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The similarity of interactions between (3+1)D spatiotemporal optical solitons in both the dispersive medium with cubic-quintic nonlinearity and the saturable medium*

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By making use of the split-step Fourier method, this paper numerically simulates dynamical behaviors, including repulsion, fusion, scattering and spiraling of colliding (3+1)D spatiotemporal solitons in both the dispersive medium with cubic-quintic and the saturable medium. Careful comparison of the colliding behaviors in these two media is presented. Although the origin of the nonlinearities is different in these two media, the obtained results show that the dynamical behaviors are very similar. This presents additional evidence to support the supposition of universality of interactions between solitons.

Keywords: spatiotemporal solitons, cubic-quintic nonlinearity, interactions
PACC: 4265S, 4265J, 4265T

1. Introduction

Solitons have attracted a great deal of attention for many decades because of their resilient, robust behaviors.[1] They arise in many areas of physics such as hydrodynamics,[2] nonlinear optics,[3] plasma physics[4] and so on.[5] Among many fascinating features of solitons, the particle-like feature appearing in the colliding process is striking. In many different systems, collisions between solitons are marvelously shown to follow the same principles.[6] As a matter of fact, one can view solitons as the bound states of their own induced potential well, thus collisions between solitons are just interactions between these bound states.[6] For this reason, the universality of collisions between solitons is thus an expectation.

Because of both the convenience in experimental studies and their potential applications, interactions between optical spatial solitons are extensively investigated.[6,7] To mention but a few, we note elastic collision in Kerr solitons[8,9] and almost elastic collision of solitons in the saturable medium[10] as well as some inelastic interactions such as fusion,[10,11] fission[12] and spiraling.[13] It is noticed that interactions between optical spatial solitons in photorefractive, quadratic and saturable media show similar features, independent of the self-trapping mechanism.[6]

More than a decade ago, spatiotemporal optical solitons were proposed.[14,15] (3+1)D spatiotemporal solitons (STSs) are nondiffracting and nondispersing wave packets localized in all transverse dimensions of space, as well as in time.[15] They are also called light bullets,[14] which will collapse in a Kerr medium.[14,16] To arrest the catastrophic collapse, various schemes have been proposed.[15] Usually, the saturable nonlinearity[17,18] and the cubic-quintic (CQ) nonlinearity[19,20] are used. The saturable medium in which the index change saturates with increasing intensity can support stable (3+1)D STSs[17,18] and interactions between (3+1)D STSs in this medium have also been investigated by numerical simulations.[21,22] In the scattering experiment, the colliding STSs demonstrate particle-like or billiard-ball-like behaviour.[22] In addition, it shows that (3+1)D STSs can exist in the CQ medium.[19,20] However, their interactions between STSs in this medium have received less attention. Since the spatial optical solitons in different media exhibit universality in

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interactions,[6] it is interesting to answer the following question: is this true for spatiotemporal optical solitons? In this paper, we investigate interactions such as repulsion, fusion, scattering and spiraling between (3+1)D STSs in the CQ medium by numerical simulations and compare the results with those in the saturable medium.

2. Theoretical mode

The evolution of the electromagnetic field envelope E in the dispersive CQ medium is governed by the CQ nonlinear Schrödinger equation:

$$2\mathrm{i}\kappa_0 E_z + \nabla_\perp^2 E + \kappa_0 D E_{\tau\tau} + 2\kappa_0^2 (n_2/n_0) |E|^2 E - 2\kappa_0^2 (n_4/n_0) |E|^4 E = 0. \quad (1)$$

Here, z is the propagation coordinate; the Laplacian $\nabla_\perp^2$ (representing the spatial diffraction) acts on the transverse coordinates x and y; the retarded time is defined as $\tau \equiv t - z/v_\mathrm{g}$, in which t is the real time and v_g is the group velocity of the carrier wave; κ_0 is the propagation constant, and $D > 0$ is the coefficient of temporal dispersion. Introducing rescaled variables $U = \sqrt{n_4/n_2}E$, $T = n_2\sqrt{2\kappa_0/Dn_0n_4}\tau$, $Z = \left(\kappa_0 n_2^2/n_0 n_4\right) z$, and $(X, Y) = \kappa_0 n_2 \sqrt{2/n_0 n_4}\,(x, y)$, we transform Eq.(1) into a normalized form:[23]

$$\mathrm{i}\,U_Z + (U_{XX} + U_{YY} + U_{TT}) + |U|^2 U - |U|^4 U = 0. \quad (2)$$

