calculated formulas. Firstly, this paper analyzes the characteristic of the matching efficiency between the fundamental mode and its Gaussian approximate field, then it presents a new method where the mode-field half-width is defined based on the maximal matching efficiency method, and the relationship between the mode-field half-width of the Gaussian approximation for the optical waveguide and normalized frequency is analyzed in detail based on the maximal matching efficiency method. Lastly, two formulas for the mode-field half-width as a function of the normalized frequency are proposed by solving linear equations. These conclusions may provide theoretical support for designing integrated

*Project supported by Natural Science Foundation of the Department of Science & Technology of Fujian Province of China (Grant No 2007F5040).

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optical devices.

2. The mode-field half-width of the planar symmetric step index optical waveguide

As the fundamental mode in the planar symmetric step refractive index optical waveguide, it can be described as follows:[19]

$$\Psi_{\text{eigen}}(x) = \cos\left(\frac{Ux}{a}\right), \qquad |x| \le a, \tag{1}$$

$$\Psi_{\text{eigen}}(x) = \cos(U)\exp\left[-W\left(\left|\frac{x}{a}\right| - 1\right)\right], \quad |x| > a, \tag{2}$$

where $U = a[(k_0 n_1)^2 - \beta^2]^{1/2}$ and $W = a[\beta^2 - (k_0 n_2)^2]^{1/2}$ are the normalized standing wave parameter and the normalized evanescent wave parameter, respectively, $k_0 = 2\pi/\lambda$ is the wave number in a vacuum, β is the propagation constant, λ is the wavelength of the electromagnetic wave in a vacuum, a is the half thickness of the core layer, and n_1 and n_2 are the refractive indices of the core and cladding, respectively. For convenience of analysis and without discounting validity, the $\exp[\mathrm{i}(\beta z - \omega t)]$ item in the above formula is omitted.

For the sake of convenience, the field distribution of the TE_0 mode is often approximated by the Gaussian shape

$$\Psi(x) = \exp\left(-\frac{x^2}{\omega_j^2}\right), \tag{3}$$

where ω_j is the mode-field half-width, the suffix j refers to the second moment, differential operator, Marcuse's and the Gaussian mode-field half-width, respectively, and they denote the relationship to the definitions of mode-field half-width. The second moment and the differential operator mode-field half-width are defined, respectively, as follows:

$$\omega_{\text{SM}} = 2\left[\frac{\int_{-\infty}^{\infty} |\Psi(x)|^2 x^2 \mathrm{d}x}{\int_{-\infty}^{\infty} |\Psi(x)|^2 \mathrm{d}x}\right]^{1/2}, \tag{4}$$

$$\omega_{\text{DO}} = \left[\frac{\int_{-\infty}^{\infty} |\Psi(x)|^2 \mathrm{d}x}{\int_{-\infty}^{\infty} \left|\frac{\mathrm{d}\Psi(x)}{\mathrm{d}x}\right|^2 \mathrm{d}x}\right]^{1/2}. \tag{5}$$

As for the TE_0 mode, its second moment mode-field half-width ω_{SM} and differential operator mode-field half-width ω_{DO} are [20]

$$\omega_{\text{SM}} = a\left[2 + \frac{2}{W} + \frac{2}{W^2} - \frac{2}{U^2} - \frac{2W}{3(1+W)}\right]^{1/2}, \tag{6}$$

$$\omega_{\text{DO}} = a\left[\frac{1+W}{U^2 W}\right]^{1/2}. \tag{7}$$

Marcuse has presented a formula of the mode-field half-width as a function of normalized frequency,[6]

$$\omega_{\text{M}} = a\left[0.00460313 + \frac{1.22083}{\sqrt{V}} + \frac{1.53597}{V^3} - \frac{0.146705}{V^5}\right]. \tag{8}$$

For example, we set $n_1 = 1.4493$, $n_2 = 1.4440$, $\lambda = 1.55$ μm and $a = 3$ μm, then $V = 1.5053$, $U = 0.9163$, $W = 1.1942$, $\omega_{\text{SM}} = 4.5815$ μm, $\omega_{\text{DO}} = 4.4380$ μm and $\omega_{\text{M}} = 4.2929$ μm.

