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Quantum superdense coding via cavity-assisted interactions*

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Quantum superdense coding (QSC) is an example of how entanglement can be used to minimize the number of carriers of classical information. This paper proposes two schemes for implementing QSC by means of cavity assisted interactions with single-photon pulses. The schemes are insensitive to the cavity decay and the thermal field, thus it might be realizable based on the current cavity QED techniques.

Keywords: quantum superdense coding, cavity QED
PACC: 4250, 3280, 0367

1. Introduction

Quantum superdense coding (QSC) is one of the important applications of quantum entangled state in quantum communication. It was first proposed by Bennett and Wiesner.[1] QSC is a process to transmit two bits of classical information through a quantum channel from a sender (Alice) to a receiver (Bob) by actually sending a single qubit. Briefly, if the receiver Bob and the sender Alice previously share a Bell-state entanglement pair and then Alice operates locally on her particle one of the four unitary transformations $\{I, \sigma_x, \mathrm{i}\sigma_y, \sigma_z\}$ before sending it to Bob, Bob can obtain two bits of classical information via measurement on both particles. This apparently doubles the information conveying capacity.

QSC has been studied extensively in recent years. It has been investigated theoretically[2,3] and experimentally[4,5] in the context of continuous variables. It has also been extended to high dimensions and multiparties.[6−11] But here we limit our discussion to the case of two parties. We use atoms to represent qubits, and photons transmit information among atoms. As the inevitable interaction between system and environment will cause decoherence, in order to avoid collective dephasing errors in decoherence-free subspace (DFS),[12−19] we utilized two atoms to encode one logical qubit, i.e., $|\widetilde{0}\rangle = |1\rangle_1|0\rangle_2 = |10\rangle$, $|\widetilde{1}\rangle = |0\rangle_1|1\rangle_2 = |01\rangle$. In this paper, two schemes are proposed for implementing QSC using the interaction between cavity and a single-photon pulse.

2. Implementation of QSC

We briefly review the realization of a controlled phase flip gate between two atoms.[20,21] There are two atoms in a cavity, each of which has three relevant levels as shown in Fig.1. The atomic transition $|1\rangle \rightarrow |e\rangle$ is resonantly coupled to the cavity mode, which has h polarization and is resonantly driven by the h polarization component of the input single-photon pulse. The $|0\rangle$ level is decoupled because of the large detuning. If the duration of the input photon pulse T is sufficiently long, for the case that both of the atoms are in the $|0\rangle$ state, the reflection of the pulse from the resonant cavity will leave the pulse shape almost unchanged but with a π phase added. On the contrary, for the case that either or both of the atoms are in the state $|1\rangle$, as the frequency of the cavity

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mode is modified by the strong coupling, the pulse will be reflected by the cavity with both its shape and phase unchanged. Therefore, by reflecting a single-photon pulse with h polarization, the two-atom controlled phase gate $U_{\rm CPF}=\exp(\mathrm{i}\pi|0\rangle_{11}\langle 0|\otimes|0\rangle_{22}\langle 0|)$ is realized.

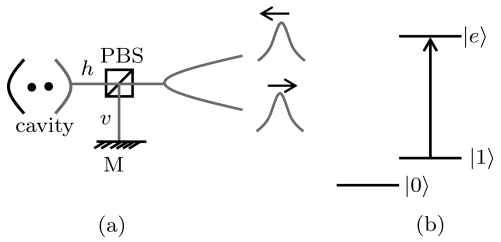


Fig.1. (a) Schematic setup to implement the controlled phase flip gate between two atoms.[22] (b) Diagram of the energy level of atoms.

To perform QSC, in the first scheme, we need a Hadamard gate of a single-logic qubit and a controlled-Z (CZ) phase-flip gate of two logic qubits constructed in Ref.[23].

Now we recall the operation process of the Hadamard gate and the CZ gate.