Spherically symmetric solitary wave solutions for Eq.(2) are in the form of $U(X, Y, Z, T) = V(r)\exp(\mathrm{i}kZ)$, where $r = \sqrt{X^2 + Y^2 + T^2}$ and k is a real propagation parameter.[19] It is assumed that the real amplitude V reaches its maximum value at $r = 0$ and $V_r \to 0$ as $r \to \infty$. With this assumed form of solution, equation (2) can be reduced to an ordinary nonlinear differential equation:

$$V'' + (2/r)V' - kV + V^3 - V^5 = 0. \quad (3)$$

The above equation is usually solved by means of the shooting method.[19] The energy of these (3+1)D zero-spin STSs is defined as follows:

$$E(k) = 4\pi \int_0^\infty V^2(r, k)\, r^2 \mathrm{d}r. \quad (4)$$

There appears a distinctive U-shaped energy curve, where the minimum energy is $E_\mathrm{min}/4\pi \approx 15.08$ at $k_\mathrm{cr} \approx 0.026$.[19] In light of the well-known Vakhitov–Kolokov criterion,[15,23] the curve is divided into two branches. The stable and unstable branches correspond to two cases $k_\mathrm{cr} > 0.026$ and $k_\mathrm{cr} < 0.026$, respectively.

On the contrary, STSs propagating in the saturable medium are governed by the following equation:[22]

$$\mathrm{i}U_Z + (U_{XX} + U_{YY} + U_{TT}) + \frac{|U|^2}{1 + a|U|^2} U = 0. \quad (5)$$

Here, a is a reciprocal measure of the saturation intensity. Employing the same way of simplification, we could get a homogeneous ordinary nonlinear differential equation:[22]

$$V'' + (2/r)V' - 2kV + 2\frac{|V|^2}{1 + a|V|^2} V = 0. \quad (6)$$

There also appears a distinctive U-shaped energy curve, where the minimum energy is 0.97 at $k_\mathrm{cr} \approx 1.0$.[22] In our numerical simulations, we choose (3+1)D STSs in the two stable branches of the two media, respectively.

3. Numerical method

With the obtained stable (3+1)D STSs, one can study interactional processes between STSs by using the split-step Fourier method that is acknowledged to be one of the most efficient ways for solving the nonlinear Schrödinger (NLS) equation.[24] The second-order derivatives with respect to X, Y and T are solved in Fourier space, so periodic boundary conditions are used for X, Y, T. Most of our numerical simulations are carried out on $512 \times 256 \times 256$ mesh (one may change the size of the mesh if necessary) and the step size is taken to be 0.005. For Eq.(2), the total energy is a conserved quantity[25] and we monitor this quantity in simulations to check the numerical accuracy. Moreover, we also change the step size to minimize the numerical artifact.

4. Results

In our simulations, the propagation constant is chosen to be $k = 0.07$ in the CQ medium and $k = 6$ in the saturable medium, in order that the selected STSs stay in the stable branch. To introduce the initial constant phase factor and initial velocity, one can multiply the individual input profile $V(r)$ by a factor $\exp(\mathrm{i}vT + \mathrm{i}\phi_0)$,[22] where v denotes the velocity relative to the group velocity and ϕ_0 represents the initial

phase factor (for simplicity, we choose $0 \leq \phi_0 < 2\pi$ in this paper). Because of the spherical symmetry of the solutions, it is informative to draw the contour plots of $|U(X, T, Y = 0, Z = Z_0)|$ in the X–T plan. However, to clearly show the whole interactional processes between the two STSs, it is necessary to present the plot of $|U(X = 0, T, Y = 0, Z)|$, where the distribution of light intensity in the cross section cannot be shown.

Figures 1(a), 1(b) and 1(c) are the contour plots of $|U(X, T, Y = 0, Z = Z_0)|$ in the X–T plan to demonstrate the repulsion of two interacting STSs in the CQ medium.

