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Influence of the virtual photon field on the squeezing properties of an atom laser*

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This paper investigates the squeezing properties of an atom laser without rotating-wave approximation in the system of a binomial states field interacting with a two-level atomic Bose–Einstein condensate. It discusses the influences of atomic eigenfrequency, the interaction intensity between the optical field and atoms，parameter of the binomial states field and virtual photon field on the squeezing properties. The results show that two quadrature components of an atom laser can be squeezed periodically. The duration and the degree of squeezing an atom laser have something to do with the atomic eigenfrequency and the parameter of the binomial states field, respectively. The collapse and revival frequency of atom laser fluctuation depends on the interaction intensity between the optical field and atoms. The effect of the virtual photon field deepens the depth of squeezing an atom laser.

Keywords: quantum optics, Bose–Einstein condensation, atom laser, virtual photon field
PACC: 4250, 3280

1. Introduction

Since the first observations of Bose–Einstein condensation (BEC) in an atomic gas in 1995,[1,2] there has been much interest in creating an atom laser and exploring its novel properties. In 1997,[3] the MIT-group first realized a pulsed atom laser, by using a radio frequency (RF) pulse to transfer the initially trapped condensate into the untrapped state. Later, successive experimental achievements in the design and amplification of the atom laser were obtained.[4−7] These achievements aroused interest resulting in theoretical works about both the output coupling and the properties of atom laser.[8,9] Moore *et al* pointed out the possibility of realizing optical control on the quantum statistics of the output matter wave in the framework of nonlinear atomic optics.[10] In recent years, a large amount of experimental and theoretical research have focused on the generation of BEC, its unique features and the interaction between atomic BEC and optical field have been carried out with great interest, and a series of important results have been achieved.[11−31] As is well known, the interaction between optical field and atoms plays an important role in atomic cooling and the preparation and detection of BEC. Thus a further study on the optical properties of BEC not only is favorable to investigate the intrinsic characteristics of BEC but also provides effective methods for its preparation and detection.

Quantum properties of the system of an optical field interacting with atomic BEC have nearly been studied with rotating-wave approximation recently. However, a mass of research results shows that the virtual photon process is actual during the interaction between optical field and atoms, which changes the quantum properties of the system and brings quantum noise.[32] Thereby the research on the virtual photon process is of much theoretical and practical value, which gradually attracts a lot of attention.

In this paper, the squeezing properties of an atom laser are investigated without rotating-wave approximation in the system of the binomial states field interacting with two-level atomic BEC. We gain the time evolution of atom laser fluctuation via numerical calculation and discuss the influences of atomic eigenfrequency, interaction intensity between the optical field and atoms, parameter of the optical field and virtual photon field on the squeezing properties of an atom

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laser.

2. Model and solution

Taking into account the system of the binomial states field interacting with atomic BEC, for simplicity, we shall assume that the atoms have two states, $|1\rangle$ and $|2\rangle$, with the initial condensation occurring in the trapped state $|1\rangle$. State $|2\rangle$, which has different trapping properties and is typically unconfined by the magnetic trap, is coupled to state $|1\rangle$ by a one-mode optical field tuned near the $|1\rangle \rightarrow |2\rangle$ transition. The interaction of the field may thus generate condensate in the state $|2\rangle$, from an initial condensate which is entirely in the state $|1\rangle$. The Hamiltonian of the system can be written as ($\hbar = 1$)

$$\begin{aligned} H =& \omega_0 b_2^\dagger b_2 + \omega_a a^\dagger a \\ &+ g' \left(a b_1 b_2^\dagger + a^\dagger b_1^\dagger b_2 + a b_1^\dagger b_2 + a^\dagger b_1 b_2^\dagger \right), \end{aligned} \quad (1)$$

in terms of the creation and annihilation operators, $b_1^\dagger$, $b_2^\dagger$, b_1 and b_2, of bosonic atoms for the magnetically trapped state $|1\rangle$ and the untrapped state $|2\rangle$ with level difference ω_0. The $a^\dagger$ and a are the creation and annihilation operators of the optical field with frequency ω_a, and g' is the interaction intensity between the optical field and atoms. The nonlinear interaction among the atoms as well as the quantized motion of the atomic center of mass in the state trapped by an inhomogeneous magnetic field has been ignored,[33] which is the main simplification of our model.

