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Quasiprobability distributions of new kinds of even and odd nonlinear coherent states*

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Using a numerical computational method, quasiprobability distributions of new kinds of even and odd nonlinear coherent states (EONLCS) are investigated. The results show that the distributions of the new even nonlinear coherent states (NLCS) are distinct from those of the new odd NLCS and imply that the new EONLCS always exhibit some different nonclassical effects. Finally, with the aid of newly introduced intermediate coordinate-momentum representation in quantum optics, the tomograms of the new EONLCS are calculated. This is a new way of obtaining the tomogram function.

Keywords: even and odd nonlinear coherent state, Wigner W-function, Husimi Q-function, tomogram

PACC: 4250, 0365

1. Introduction

The phase space technique has proved to be very important in various branches of physics. Quasiprobability distribution functions are extremely useful in studing the statistical properties of quantized light fields from the view point of the quasiprobability phase space because these functions are related to the density matrix which provides a complete statistical description of some physical systems. Generally, in order to examine some nonclassical effects of physical systems, we shall exploit the following quasiprobability distribution functions: Wigner W-, Husimi Q- and Glauber P-functions.[1,2] However, the Glauber P-function is highly singular, therefore we shall usually only consider the Husimi Q-function and Wigner W-function. Under any state the negative values of the Wigner W-function and the stretching of the Husimi Q-function are looked at as signatures of nonclassical effects.

It is well known that nonclassical states of light fields are important in quantum optics due to their significant nonclassical properties. In recent years, many nonclassical states[3–5] have been constructed in terms of the fundamental principles of quantum mechanics. Various nonclassical effects of these states have been studied in detail. Moreover, some important types of nonclassical states such as the Fock state, coherent state and NLCS, etc., have already been realized in experimental demonstrations by means of various physical schemes.[6–8] In the past two years we have also constructed types of representative nonclassical states[9,10] and discussed the phase properties,[9] squeezing properties[10] and quasiprobability functions[11,12] for these states. In this paper, based on the new EONLCS constructed by us in Ref.[9], we want to continue to study quasiprobability distribution functions, including the Husimi Q-function and Wigner W-function, associated with the new EONLCS. Then, using newly introduced intermediate coordinate-momentum representation in quantum optics, we obtain the tomograms of the new EONLCS.

2. A new EONLCS

In Ref.[9] we have obtained a new kind of EONLCS by the induction method in terms of the right eigenstates[13] of a two-photon annihilation operator $\left(a^{\dagger}\right)^{-1} a$. Here we shall give a brief review of the new EONLCS. To obtain the new EONLCS,[7] we denote the right eigenstates as the state $|\lambda, f\rangle$ of the operator $\left(a^{\dagger}\right)^{-1} a \dfrac{1}{f\left(N\right)}$, i.e.,

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$$\left(a^{\dagger}\right)^{-1} a \frac{1}{f(N)} |\lambda, f\rangle = \lambda |\lambda, f\rangle , \tag{1}$$

where $\lambda = r \exp(\mathrm{i}\varphi)$ is a complex number. According to the number states $|n\rangle$ of the harmonic oscillator, we can express the state $|\lambda, f\rangle$ as

$$|\lambda, f\rangle = \sum_{n=0}^{\infty} C_n |n\rangle . \tag{2}$$

Therefore, we get the expansion constants satisfied with

$$\begin{aligned} C_{2n} &= \sqrt{\frac{(2n-1)!!}{(2n)!!}} f(2n)!! \lambda^n C_0, \\ C_{2n+1} &= \sqrt{\frac{(2n)!!}{(2n+1)!!}} f(2n+1)!! \lambda^n C_1, \end{aligned} \tag{3}$$

where the function $f(n)$ is obtained by replacing the number operator N in $f(N)$ by the integer n,

$$\begin{aligned} f(2n)!! &= f(0) f(2) \ldots f(2n), \ f(0) = 1, \\ f(2n+1)!! &= f(1) f(3) \cdots f(2n+1). \end{aligned} \tag{4}$$

The constants C_0 and C_1 are determined by normalization of the states $|\lambda, f\rangle$.