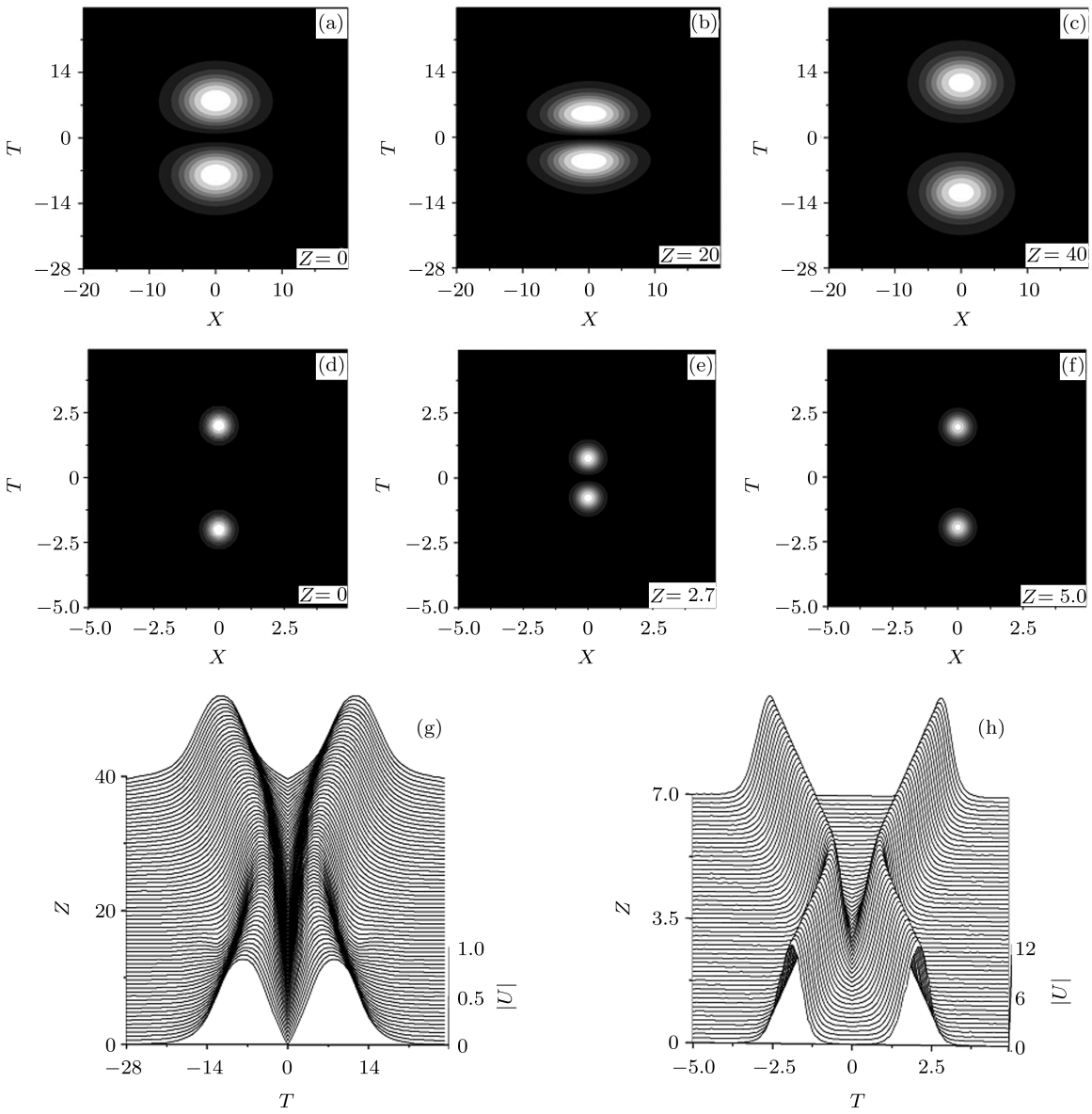


Fig.1. (a)–(c) are the contour plots of $|U(X, T, Y = 0, Z = Z_0)|$ showing head-on collision of two STSs in the CQ media. (d)–(f) show a similar case in the saturable media. The two colliding STSs are of equal initial speeds and initial π phase difference in both collisions where the initial speeds are 0.2 and 0.5, respectively. The luminosity is proportional to the intensity of the light. The plots of $|U(X = 0, T, Y = 0, Z)|$ (g) and (h) show the whole interacting processes of the two collisions.