Numerical data show that there are some differences. It is difficult for us to take one of them as the mode-field half-width so that the Gaussian field can be more closed to its principal field. Furthermore, our research indicates that they are not optimal based on matching efficiency, especially, large deviation exists in ω_{M}. So we present a new definition of the mode-field half-width. That is, the mode-field half-width is defined based on the maximal matching efficiency method and is named the Gaussian mode-field half-width ω_{G}.

The matching efficiency can be evaluated through an overlap integral,[21]

$$\eta = \frac{\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Psi_1(x,y)\,\Psi_2(x,y)\mathrm{d}x\mathrm{d}y\right]^2}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Psi_1^2(x,y)\mathrm{d}x\mathrm{d}y \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Psi_2^2(x,y)\mathrm{d}x\mathrm{d}y}, \tag{9}$$

where $\Psi_1(x)$ and $\Psi_2(x)$ refer to the eigenmode field and its Gaussian field, respectively.

For the planar symmetric waveguide, the two dimensions of Eq.(9) can be simplified to one dimension. Substituting Eqs.(1)–(3) into Eq.(9), we can obtain the matching efficiency η between $\Psi_{\text{eigen}}(x)$ and $\Psi(x)$ displayed in Fig.1. It shows that η varies with increasing ω and when the matching efficiency reaches the maximum $\eta_{\max} = 99.66\%$, the mode-field half-width is only defined as the Gaussian mode-field half-width ω_{G}. Compared with $\omega_{\text{SM}} = 4.5815$ μm,

$\omega_{\rm DO} = 4.4380\ \mu$m and $\omega_{\rm M} = 4.2929\ \mu$m, it is clear that they have a certain deviation from $\omega_{\rm G} = 4.5000\ \mu$m, especially, $\omega_{\rm M}$ is significantly different to $\omega_{\rm G}$. So $\omega_{\rm G}$ is optimal only.

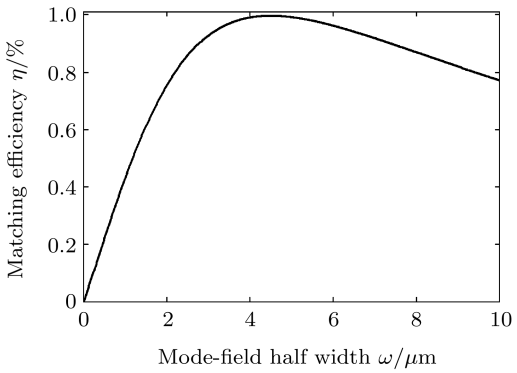


Fig.1. The characteristic curve of matching efficiency versus the mode-field half-width.

Through a similar procedure, a relational curve between the Gaussian mode-field half-width $\omega_{\rm G}$ and normalized frequency V is obtained from the maximal matching efficiency method as the solid line shown in Fig.2. It is seen that $\omega_{\rm G}$ decreases with increasing V.

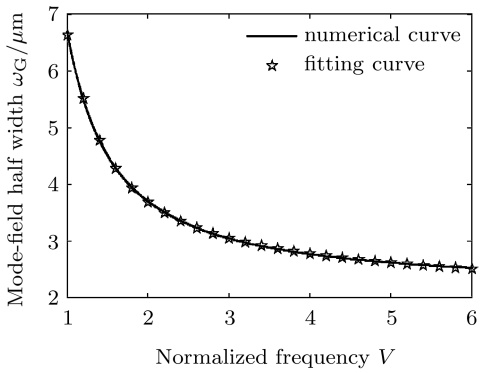


Fig.2. The relational curve of the mode-field half-width as a function of normalized frequency V.

For convenience of calculation and analysis, we would like to establish an equation between the Gaussian mode-field half-width and normalized frequency. An approach of solving linear equations is presented based on the mathematical model as follows:

$$\omega = A + \frac{B}{\sqrt{V}} + \frac{C}{V^3} + \frac{D}{V^5}, \tag{10}$$

where A, B, C and D are coefficients. In order to solve Eq.(10), we take four values of V into account and assume that they are V_1, V_2, V_3 and V_4, respectively. Accordingly, four values of ω_1, ω_2, ω_3 and ω_4 are taken for ω. Consequently, equation (10) can be translated into a matrix form as

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \begin{bmatrix} 1 & V_1^{-1/2} & V_1^{-3} & V_1^{-5} \\ 1 & V_2^{-1/2} & V_2^{-3} & V_2^{-5} \\ 1 & V_3^{-1/2} & V_3^{-3} & V_3^{-5} \\ 1 & V_4^{-1/2} & V_4^{-3} & V_4^{-5} \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}, \tag{11}$$

where coefficients A, B, C and D can be obtained according to solving matrix equation (11).