Assume the bases $\{|\widetilde{0}\rangle,\ |\widetilde{1}\rangle\}$ for atoms, and $\{|h\rangle,|v\rangle\}$ for photons, the initial state of the logic qubit is $|\widetilde{1}\rangle$, and the single-photon pulse is in state $(1/\sqrt{2})(|h\rangle+|v\rangle)$. The steps to realize the $\widetilde{H}$ gate are as follows:

$$\begin{aligned}&\frac{1}{\sqrt{2}}(|h\rangle+|v\rangle)|\widetilde{1}\rangle\xrightarrow{\widetilde{X}}\frac{1}{\sqrt{2}}(|h\rangle|\widetilde{0}\rangle+|v\rangle|\widetilde{1}\rangle)\xrightarrow{\rm HWP}\\&\frac{1}{\sqrt{2}}(|v\rangle|\widetilde{0}\rangle+|h\rangle|\widetilde{1}\rangle)\xrightarrow{\sigma_x^2U_{\rm CPF}\sigma_x^2}\frac{1}{\sqrt{2}}(|v\rangle|\widetilde{0}\rangle-|h\rangle|\widetilde{1}\rangle)\\&\xrightarrow{P_{45}}\frac{|h\rangle+|v\rangle}{\sqrt{2}}(|\widetilde{0}\rangle-|\widetilde{1}\rangle),\end{aligned}\tag{1}$$

where $\widetilde{X}=C_{\rm NOT}^{2,1}H_2U_{\rm CPF}H_2C_{\rm NOT}^{2,1}$,[24] whose function is to flip the logic qubit $|\widetilde{0}\rangle\leftrightarrow|\widetilde{1}\rangle$ (being different from the standard controlled-NOT gate operation, the $C_{\rm NOT}^{2,1}$ here is designed to flip the state of atom 1 only when the atom 2 is in state $|0\rangle$), half wave plate (HWP) rotates the photon polarization state as $|h\rangle\leftrightarrow|v\rangle$, σ_x^2 is the σ_x operation on atom 2, and P_{45} is a 45° polarizer projecting the polarization $\dfrac{1}{\sqrt{2}}(|h\rangle+|v\rangle)$. In the same way, if the initial state of the logic qubit is $|\widetilde{0}\rangle$, we can get $\dfrac{1}{\sqrt{2}}(|\widetilde{0}\rangle+|\widetilde{\widetilde{1}}\rangle)$.

The operation process of the CZ gate of two logic qubit $\widetilde{U}_{\rm CZ}^{i,j}$, with the subscripts i and j denoting the control and target logic qubits in cavity i and j, can be shown more specifically as follows:[23]

Assume that the single-photon pulse is in state $(1/\sqrt{2})(|h\rangle+|v\rangle)$, and the two logic qubits are in a superposition state $a|\widetilde{0}\rangle_i|\widetilde{0}\rangle_j+b|\widetilde{0}\rangle_i|\widetilde{1}\rangle_j+c|\widetilde{1}\rangle_i|\widetilde{0}\rangle_j+d|\widetilde{1}\rangle_i|\widetilde{1}\rangle_j$, where a, b, c, and d are normalized constants.

$$\begin{aligned}&\frac{1}{\sqrt{2}}(|h\rangle+|v\rangle)\otimes(a|\widetilde{0}\rangle_i|\widetilde{0}\rangle_j+b|\widetilde{0}\rangle_i|\widetilde{1}\rangle_j+c|\widetilde{1}\rangle_i|\widetilde{0}\rangle_j\\&+d|\widetilde{1}\rangle_i|\widetilde{1}\rangle_j)\xrightarrow{\sigma_{x,i}^1U_{\rm CPF}\sigma_{x,i}^1}\frac{1}{\sqrt{2}}[a(-|h\rangle+|v\rangle)|\widetilde{0}\rangle_i|\widetilde{0}\rangle_j\\&+b(-|h\rangle+|v\rangle)|\widetilde{0}\rangle_i|\widetilde{1}\rangle_j+c(|h\rangle+|v\rangle)|\widetilde{1}\rangle_i|\widetilde{0}\rangle_j\\&+d(|h\rangle+|v\rangle)|\widetilde{1}\rangle_i|\widetilde{1}\rangle_j]\xrightarrow{\rm HWP1,HWP2}a|v\rangle|\widetilde{0}\rangle_i|\widetilde{0}\rangle_j\\&+b|v\rangle|\widetilde{0}\rangle_i|\widetilde{1}\rangle_j+c|h\rangle|\widetilde{1}\rangle_i|\widetilde{0}\rangle_j+d|h\rangle|\widetilde{1}\rangle_i|\widetilde{1}\rangle_j\\&\xrightarrow{\sigma_{x,j}^2U_{\rm CPF}\sigma_{x,j}^2,P_{45}}\frac{1}{\sqrt{2}}(|h\rangle+|v\rangle)\otimes(a|\widetilde{0}\rangle_i|\widetilde{0}\rangle_j\\&+b|\widetilde{0}\rangle_i|\widetilde{1}\rangle_j+c|\widetilde{1}\rangle_i|\widetilde{0}\rangle_j-d|\widetilde{1}\rangle_i|\widetilde{1}\rangle_j),\end{aligned}\tag{2}$$