Figure 1(g) shows the whole interacting process. The two STSs are initially moving in opposite directions with equal speed and their initial phase difference is π. Firstly, they are moving head-on. Meanwhile, a repulsive force, arising and increasing between them， causes them to slow down and obviously change their shapes (change

the light intensity distribution of STSs). Later, they rebound away from each other and accelerate in opposite directions and regain their original shape at approximately $Z = 30.0$. There is almost no energy exchange in this process. Figures 1(d), 1(e), 1(f) and 1(h) show the head-on collision of two STSs with π phase difference in the saturable media. From Figs.1(a)–1(h), it is clearly shown that the two collisions of STSs in both the CQ and the saturable media interact in the same way.

Collisions of STSs in the CQ medium should belong to coherent collision, because the force between STSs in this medium, whether repulsive or attractive, only depends on the phase difference between STSs. When the phase difference is π, the force is repulsive, which has been demonstrated in Figs.1(a)–1(c) and 1(g).

Figures 2(a)–2(c) and 2(g) show the fusion of two head-on colliding STSs with equal speed 0.2 and 0 phase difference in the CQ medium. They move towards each other and then pass through each other. An attractive force, caused by their 0 phase difference, drags them back. In Fig.2(c), a light bullet arises in the medium after the second oscillation, but it will be broadened after a while. The two STSs will fuse to one after a finite number of oscillations of decreasing period. During the fusing process, a small amount of energy of STSs is lost to radiation.