The new even NLCS are given by

$$|\lambda, f\rangle_{+} = C_0 \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{2^n n!} f(2n)!! \lambda^n |2n\rangle , \tag{5}$$

$$C_0 = \left(\sum_{n=0}^{\infty} \frac{(2n)!}{\left(2^n n!\right)^2} \left(f(2n)!!\right)^2 |\lambda|^{2n} \right)^{-1/2}, \tag{6}$$

where the range of λ for $|\lambda, f\rangle_{+}$ is given by the following inequality

$$|\lambda|^2 \leq \lim_{n\to\infty} \frac{2(n+1)}{(2n+1)\left(f(2n+2)\right)^2}. \tag{7}$$

In the same way, we can obtain the new odd NLCS expressed as

$$\begin{aligned} |\lambda, f\rangle_{-} = C_1 \sum_{n=0}^{\infty} & \frac{2^n n!}{\sqrt{(2n+1)!}} \\ & \times f(2n+1)!! \lambda^n |2n+1\rangle , \end{aligned} \tag{8}$$

$$C_1 = \left(\sum_{n=0}^{\infty} \frac{\left(2^n n!\right)^2}{(2n+1)!} \left(f(2n+1)!!\right)^2 |\lambda|^{2n} \right)^{-1/2}, \tag{9}$$

so we have

$$|\lambda|^2 \leq \lim_{n\to\infty} \frac{(2n+3)}{2(n+1)(f(2n+3))^2}. \tag{10}$$

Obviously, if setting $f(n) = 1$, the new EONLCS will become usual even and odd coherent states in Ref.[13]. We can also find that the nonclassical properties of the new EONLCS depend on the form of the nonlinearity function $f(n)$. In this paper, we shall choose the nonlinearity function $f(n)$ given by[14]

$$f(n) = \mathrm{L}_n^1\left(\eta^2\right) \left[(n+1)\, \mathrm{L}_n^0\left(\eta^2\right)\right]^{-1}, \tag{11}$$

which is usually used in the description of the motion of a trapped ion and where η is the Lamb–Dicke parameter and $\mathrm{L}_n^m(x)$ is the associated Laguerre polynomials. By using the nonlinearity function $f(n)$, it is very interesting to study quasiprobability distributions and tomograms of the new EONLCS in the following sections.

3. Quasiprobability functions for the new EONLCS

In this section we shall consider the Husimi Q-function and Wigner W-function associated with the new EONLCS.

The Husimi Q-function is positive-definite at any point in the phase space for any quantum state and can be also calculated in terms of the following formula[15]

$$Q(\alpha) = \pi^{-1} \langle \alpha | \rho | \alpha \rangle , \tag{12}$$

where ρ is the density matrix for the new EONLCS and $|\alpha\rangle$ is the usual coherent state.[16] Thus the Q-function for light fields in the new EONLCS will be given by, respectively

(i) for the new even NLCS

$$\begin{aligned} Q_{+}(\alpha) = \pi^{-1} |C_0|^2 & \exp\left(-|\alpha|^2\right) \\ & \times \left| \sum_{n=0}^{\infty} \frac{\alpha^{2n}}{2^n n!} \lambda^{*n} f(2n)!! \right|^2 ; \end{aligned} \tag{13}$$

(ii) for the new odd NLCS

$$\begin{aligned} Q_{-}(\alpha) = \pi^{-1} |C_1|^2 & \exp\left(-|\alpha|^2\right) \\ & \times \left| \sum_{n=0}^{\infty} \frac{2^n n!}{(2n+1)!} \lambda^{*n} f(2n+1)!! \alpha^{2n+1} \right|^2 . \end{aligned} \tag{14}$$

Here the constants C_0 and C_1 are given by Eqs.(6) and (9), respectively.