We suppose that the initial state of the system is theoretically described as

$$|\psi(0)\rangle = |\alpha\rangle_1 \otimes |\Phi(0)\rangle_s \quad (2)$$

with $|\Phi(0)\rangle_s = |0\rangle_2 \otimes |\eta, M\rangle$. Here $|\alpha\rangle_1$ is a Glauber coherent state of the operator b_1 characterizing the condensed atoms in the trapped state $|1\rangle$, namely, $b_1 |\alpha\rangle_1 = \sqrt{N_0} \mathrm{e}^{-\mathrm{i}\theta} |\alpha\rangle_1$, where N_0 is the average particle number of the trapped state $|1\rangle$. The state $|0\rangle_2$ represents that the initial untrapped state $|2\rangle$ is a vacuum state since there is no occupying atoms in it. The initial state of the input optical field is binomial states[34]

$$|\eta, M\rangle = \sum_{n=0}^{M} \beta_n^M |n\rangle, \quad (0 < \eta < 1, M > 0), \quad (3)$$

where $\beta_n^M = \left[\frac{M!}{n!(M-n)!} \eta^n (1-\eta)^{M-n} \right]^{1/2}$, which is the photon distributing function of the binomial states field representing sub-Poisson distribution. The state $|n\rangle$ denotes the Fock state of the optical field. The $\{\eta, (1-\eta)\}$ denote two possible outputing probabilities of the Bernoulli experiment and M is the maximum photon number.

Here we only discuss the case of weak optical field. In order to facilitate the solution of the dynamic equation for the system, the Bogoliubov's approximation is adopted. Namely, it is assumed that the atom number of BEC is so large at the initial time that the slow change of the atom number in the ground state can be neglected in the process of the interaction with the optical field. So we can replace b_1 and $b_1^\dagger$ with $\sqrt{N_0}\mathrm{e}^{-\mathrm{i}\theta}$ and $\sqrt{N_0}\mathrm{e}^{\mathrm{i}\theta}$. For simplicity, we only consider the case of resonance, namely, $\omega_a = \omega_0 = \omega$, and replace b_2, $b_2^\dagger$ and $g'\sqrt{N_0}$ with b, $b^\dagger$ and g. Then, the Hamiltonian of the system can be written as

$$\begin{aligned} H =& \omega b^\dagger b + \omega a^\dagger a \\ &+ g \left(a b^\dagger + a^\dagger b + a b + a^\dagger b^\dagger \right). \end{aligned} \quad (4)$$

According to Eq.(4), the matrix format of the Heisenberg equations on the operators b, a, $b^\dagger$ and $a^\dagger$ is

$$\mathrm{i}\frac{\partial}{\partial t} \begin{pmatrix} b(t) \\ a(t) \\ b^\dagger(t) \\ a^\dagger(t) \end{pmatrix} = \begin{pmatrix} \omega & g & 0 & g \\ g & \omega & g & 0 \\ 0 & -g & -\omega & -g \\ -g & 0 & -g & -\omega \end{pmatrix} \begin{pmatrix} b(t) \\ a(t) \\ b^\dagger(t) \\ a^\dagger(t) \end{pmatrix}. \quad (5)$$