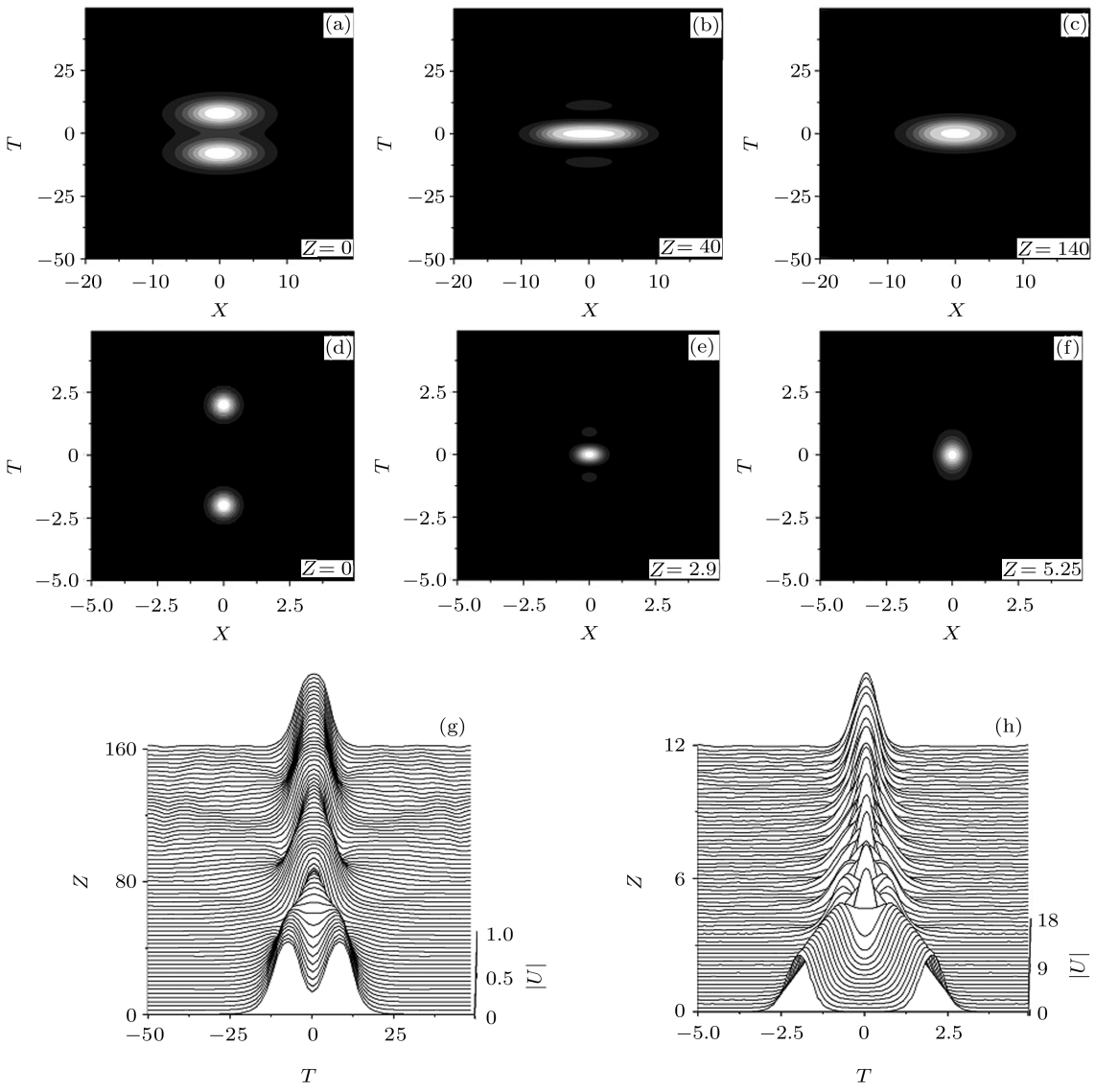


Fig.2. (a)–(c) display the fusion of two STSs in the CQ media. (d)–(f) are the cases in the saturable media. The two head-on interacting STSs are of equal initial speeds and initial 0 phase difference in both fusions where the initial speeds are 0.01 and 0.5, respectively. (g) and (h) show the detailed processes of the two fusions.

The corresponding collision occurs in the saturable media and is shown in Figs.2(d)–2(f) and 2(h). Obvi-

ously, the two collisions behave in the same way. Interestingly, these (3+1)D collisions also resemble the (1+1)D collision that happened in the saturable medium.[6] Stimulated by the universality of interactions exhibited in optical spatial solitons, there may be similarity in the interactions between (3+1)D STSs in these two media.