For the new EONLCS, the numerical computation results of the Q-functions are presented in figures 1 and 2. In figures 1 and 2 we show the Q-functions

as a function of α for different values of the parameter $|\lambda|$ ($|\lambda| = 0.1$ and 0.76) and fixed $\eta^2 = 0.7$. In figure 1 for the new even NLCS, while for $|\lambda| = 0.1$ we can see a single compressed peak of $Q_+(\alpha)$ function, and as $|\lambda|$ increases, the peaks can always be compressed. It signifies that the new even NLCS always exhibit a squeezing effect in the p direction. With $|\lambda|$ increasing from 0.1 to 0.76 the peak of $Q_+(\alpha)$ functions begins to split into two peaks. The greater the value of $|\lambda|$, the further the distance between the two peaks, but the value of the two peaks cannot vary remarkably. When the value of $|\lambda|$ is large enough, the two compressed peaks will be surrounded by an inverted deep crater at the origin. However, from figure 2 we can also see a single cylindraceous peak of $Q_-(\alpha)$ functions for the new odd NLCS while for $|\lambda| = 0.1$. With $|\lambda|$ increasing the single cylindraceous peak gradually becomes thinner, however it cannot be compressed. It signifies that the new odd NLCS cannot exhibit the squeezing effect in both directions. And as $|\lambda|$ increases, the inverted crater at the origin of the single peak appears and subsequently two peaks of $Q_-(\alpha)$ functions begin to develop remarkably. This result is in sharp contrast to that of the new even NLCS. When the value of $|\lambda|$ is large enough, the $Q_-(\alpha)$ functions of the new EONLCS are similar to those of the combination number states.

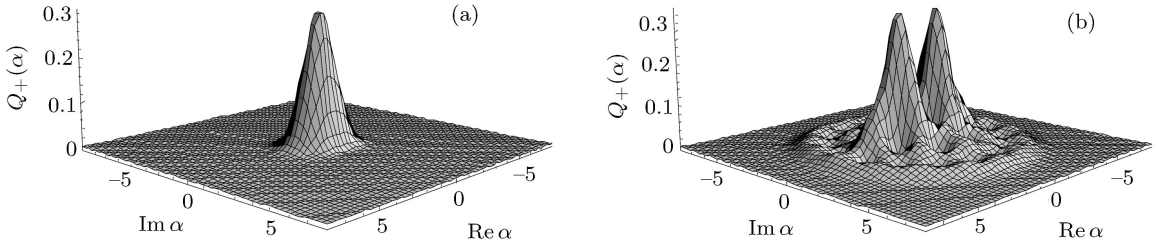


Fig.1. Husimi Q-functions for the new even NLCS for $\eta^2 = 0.7$ and (a) $\lambda = 0.1$, (b) $\lambda = 0.76$.

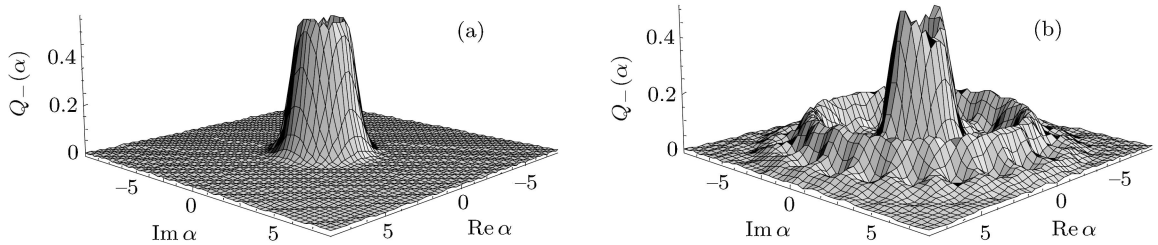


Fig.2. Husimi Q-functions for the new odd NLCS for $\eta^2 = 0.7$ and (a) $\lambda = 0.1$, (b) $\lambda = 0.76$.