Therefore, we set V to be 1.0716, 1.5094, 2.4764 and 5.6249. Then $\omega_{\rm G}$ is 6.1880, 4.4850, 3.3020 and 2.5480 accordingly. Substituting these data into Eq.(11), we can obtain a formula of the Gaussian mode-field half-width as a function of the normalized frequency as follows:

$$\omega_{\rm G} = 1.4826 + \frac{2.4746}{\sqrt{V}} + \frac{3.9568}{V^3} - \frac{1.2726}{V^5}. \tag{12}$$

From Eq.(12), the pentagram marker is displayed in Fig.2 and it is clear that it is in excellent agreement with the solid line. Furthermore, we calculate the relative error between numerical data and those from Eq.(12) to explain the accuracy of Eq.(12). The relative error based on its definition can be expressed as follows:

$$\delta = \frac{|\beta - \alpha|}{\alpha} \times 100\%, \tag{13}$$

where α is the numerical datum from the maximal matching method and β is the calculated result by Eq.(12).

Further study has shown that the relative error varies with increasing normalized frequency, but it varies within 0.6% as V changes from 1.0190 to 5.8810. Therefore, equation (12) is accurate within a reasonable parameter range.

For the purpose of comparison, we calculate the matching efficiency $\eta(\Psi_{\rm eigen}, \Psi_{\rm G})$ and $\eta(\Psi_{\rm eigen}, \Psi_{\rm M})$, where $\Psi_{\rm G}(x) = \exp(-x^2/\omega_{\rm G}^2)$ and $\Psi_{\rm M}(x) = \exp(-x^2/\omega_{\rm M}^2)$. The results are displayed in Fig.3.

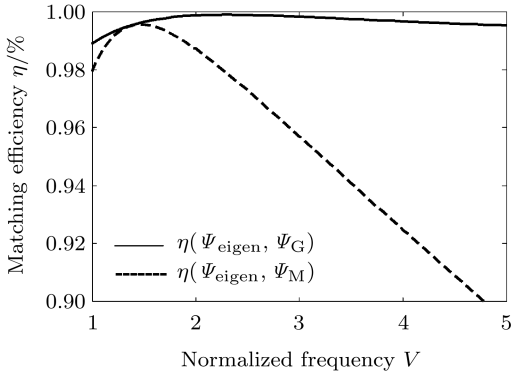


Fig.3. The comparative curve of two kinds of matching efficiency.

As shown in fig.3, we can see that the two lines are significantly different. Calculative results indicate that the matching efficiency $\eta(\Psi_{\text{eigen}}, \Psi_{\text{M}})$ is not good except for a small range $1.17 < V < 1.88$, in that $\eta(\Psi_{\text{eigen}}, \Psi_{\text{M}}) \geq 99\%$. However, the matching efficiency $\eta(\Psi_{\text{eigen}}, \Psi_{\text{G}})$ is satisfying: it always satisfies $\eta(\Psi_{\text{eigen}}, \Psi_{\text{G}}) \geq 99\%$ within $1.04 < V < 12$. Thus, ω_{G} is more accurate than ω_{M} if they are used for the mode-field half-width in the Gaussian approximation.

3. The mode-field radius of circular-symmetric step index optical fiber

As for circular-symmetric step index optical fiber, the $\exp[\mathrm{i}(\beta z - \omega t)]$ item in the following formula is omitted. The normalized eigenmode-field distribution of the LP_{01} mode can be expressed as follows under weakly guiding approximation:[19]

$$\Psi_{\text{eign}}(r) = J_0\left(U\frac{r}{a}\right), \qquad r \leq a, \tag{14}$$

$$\Psi_{\text{eign}}(r) = \frac{J_0(U)}{K_0(W)} K_0\left(W\frac{r}{a}\right), \qquad r > a. \tag{15}$$

All the same, the field distribution of the LP_{01} mode is often expressed approximately by the Gaussian field,

$$\Psi(r) = \exp\left(-\frac{r^2}{\omega_j^2}\right), \tag{16}$$

where ω_j is the mode-field radius, the suffix j is the second moment, differential operator, Marcuse's and the Gaussian mode-field radius respectively, and they denote the relationship to the definitions of the mode-field radius.