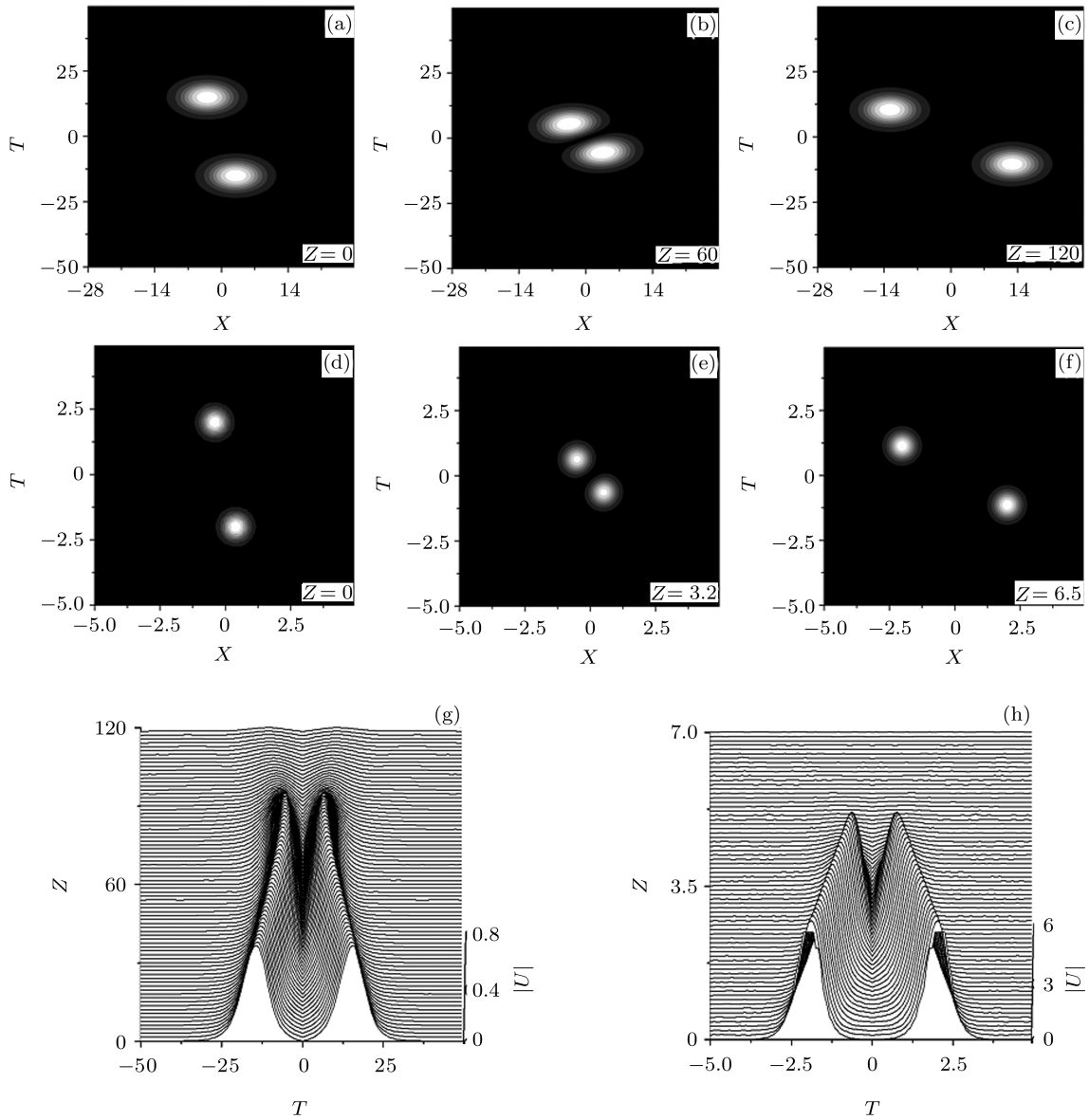


Fig.3. (a)–(c) display the scattering of two STSs in the CQ media. (d)–(f) are the cases in the saturable media. In the two scatterings, the initial velocities and initial phase difference of two colliding STSs are the same as those in Fig.1. The impact parameters of the two scatterings are 6 and 0.8, respectively. (g) and (h) show the whole processes of the two scatterings.

We also investigate other interactions between (3+1)D STSs, such as scattering and spiraling in the CQ medium, and compare them with the corresponding cases in the saturable medium. Figures 3(a)–3(h) show scatterings of STSs in the two media, corresponding to impact parameter[22] 6 and 0.8, respectively. The colliding STSs experience an almost elastic collision, compressing each other when they approach and regain their original shape when they rebound in another direction. In Figs.3(g) and 3(h), we find that the STSs disappear at last, because the scatterings change the way of colliding STSs, and Figs.3(g) and 3(h) of $|U\,(X=0,T,Y=0,Z)|$ only demonstrate the intensity changes along the coordinate, from which the STSs move away after the scatterings. Incidentally, the particle-like behavior of (3+1)D STSs is exhibited among repulsions (in Fig.1) and scatterings (in Fig.3), where the collisions are almost elastic and there is less energy loss. Figures 4(a)–4(h) show the spiralings of two STSs in the two media. When the separation between two STSs is relevant, they spiral on a

converging orbit and lead to fuse finally. Both the satterings and the spiralings demonstrate that corresponding interactions in these two media are similar to each other.