On the other hand, in Ref.[17] in terms of the coherent state representation the Wigner operator $\Delta(\alpha, \alpha^*)$ is expressed as

$$\Delta(\alpha, \alpha^*) = \int \frac{\mathrm{d}^2 z}{\pi^2} |\alpha + z\rangle \langle \alpha - z| \, \mathrm{e}^{\alpha z^* - z\alpha^*}. \quad (15)$$

Then by virtue of the technique of integration within an ordered product (IWOP) of operators,[17] the Wigner operator $\Delta(\alpha, \alpha^*)$ reduces to the following form

$$\Delta(\alpha, \alpha^*) = \frac{2}{\pi} D(2\alpha)(-)^N, \quad (16)$$

where N is the number operator and $D(2\alpha)$ is the displaced operator, $\alpha = \alpha_1 + \mathrm{i}\alpha_2$. The Wigner function $W(\alpha, \alpha^*)$ associated with a pure superposition state

$$|\phi\rangle = \sum_{n=0}^{\infty} f_n |n\rangle \quad (17)$$

is given by

$$W(\alpha, \alpha^*) = \frac{2}{\pi} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} f_m^* f_n (-1)^n \chi_{mn}(2\alpha), \quad (18)$$

where

$$\chi_{mn}(\alpha)=\langle m|\,\Delta(\alpha,\alpha^*)\,|n\rangle=\begin{cases}\sqrt{\dfrac{n!}{m!}}\alpha^{m-n}\mathrm{e}^{-|\alpha|^2/2}\mathrm{L}_n^{(m-n)}\left(|\alpha|^2\right), & n\leqslant m\\ \sqrt{\dfrac{m!}{n!}}\left(-\alpha^*\right)^{n-m}\mathrm{e}^{-|\alpha|^2/2}\mathrm{L}_n^{(n-m)}\left(|\alpha|^2\right), & n>m\end{cases}\tag{19}$$

and $\mathrm{L}_n^{(\rho)}(x)$ is the generalized Laguerre polynomials[18]

$$\mathrm{L}_n^{(\rho)}(x)=\sum_{k=0}^{n}\binom{n+\rho}{n-k}\frac{1}{k!}(-x)^k.\tag{20}$$

Using Eqs.(5), (8), (18) and (19), we can obtain the W-functions of the new EONLCS as follows:

$$W_{\pm}(\alpha,\alpha^*)=\frac{2}{\pi}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(b_{\pm}^*)_m(b_{\pm})_n(-1)^n\chi_{mn}(2\alpha),\tag{21}$$

where

$$\begin{aligned}(b_+)_n&=C_0\frac{\sqrt{(2n)!}}{2^n n!}f(2n)!!\lambda^n,\\ (b_-)_n&=C_1\frac{2^n n!}{\sqrt{(2n+1)!}}f(2n+1)!!\lambda^n.\end{aligned}\tag{22}$$

We have studied numerically the behaviour of the Wigner functions $W_{\pm}(\alpha)$ for Eq.(21) as a function of α for different values of the parameter $|\lambda|$ ($|\lambda|=0.1$ and 0.6) and fixed $\eta^2=0.7$. The results are shown in figures 3 and 4. From figure 3 for the new even NLCS it is seen that for the small value of $|\lambda|$ ($|\lambda|$=0.1) the Wigner function $W_+(\alpha)$ displays a single peak and the function has an almost Gaussian shape centered at the origin of phase space, which is analogous to the Wigner function for the vacuum state. This is reasonable and accurate owing to the dominance of the effects in the vacuum state over the effects in the higher excitations. As the value of $|\lambda|$ increases, the vacuum state begins to lose its higher probability in the number state expansion, a little single peak appears in the negative part of the distribution. The negativity of the Wigner function $W_+(\alpha)$ means the new even NLCS are nonclassical states and exhibit some nonclassical effects. When $|\lambda|$ is large enough, the negative part is larger, and the little single peak in the negative part of the distribution begins to split into two peaks and the distance between the two peaks is greater. For the new even NLCS $W_+(\alpha)$ deviates far from the Gaussian distribution, the ring feature starts being formed.