where $\sigma_{x,i}^1$ is the σ_x operation on atom 1 in cavity i, HWP1, at an angle of 45° to the horizontal direction, flips the photon polarization state $|h\rangle\leftrightarrow|v\rangle$, and HWP2, at an angle of 22.5° to the horizontal direction, performs a Hadamard gate on the photon polarization states, i.e., $|h\rangle\rightarrow\dfrac{1}{\sqrt{2}}(|h\rangle+|v\rangle)$, $|v\rangle\rightarrow\dfrac{1}{\sqrt{2}}(|h\rangle-|v\rangle)$.

The $\widetilde{C}_{\rm NOT}$ gate for the logic qubits in a DFS is given by:[23]

$$\begin{aligned}\widetilde{C}_{\rm NOT}^{i,j}&=\widetilde{H}_j\otimes\widetilde{U}_{\rm CZ}^{i,j}\otimes\widetilde{H}_j,\\\widetilde{C}_{\rm NOT}^{j,i}&=\widetilde{H}_i\otimes\widetilde{U}_{\rm CZ}^{j,i}\otimes\widetilde{H}_i.\end{aligned}\tag{3}$$

Now we turn to the proposal for QSC for the logic qubits described above. Assume that Alice and Bob have two logic qubits in states i, j, respectively, and $|\widetilde{\psi}\rangle_{ij}$ is a Bell-style state. We take $|\widetilde{\psi}\rangle_{ij}=(1/\sqrt{2})(|\widetilde{0}\rangle_i|\widetilde{0}\rangle_j+|\widetilde{1}\rangle_i|\widetilde{1}\rangle_j)$ as an example, which can be attained by $\widetilde{H}$ and $\widetilde{C}_{\rm NOT}$ operation from the initial state $|\widetilde{0}\rangle_i|\widetilde{0}\rangle_j$, i.e.,

$$\begin{aligned}&|\widetilde{0}\rangle_i|\widetilde{0}\rangle_j\xrightarrow{\widetilde{H}_i}\frac{1}{\sqrt{2}}(|\widetilde{0}\rangle_i+|\widetilde{1}\rangle_i)|\widetilde{0}\rangle_j\\&\xrightarrow{\widetilde{C}_{\rm NOT}^{i,j}}\frac{1}{\sqrt{2}}(|\widetilde{0}\rangle_i|\widetilde{0}\rangle_j+|\widetilde{1}\rangle_i|\widetilde{1}\rangle_j).\end{aligned}\tag{4}$$

Alice operates on the logic qubit i via one of the four local operations $\{\widetilde{I},\widetilde{\sigma}_x,\mathrm{i}\widetilde{\sigma}_y,\widetilde{\sigma}_z\}$, so as to put the logic qubits into one of the Bell-style states (denoted by $|\widetilde{\psi}^{\pm}\rangle,|\widetilde{\phi}^{\pm}\rangle$), where $\widetilde{I}$ is identical operation, $\widetilde{\sigma}_x=\sigma_x^1\sigma_x^2$ means σ_x operation on each of the atoms of the logic qubit i, $\widetilde{\sigma}_z=\sigma_z^2$ means σ_z operation on atom 2, and $i\widetilde{\sigma}_y=\sigma_z^2\sigma_x^1\sigma_x^2$ means σ_x operation on atom 1 and σ_x, σ_z operations on atom 2. Then the logic qubit will be sent to Bob. Bob can discriminate the operation of Alice by using only local operations as follows:

Firstly, Bob applies a $\widetilde{C}_{\rm NOT}^{i,j}$ operation to the Bell-style state ($|\widetilde{\psi}^+\rangle, |\widetilde{\psi}^-\rangle, |\widetilde{\phi}^+\rangle$ or $|\widetilde{\phi}^-\rangle$) possessed by him. It can be implemented in Fig.2. An $\widetilde{H}$ operation on the logic qubit j is followed by a $\widetilde{U}_{\rm CZ}^{i,j}$ operation, and an $\widetilde{H}$ operation is applied on the logic qubit j again. That is to say, $\widetilde{C}_{\rm NOT}^{i,j} = \widetilde{H}_j \otimes \widetilde{U}_{\rm CZ}^{i,j} \otimes \widetilde{H}_j$,

$$\widetilde{C}_{\rm NOT}^{i,j}|\widetilde{\psi}^{\pm}\rangle = \frac{1}{\sqrt{2}}(|\widetilde{0}\rangle_i \pm |\widetilde{1}\rangle_i)|\widetilde{1}\rangle_j,$$
$$\widetilde{C}_{\rm NOT}^{i,j}|\widetilde{\phi}^{\pm}\rangle = \frac{1}{\sqrt{2}}(|\widetilde{0}\rangle_i \pm |\widetilde{1}\rangle_i)|\widetilde{0}\rangle_j. \quad (5)$$

With the measurement on the target logic qubit j, Bob can discriminate $|\widetilde{\psi}^{\pm}\rangle$ from $|\widetilde{\phi}^{\pm}\rangle$. Then an $\widetilde{H}$ operation is applied on the qubit i:

$$\widetilde{H}_i[\frac{1}{\sqrt{2}}(|\widetilde{0}\rangle_i \pm |\widetilde{1}\rangle_i)] = \begin{cases} |\widetilde{0}\rangle_i, & +, \\ |\widetilde{1}\rangle_i, & -. \end{cases} \quad (6)$$

After measuring the controlled logic qubit i, Bob can discriminate the four Bell states totally, and thus reliably learns which operation Alice performed.