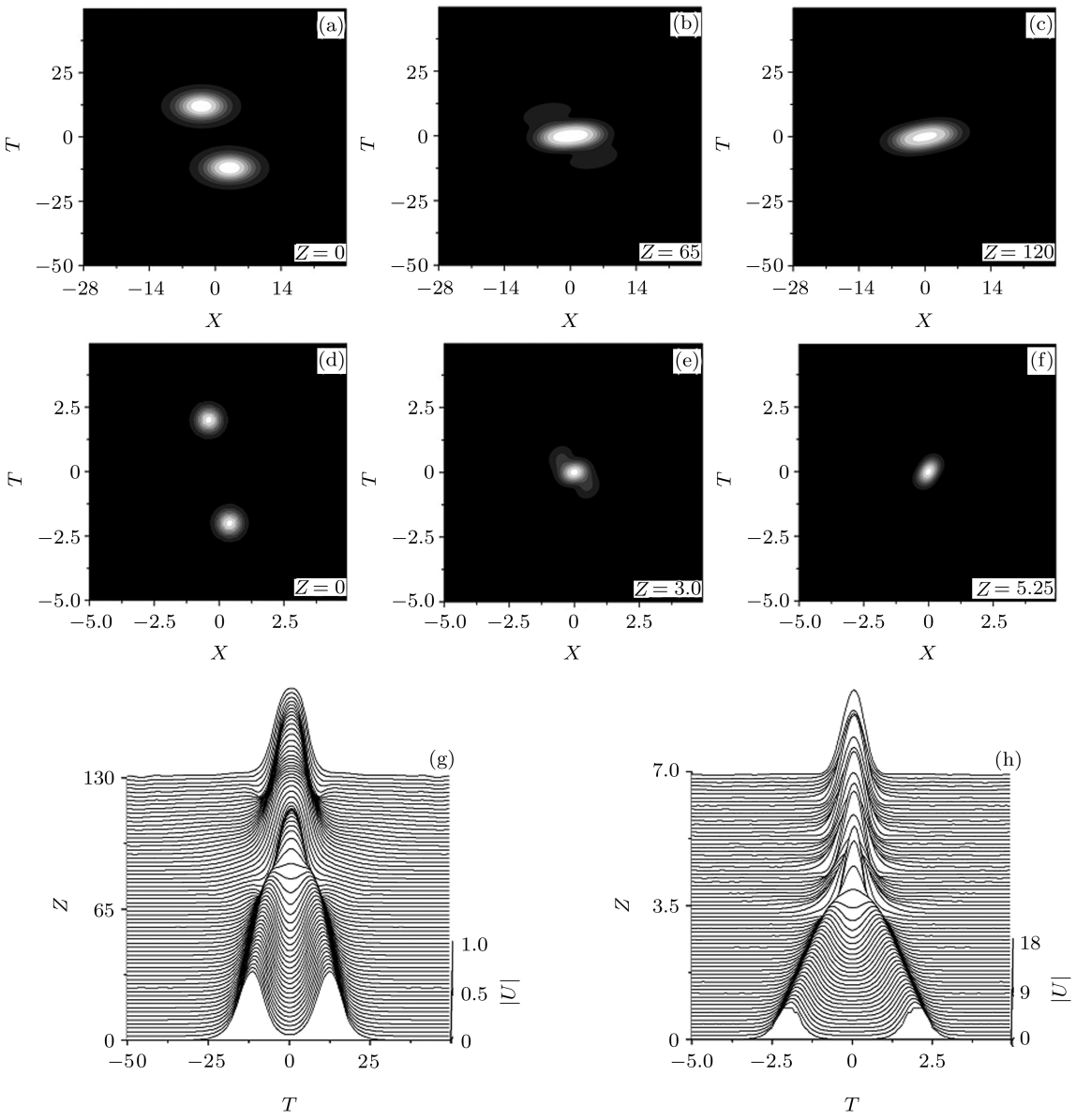


Fig.4. (a)–(c) display the spiraling of two STSs in the CQ media. (d)–(f) are the case in the saturable media. The initial velocities and initial phase difference of two colliding STSs in the two spiralings are the same as those in Fig.2. The impact parameters of the two scatterings are 6 and 0.8, respectively. (g) and (h) show the whole processes of the two spiralings.

When the phase difference between STSs with equal speed is intermediate in 0 and π or π and 2π, the interactions between STSs are more complicated, for they are always accompanied by energy exchange between colliding (3+1)D STSs. Figure 5(a) shows the head-on collision of two STSs with equal speed and the initial $\pi/2$ phase difference in the CQ medium. The energy is seen to be transferred from the light bullet initially lagging in phase to the one initially leading in phase. When the phase difference between the two STSs is $3\pi/2$ (in Fig.5(b)), it is found that the energy exchange will reverse. Energy is transferred from the light bullet leading in phase to the one lagging in phase. Similar cases in the saturable media are shown in Figs.5(d) and 5(e). Moreover, extensive simulations reveal that, both in the CQ and the staturable media, when the phase difference between two colliding STSs being of equal and opposite velocities varies from 0 to π, the light bullet

leading in phase gains energy. While the phase difference varies from π to 2π, the one leading in phase loses energy.