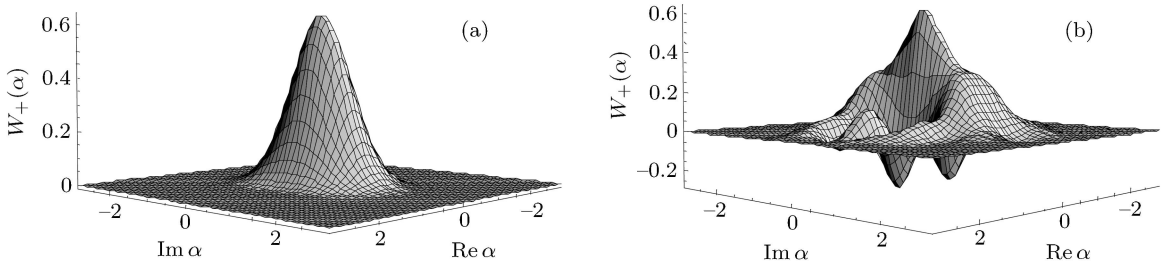


Fig.3. Wigner functions for the new even NLCS for $\eta^2=0.7$ and (a) $\lambda=0.1$, (b) $\lambda=0.6$.

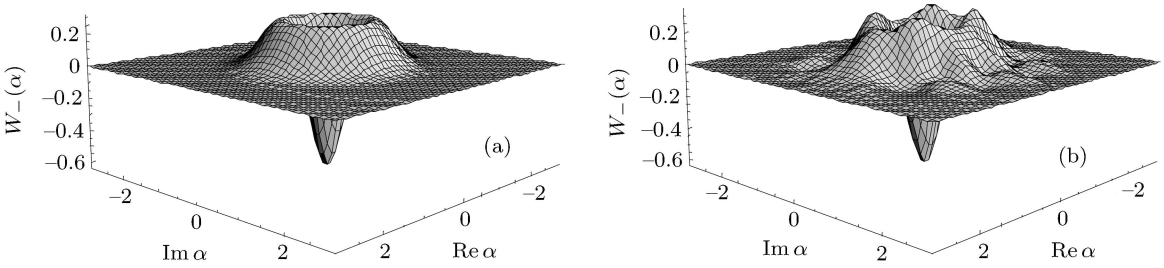


Fig.4. Wigner functions for the new odd NLCS for $\eta^2=0.7$ and (a) $\lambda=0.1$, (b) $\lambda=0.6$.

As shown in figure 4, the new odd NLCS the Wigner function $W_-(\alpha)$ always has a single peak, which is surrounded by an inverted crater at the origin, in the negative part of the distribution as increasing $|\lambda|$, it means the new odd NLCS are also nonclassical states and exhibit some nonclassical effects. When $|\lambda|$ is large enough, there is a multiple peak structure at the origin of the carter in the positive part of phase space. This signifies that the variations of $W_-(\alpha)$ are not regular and more sensitive to the greater $|\lambda|$. From the above results we can see that the Wigner distributions for the new even NLCS distinct from those of the new odd NLCS.

4. Tomograms of the new EONLCS

In the following section we want to derive the tomograms, which can provide the possibility of describing quantum states with a positive probability distribution, of the states $|\lambda, f\rangle_\pm$. The tomograms of the states are defined as[19]

$$T_\pm(x,\mu,\nu) = \iint \mathrm{d}p'\mathrm{d}x'\delta(x-\mu x'-\nu p')W_\pm(x',p'), \tag{23}$$

where $W_\pm(x',p')$ are the Wigner function for the states given by equation (21). Since a direct description of quantum states by means of quantum tomograms for the system observables is interesting from both the theoretical and experimental points of view, in recent years the tomogram approach has attracted much interest from physicists. However, it will be very diffcult to evaluate the integration if we directly substitute equations (21) and (22) into Eq.(23), fortunately we use the following relation between the Wigner operator and the projector of intermediate coordinate-momentum state[19]

$$|x\rangle_{\mu,\nu\mu,\nu}\langle x| = \iint \mathrm{d}p'\mathrm{d}x'\delta(x-\mu x'-\nu p')\Delta(x',p'), \tag{24}$$