By applying the technique of diagonalizing the coefficient matrix, we can obtain the exact solutions of the coupling equations as follows:

$$\begin{aligned} b(t) &= \gamma_1 b(0) + \gamma_2 b^\dagger(0) + \gamma_3 a(0) + \gamma_4 a^\dagger(0), \\ a(t) &= \gamma_1 a(0) + \gamma_2 a^\dagger(0) + \gamma_3 b(0) + \gamma_4 b^\dagger(0), \end{aligned} \quad (6)$$

where

$$\gamma_1 = \frac{1}{2} \left(\cos(\lambda_1 t) + \cos(\lambda_2 t) - \frac{\mathrm{i}(\omega - g)}{\lambda_1} \sin(\lambda_1 t) - \frac{\mathrm{i}(\omega + g)}{\lambda_2} \sin(\lambda_2 t) \right),$$

$$\gamma_2 = \frac{\mathrm{i}g}{2}\left(\frac{\sin(\lambda_1 t)}{\lambda_1} - \frac{\sin(\lambda_2 t)}{\lambda_2}\right),$$
$$\gamma_3 = \frac{1}{2}\left(-\cos(\lambda_1 t) + \cos(\lambda_2 t) + \frac{\mathrm{i}(\omega - g)}{\lambda_1}\sin(\lambda_1 t) - \frac{\mathrm{i}(\omega + g)}{\lambda_2}\sin(\lambda_2 t)\right),$$
$$\gamma_4 = -\frac{\mathrm{i}g}{2}\left(\frac{\sin(\lambda_1 t)}{\lambda_1} + \frac{\sin(\lambda_2 t)}{\lambda_2}\right),$$
$$\lambda_1 = \sqrt{\omega^2 - 2\omega g}, \quad \lambda_2 = \sqrt{\omega^2 + 2\omega g}.$$

3. Squeezing of an atom laser

In order to investigate the squeezing properties of an atom laser, we define its quadrature component operators as

$$U_1 = \frac{1}{2}(b + b^\dagger), \tag{7}$$

$$U_2 = \frac{1}{2\mathrm{i}}(b - b^\dagger), \tag{8}$$

where U_1 and U_2 satisfy

$$[U_1, U_2] = \frac{\mathrm{i}}{2}, \tag{9}$$

and the uncertainty relationship is

$$(\Delta U_1)^2 (\Delta U_2)^2 \geq 1/16. \tag{10}$$

For analysing the quantum fluctuation of the atom laser, we introduce

$$Q_i = (\Delta U_i)^2 - \frac{1}{4}, \quad (i = 1, 2). \tag{11}$$

Quantum noise of the i-th quatrature component will be squeezed, if $Q_i < 0$ $(i = 1, 2)$. From Eqs.(2) and (6), one can get

$$\langle a(t)\rangle = (\gamma_1 + \gamma_2)\xi_1, \tag{12}$$
$$\langle a^2(t)\rangle = \left(\gamma_1^2 + \gamma_2^2\right)\xi_2 + \gamma_1\gamma_2(2\eta M + 1) + \gamma_3\gamma_4, \tag{13}$$
$$\langle a^\dagger(t)\, a(t)\rangle = \eta M\left(|\gamma_1|^2 + |\gamma_2|^2\right) + \xi_2(\gamma_1^*\gamma_2 + \gamma_1\gamma_2^*) + |\gamma_2|^2 + |\gamma_4|^2, \tag{14}$$

where

$$\xi_1 = \sum_{n=0}^{M} \frac{M!\eta^n (1-\eta)^{M-n-1}}{n!\,(M-n)!}\sqrt{\eta(1-\eta)(M-n)}, \quad \xi_2 = \sum_{n=0}^{M} \frac{M!\eta^{n+1}(1-\eta)^{M-n-1}}{n!\,(M-n)!}\sqrt{(M-n)(M-n-1)}.$$