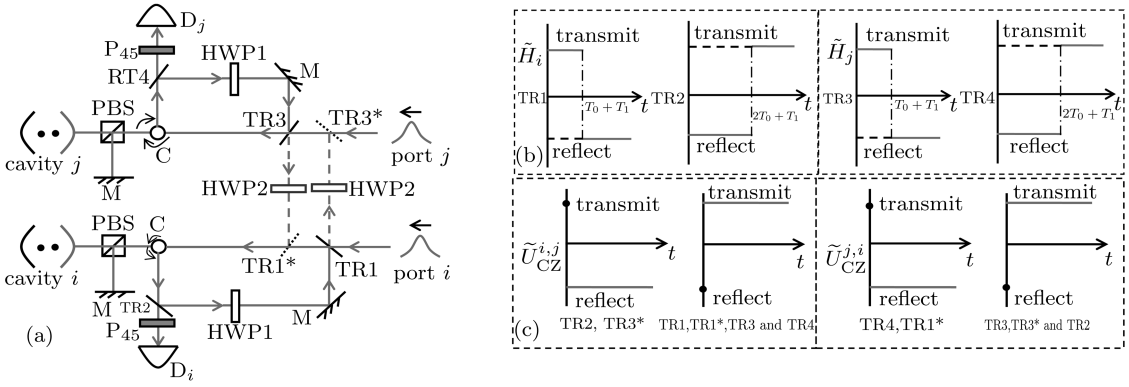


Fig.2. (a) Schematic setup for realizing QSC. D_i is a detector, C is a circulator, HWP1 rotates the photon polarization as $|h\rangle \leftrightarrow |v\rangle$, HWP2 performs a Hadamard gate on the photon polarization states. TR can be controlled exactly as needfulness to transmit or reflect a photon. The single-photon pulse's path for $\widetilde{H}_i$ is port i-TR1-TR1*-C-PBS-cavity i-PBS-C-TR2-HWP1-M-TR1-TR1*-C-PBS-cavity i-PBS-C-TR2-P_{45}-D_i. The single-photon pulse's path for $\widetilde{U}_{\rm CZ}^{i,j}$ is port i-TR1-TR1*-C-PBS-cavity i-PBS-C-TR2-HWP1-M-TR1-HWP2-TR3*-TR3-C-PBS-cavity j-PBS-C-TR4-P_{45}-D_j. The path for $\widetilde{H}_j$ and $\widetilde{U}_{\rm CZ}^{j,i}$ is similar. (b) Time control sequence TR1 (TR3) and TR2 (TR4) for $\widetilde{H}$. T_0 is the time for the single-photon pulse process TR1 (TR3)-C-PBS-Cavity-PBS-C-TR2 (TR4), while T_1 is the time for TR2 (TR4)-HWP1-M-TR1 (TR3). (c) Time control sequence TR for $\widetilde{U}_{\rm CZ}$.[23]

Here we conclude the first proposal for the QSC. Next, we will present an alternative one, and then we will make a comparison between these two proposals.

In the second scheme, we need to construct a SWAP gate between a single logic qubit of two atoms and a qubit of one photon.[25] The quantum SWAP gate can be written as

$$\widetilde{U}_{ap}^{\rm SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (7)$$

in the bases $\{|\widetilde{0}\rangle h,\ |\widetilde{0}\rangle v,\ |\widetilde{1}\rangle h,\ |\widetilde{1}\rangle v\}$, which has the following operator identity:

$$\widetilde{U}_{ap}^{\rm SWAP} = R_p\sigma_x^1 U_{ap}^{\rm CPF}\sigma_x^1 R_p R_a \sigma_x^1 U_{ap}^{\rm CPF}\sigma_x^1 R_a \times R_p\sigma_x^1 U_{ap}^{\rm CPF}\sigma_x^1 R_p, \quad (8)$$

where σ_x^1 is the not operation on atom 1, $R = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$, R_a is a single-logic qubit rotation on two atoms, which can be done easily by a similar process to Eq.(1).[23] R_p is a single qubit rotation on one photon, which can be implemented by an HWP.

The SWAP gate will be implemented in Fig.3. It can be used to exchange information between a single-logic qubit of two atoms and a qubit of one photon, i.e., $(a|\widetilde{0}\rangle + b|\widetilde{1}\rangle)(c|h\rangle + d|v\rangle) \rightarrow (a|h\rangle + b|v\rangle)(c|\widetilde{0}\rangle + d|\widetilde{1}\rangle)$. With its help, we can accomplish teleportation, dense coding of atomic or ionic states, etc. This kind of gate can be used widely in quantum computation and quantum information.