The problem is how to obtain an accurate mode-field radius. There is the second moment mode-field radius ω_{SM} and differential operator mode-field radius ω_{DO} too,[22]

$$\omega_{\text{SM}} = \frac{\sqrt{6}a}{3}\left[1 - \frac{2}{U^2} + \frac{2}{W^2} + \frac{2J_0(U)}{UJ_1(U)}\right]^{1/2}, \tag{17}$$

$$\omega_{\text{DO}} = \frac{\sqrt{2}aJ_1(U)}{WJ_0(U)}. \tag{18}$$

In the same way, Marcuse has presented a formula of the mode-field radius as a function of the normalized frequency ω_{M},[16]

$$\omega_{\text{M}} = a[0.65 + 1.619V^{-3/2} + 2.879V^{-6}]. \tag{19}$$

Taking an example, we assume that the basic parameters of the optical fiber are as follows: the refractive index of the core layer $n_1 = 1.4569$, the refractive index of the cladding layer $n_2 = 1.4440$, the core layer radius $a = 3$ μm, the wavelength in a vacuum $\lambda = 1.55$ μm, then the normalized frequency $V = 2.3539$, the normalized standing wave parameter $U = 1.6335$ and the normalized evanescent wave parameter $W = 1.6949$.

From Eqs.(17)–(19), we can obtain $\omega_{\text{SM}} = 3.3576$ μm, $\omega_{\text{DO}} = 3.2878$ μm and $\omega_{\text{M}} = 3.3457$ μm. They are different to one other and, furthermore, referring to the matching efficiency between the LP_{01} mode and its Gaussian field, we find that they are also not optimal.

Substituting Eqs.(14), (15) and (16) into Eq.(9), we can obtain a characteristic curve of the matching efficiency η between the field of the LP_{01} mode and its Gaussian field. It is displayed in Fig.4.

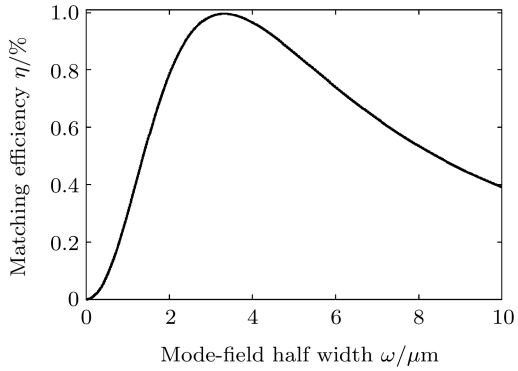


Fig.4. The characteristic curve of matching efficiency versus the mode-field radius.

It is obvious that the matching efficiency η varies with increasing ω and η reaches the maximum $\eta_{\max} = 99.63\%$ while $\omega_{\text{G}} = 3.3193$ μm. Compared with the other three kinds of mode-field radius $\omega_{\text{SM}} = 3.3576$ μm, $\omega_{\text{DO}} = 3.2878$ μm and $\omega_{\text{M}} = 3.3457$ μm, we can draw the conclusion that they are not optimal: only if the matching efficiency is maximal, the mode-field radius is optimal.

We can obtain all the numerical data of the Gaussian mode-field radius ω_{G} along with the increase of V based on the maximal matching efficiency method. The solid line in Fig. 5 shows that the Gaussian mode-field radius ω_{G} decreases with increasing normalized frequency V, but there was not a direct equation between them.