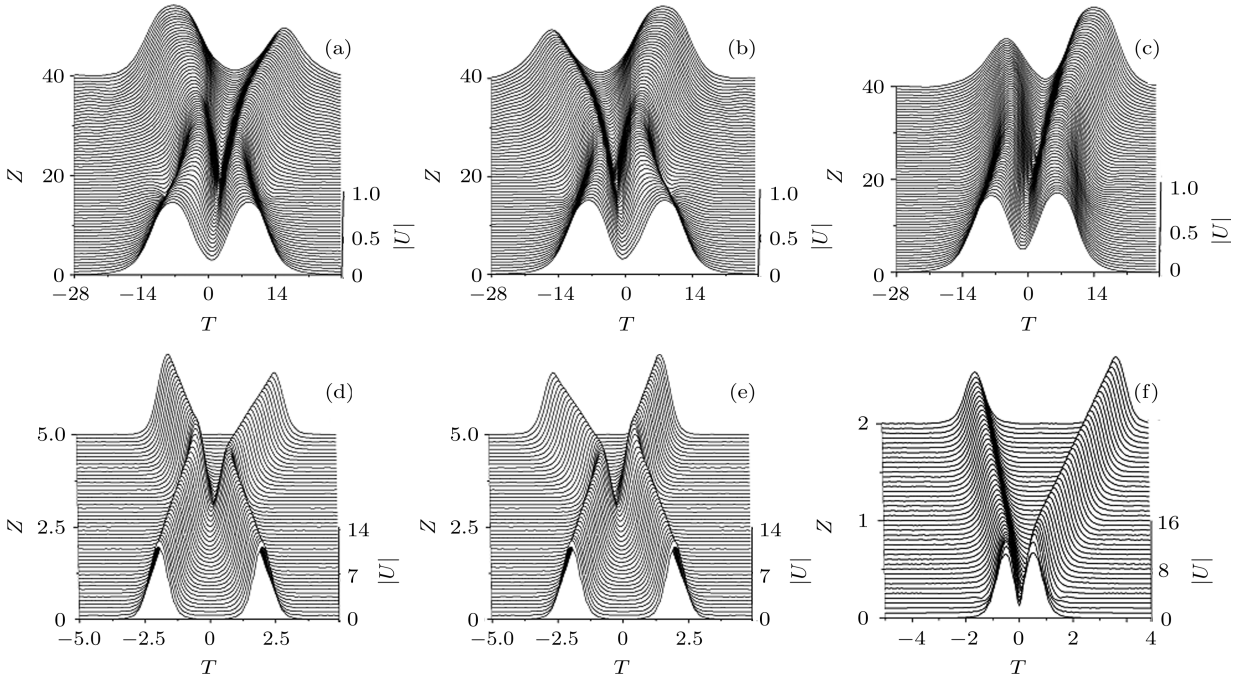


Fig.5. (a)–(c) display the evolutions of two head-on colliding STSs in the CQ media, in which the initial phase differences are $\pi/2$, $3\pi/2$ and π, respectively. (d)–(f) show similar evolutions of two colliding STSs in the saturable media, in which the initial phase differences are $\pi/2$, $3\pi/2$ and π. In (a)–(f), the speeds of STSs in the left-hand side are 0.2, 0.2, 0.2, 0.5, 0.5 and 0.5 while the speeds of STSs in the right-hand side are 0.2, 0.2, 0, 0.5, 0.5 and 0.

The cases discussed above all concern two STSs with equal speed. As a matter of fact, colliding STSs with different speeds receive less attention. Here, we also examine collisions of two STSs with different initial speeds and π-phase difference. Figure 5(c) clearly show that there exists energy exchange between the two STSs. From Fig.5(f), a similar phenomenon in the saturable medium can also be easily seen.

5. Conclusion

To summarize, we systematically simulate interactions between (3+1)D STSs, including repulsion, fusion, scattering and spiraling, in the CQ medium. Inspired by the universality in interactions between spatial optical solitons in different media, we carefully compare the results in the CQ medium with those in the saturable medium and find that the interactions between (3+1)D STS are independent of the media in which the solitons exist. This provides strong support for the viewpoint which recognizes solitons as bound states of their own induced potential well. In addition, Collisions accompanied by energy exchange, which has received less attention in previous studies, are investigated in these two media. The results show that the energy exchange between STSs in the two media exhibits similar features as well. Finally, although the similarity of interactions between spatiotemporal optical solitons is revealed in the present paper, confirmation of the universality of such similarity still needs further study in other optical media.

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