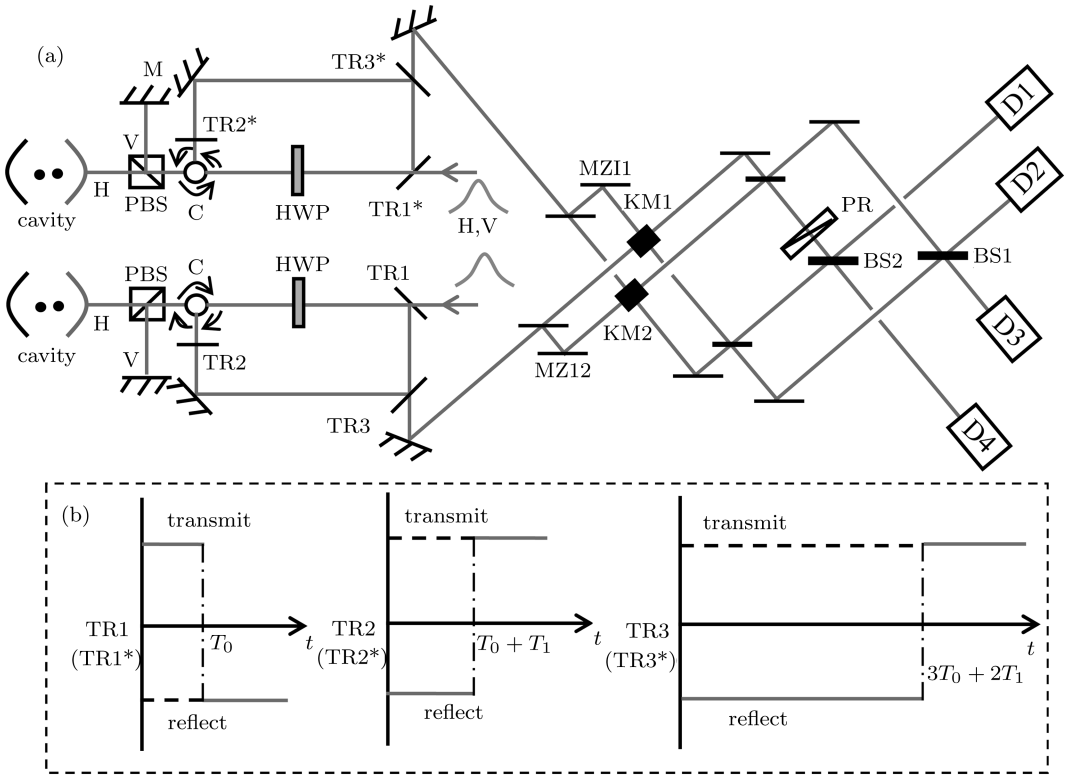


Fig.3. (a) Schematic setup for realizing QSC via the SWAP gate and Bell-state analyzer. HWP is a half wave plate; TR1(TR1*), TR2(TR2*) and TR3(TR3*) are controlled to transmit or reflect one photon according to (b). C is a circulator. The single-photon pulse's path for $\widetilde{U}_{ap}^{\mathrm{SWAP}}$ is TR1(TR1*)-HWP-C-PBS-Cavity(M)-PBS-HWP-TR1(TR1*)-TR3(TR3*)-M-C-PBS-Cavity(M)-PBS-HWP-TR1(TR1*)-TR3(TR3*)-M-C-PBS-Cavity(M)-PBS-HWP-TR1(TR1*)-TR3(TR3*). (b) Time control sequence TR1(TR1*), TR2(TR2*) and TR3(TR3*). T_0 is the time for the single-photon pulse process C-PBS-Cavity(M)-PBS-C-HWP-TR1(TR1*), while T_1 is the time for TR1(TR1*)-TR3(TR3*)-M-TR2(TR2*).

Next we will introduce how to use the SWAP gate to accomplish QSC of atomic states. Our scheme consists of two main steps. Firstly, map the information of atoms to photons under the SWAP gate realized in Fig.3. Then distinguish four different entangled Bell states of the photons through a photonic bell-state analyzer.[26] Compared with the former QSC of atomic states, the advantage of the scheme lies in transferring entanglement between atoms into that of photons, for the photons are the best carriers of information in DFS, and the entanglement of photons can be identified more easily.

Assume that Alice and Bob each hold one logic qubit of a maximally entanglement state that has been previously prepared and distributed. After Alice performs one of the four local operations on her qubit as above, the two logic qubits turn into one of the Bell-style states with a theoretical probability of 1/4. Then Alice maps the information of her qubit to a photon state under the SWAP gate, and sends the photon to Bob. Bob maps the information of his qubit to another photon through a similar operation, and then inputs the two photons to a photonic Bell-state analyzer simultaneously,[26] in which the photons are counted by single-photon detectors numbered D1–D4. Bob can reliably learn which operation Alice applied by registering coincidence counting of them. The overall process can be read clearly in Fig.3.

3. Conclusions

In summary, we have proposed two schemes for realizing QSC via cavity QED. These schemes enhance the information capacity of the transmission channel to two bits in comparison with the classical maximum of one bit. With the help of the cavity assisted interactions, the use of atoms interacting with single-photon pulses has been very promising for handling informa-

tion storage, communication, and computation. As the cavity QED technique is developing very quickly, we hope for more applications by means of cavity assisted interactions in quantum operation.

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