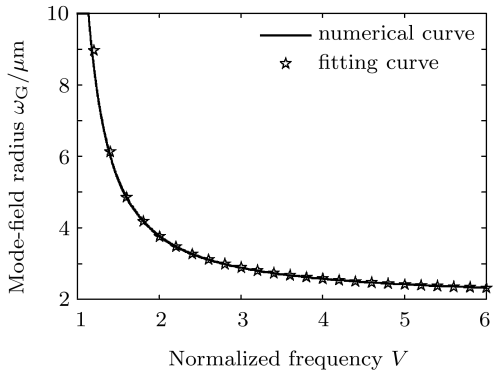


Fig.5. The relational curve of the mode-field radius as a function of normalized frequency.

In view of this, the method mentioned above is taken based on another mathematical model as follows:

$$\omega = A + \frac{B}{V^{3/2}} + \frac{C}{V^6}, \tag{20}$$

where A, B and C are coefficients. All the same, in order to obtain the coefficients, three values of V_1, V_2 and V_3 must be taken into account, and ω takes ω_1, ω_2 and ω_3 accordingly. Consequently, equation (20) can be translated into a matrix form,

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} 1 & V_1^{-3/2} & V_1^{-6} \\ 1 & V_2^{-3/2} & V_2^{-6} \\ 1 & V_3^{-3/2} & V_3^{-6} \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix}, \tag{21}$$

where coefficients A, B and C can be obtained according to solving matrix equation (21).

Therefore, we set V to be 1.6220, 2.2480 and 3.8200, and then ω_G is 4.7700, 3.4300, and 2.6200 accordingly. Substituting these data into Eq.(21), we get

$$\omega_G = 2.0145 + \frac{4.4955}{V^{3/2}} + \frac{10.5491}{V^6}. \tag{22}$$

The pentagram marker is displayed in Fig.5 from Eq.(22). It is clear that it is in excellent agreement with the solid line. Numerical calculation has shown that the relative error fluctuates along with increase of the normalized frequency, but it varies within 0.5% as V changes from 1.5160 to 9.7750.

Furthermore, the matching efficiency $\eta(\Psi_{\text{eigen}}, \Psi_G)$ as well as $\eta(\Psi_{\text{eigen}}, \Psi_M)$ are given so as to explain the accuracy of Eq.(22), where $\Psi_G(r) = \exp(-r^2/\omega_G^2)$ and $\Psi_M(r) = \exp(-r^2/\omega_M^2)$. The results are displayed in Fig.6.

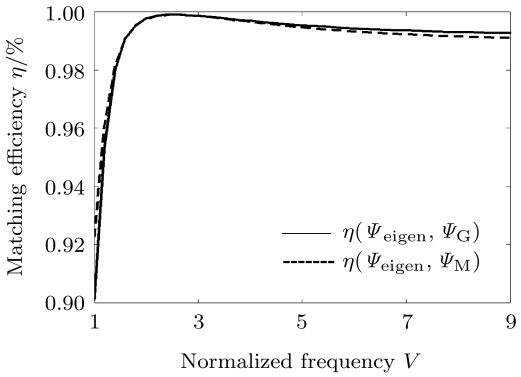


Fig.6. The comparative curve of two kinds of matching efficiency.

Figure 6 shows that there is some difference: the matching efficiency $\eta(\Psi_{\text{eigen}}, \Psi_G)$ is greater than $\eta(\Psi_{\text{eigen}}, \Psi_M)$ as a whole. Further study proves that only if $1.58 < V < 15.53$, $\eta(\Psi_{\text{eigen}}, \Psi_M) \geq 99\%$, however, the matching efficiency $\eta(\Psi_{\text{eigen}}, \Psi_G)$ always satisfies $\eta(\Psi_{\text{eigen}}, \Psi_G) \geq 99\%$ while $V > 1.6$. Consequently, ω_G is more apt to be used for the mode-field radius than ω_M in the Gaussian approximation.

4. Conclusions

For the symmetric step index optical waveguide, we investigate the characteristic of the matching efficiency between the fundamental mode and its Gaussian approximate field by the numerical analysis method. Based on that, we present a new method that the mode-field half-width should be defined according to the maximal matching efficiency method and it is called the Gaussian mode-field half-width. The relationship between the Gaussian mode-field half-width obtained from the maximal matching efficiency and normalized frequency is analyzed. Furthermore, formulas for the Gaussian mode-field half-widths as a function of normalized frequency are introduced according to solving linear equations. Numerical data prove that they have high accuracy in a reasonable parameter range.

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