where the state $|x\rangle_{\mu,\nu}$ is expressed as

$$\begin{aligned}|x\rangle_{\mu,\nu} = {} & \frac{1}{[\pi(\mu^2+\nu^2)]^{1/4}} \\ & \times \exp\left[-\frac{x^2}{2(\mu^2+\nu^2)}+\frac{\sqrt{2}x}{\mu-\mathrm{i}\nu}a^\dagger \right. \\ & \left. -\frac{\mu+\mathrm{i}\nu}{2(\mu-\mathrm{i}\nu)}a^{\dagger 2}\right]|0\rangle, \end{aligned} \tag{25}$$

which is the eigenvector of the operator $(\mu X+\nu P)$ corresponding to the eigenvalue x, μ and ν are two real parameters. $|x\rangle_{\mu,\nu}$ becomes the coordinate (momentum) eigenstate respectively when $\mu=1$, $\nu=0$ ($\mu=0$, $\nu=1$), for this reason we assign $|x\rangle_{\mu,\nu}$ the intermediate coordinate-momentum state. Thus the tomograms of the states $|\lambda, f\rangle_\pm$ are

$$\begin{aligned} T_\pm(x,\mu,\nu) = {} & \iint \mathrm{d}p'\mathrm{d}x'\delta(x-\mu x'-\nu p') \\ & \times {}_\pm\langle\lambda, f|\,\Delta(x',p')\,|\lambda,f\rangle_\pm \\ = {} & \left|{}_{\mu,\nu}\langle x|\,\lambda,f\rangle_\pm\right|^2, \end{aligned} \tag{26}$$

which shows that for the tomographic approach there exists an intermediate coordinate-momentum state $|x\rangle_{\mu,\nu}$, and the Radon transforms of the Wigner operator are just the pure-state density matrices $|x\rangle_{\mu,\nu\mu,\nu}\langle x|$. As a result, the tomogram functions for quantum states can be considered as the module-square of the states' wave function in this representation. This is a new way to derive the tomogram functions of quantum states. Using the coherent state's completeness relation[16]

$$\int \frac{\mathrm{d}^2 z}{\pi}\,|z\rangle\langle z| = 1 \tag{27}$$

and the following formula[19]

$$\begin{aligned} & \int \frac{\mathrm{d}^2 z}{\pi}\exp\left[\zeta|z|^2+\xi z+\eta z^*+fz^2+gz^{*2}\right] \\ & = \frac{1}{\sqrt{\zeta^2-4fg}}\exp\left[\frac{-\zeta\xi\eta+\xi^2 g+\eta^2 f}{\zeta^2-4fg}\right] \end{aligned} \tag{28}$$

with

$$\mathrm{Re}\,(\zeta\pm f\pm g)<0, \quad \mathrm{Re}\left(\frac{\zeta^2-4fg}{\zeta\pm f\pm g}\right)<0,$$

as well as the generating function of $H_n(x)$[18]

$$H_n(x) = \frac{\mathrm{d}^n}{\mathrm{d}x^n}\mathrm{e}^{2xt-t^2}\bigg|_{t=0}, \tag{29}$$

we can obtain the tomogram amplitude of the state $|\lambda, f\rangle_+$

$$
\begin{aligned}
{}_{\mu,\nu}\langle x|\ \lambda,f\rangle_{+} &= {}_{\mu,\nu}\langle x|\int\frac{\mathrm{d}^2z}{\pi}\,|z\rangle\,\langle z|\ \lambda,f\rangle_{+}\\
&=\frac{C_0}{[\pi(\mu^2+\nu^2)]^{1/4}}\sum_{n=0}^{\infty}\frac{f\,(2n)!!\lambda^n}{2^n n!}\int\frac{\mathrm{d}^2z}{\pi}z^{*2n}\\
&\quad\times\exp\left[-|z|^2+\frac{\sqrt{2}x}{\mu+\mathrm{i}\nu}z-\frac{\mu-\mathrm{i}\nu}{2\,(\mu+\mathrm{i}\nu)}z^2-\frac{x^2}{2(\mu^2+\nu^2)}\right]\\
&=\frac{C_0}{[\pi(\mu^2+\nu^2)]^{1/4}}\sum_{n=0}^{\infty}\frac{f\,(2n)!!\lambda^n}{2^n n!}\frac{\partial^{2n}}{\partial k^{2n}}\\
&\quad\times\int\frac{\mathrm{d}^2z}{\pi}\exp\left[-|z|^2+\frac{\sqrt{2}x}{\mu+\mathrm{i}\nu}z+kz^*\,\frac{\mu-\mathrm{i}\nu}{2\,(\mu+\mathrm{i}\nu)}z^2-\frac{x^2}{2(\mu^2+\nu^2)}\right]\Bigg|_{k=0}\\
&=\frac{C_0}{[\pi(\mu^2+\nu^2)]^{1/4}}\sum_{n=0}^{\infty}\frac{f\,(2n)!!\lambda^n}{2^n n!}\left(\frac{\mu-\mathrm{i}\nu}{2\,(\mu+\mathrm{i}\nu)}\right)^n\\
&\quad\times H_{2n}\left(\frac{x}{\sqrt{\mu^2+\nu^2}}\right)\exp\left[-\frac{x^2}{2(\mu^2+\nu^2)}\right].
\end{aligned}
\tag{30}
$$

Then using Eq.(26) the tomogram of the state $|\lambda,f\rangle_{+}$ is

$$
\begin{aligned}
T_{+}(x,u,v) &= \frac{|C_0|^2}{\sqrt{\pi(\mu^2+\nu^2)}}\left|\sum_{n=0}^{\infty}\frac{f\,(2n)!!\lambda^n}{2^n n!}\left(\frac{\mu-\mathrm{i}\nu}{2\,(\mu+\mathrm{i}\nu)}\right)^n\right.\\
&\quad\left.\times H_{2n}\left(\frac{x}{\sqrt{\mu^2+\nu^2}}\right)\exp\left[-\frac{x^2}{2(\mu^2+\nu^2)}\right]\right|^2.
\end{aligned}
\tag{31}
$$

In the same way, we can also derive the tomogram of the state $|\lambda,f\rangle_{-}$

$$
\begin{aligned}
T_{-}(x,u,v) &= \frac{|C_1|^2}{\sqrt{\pi(\mu^2+\nu^2)}}\left|\sum_{n=0}^{\infty}\frac{2^n n!f\,(2n+1)!!\lambda^n}{(2n+1)!}\left(\sqrt{\frac{\mu-\mathrm{i}\nu}{2\,(\mu+\mathrm{i}\nu)}}\right)^{2n+1}\right.\\
&\quad\left.\times H_{2n+1}\left(\frac{x}{\sqrt{\mu^2+\nu^2}}\right)\exp\left[-\frac{x^2}{2(\mu^2+\nu^2)}\right]\right|^2.
\end{aligned}
\tag{32}
$$

It is a good function of describing a quantum state with a positive probability distribution function like the Husimi Q-function. Therefore, in an experiment if we can measure $\left|{}_{\mu,\nu}\langle x|\ \lambda,f\rangle_{\pm}\right|^2$ in the intermediate coordinate-momentum representation, the tomograms of the states $|\lambda,f\rangle_{\pm}$ are obtained and further the states $|\lambda,f\rangle_{\pm}$ are measured.

In conclusion, by making use of numerical computation we have studied the Wigner W-function and Husimi Q-function for the new EONLCS. We find that the distributions for the new even NLCS distinct from those of the new odd NLCS and these states can always exhibit some different nonclassical behaviour. Finally, with the aid of newly introduced intermediate coordinate-momentum representation in quantum optics, the tomograms of the states are calculated. This is a new way that is also suitable to obtain tomograms of other quantum states. Moreover, the results in this paper may be useful for experiments as reference.

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