According to Eqs.(11)–(14), we have the results:

$$Q_1 = \frac{1}{4}\left\{\begin{array}{l} -\xi_1^2(\gamma_1 + \gamma_1^* + \gamma_2 + \gamma_2^*)^2 + \xi_2\left[(\gamma_1 + \gamma_2^*)^2 + (\gamma_1^* + \gamma_2)^2\right] + 2\eta M \times \\ (\gamma_1 + \gamma_2^*)(\gamma_1^* + \gamma_2) + \gamma_1\gamma_2 + \gamma_1^*\gamma_2^* + \gamma_3\gamma_4 + \gamma_3^*\gamma_4^* + 2\gamma_2\gamma_2^* + 2\gamma_4\gamma_4^* \end{array}\right\}, \tag{15}$$

$$Q_2 = -\frac{1}{4}\left\{\begin{array}{l} -\xi_1^2(\gamma_1 - \gamma_1^* + \gamma_2 - \gamma_2^*)^2 + \xi_2\left[(\gamma_1 - \gamma_2^*)^2 + (\gamma_1^* - \gamma_2)^2\right] + 2\eta M \times \\ (\gamma_1 - \gamma_2^*)(\gamma_2 - \gamma_1^*) + \gamma_1\gamma_2 + \gamma_1^*\gamma_2^* + \gamma_3\gamma_4 + \gamma_3^*\gamma_4^* - 2\gamma_2\gamma_2^* - 2\gamma_4\gamma_4^* \end{array}\right\}. \tag{16}$$

Because the inherent laws of Q_1, Q_2 are difficult to find from Eqs.(15) and (16), we analyze the squeezing properties of an atom laser via numerical calculation. We only give the time evolution curve of Q_1, as shown in Fig.1, for the relationship of Q_1 and Q_2 is symmetrical.

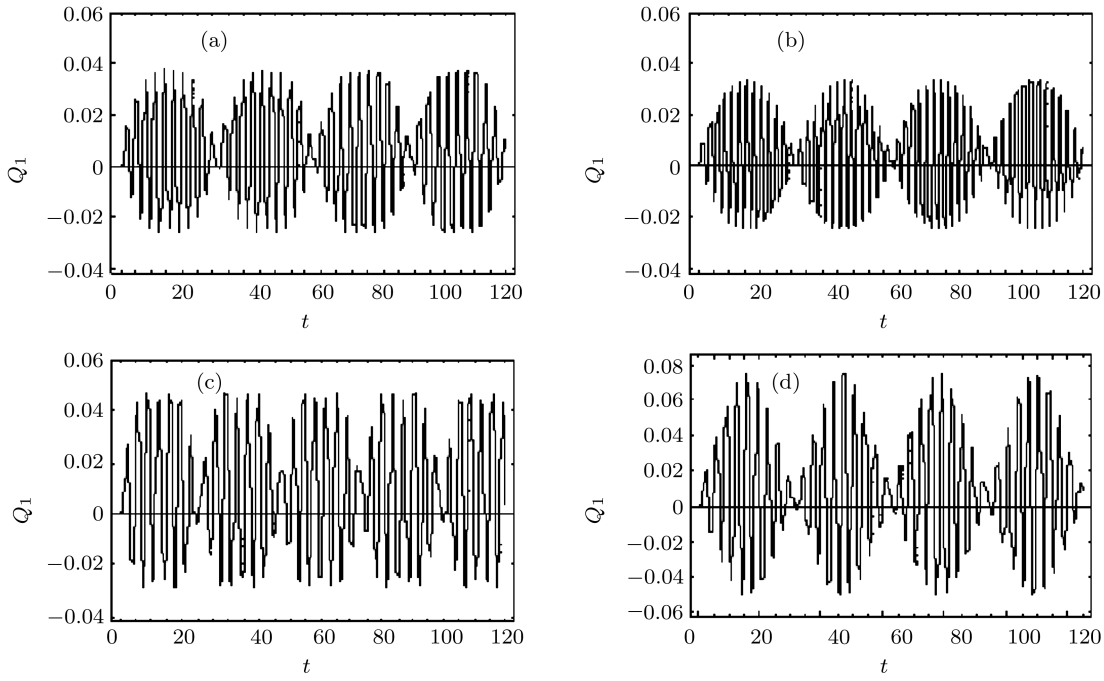
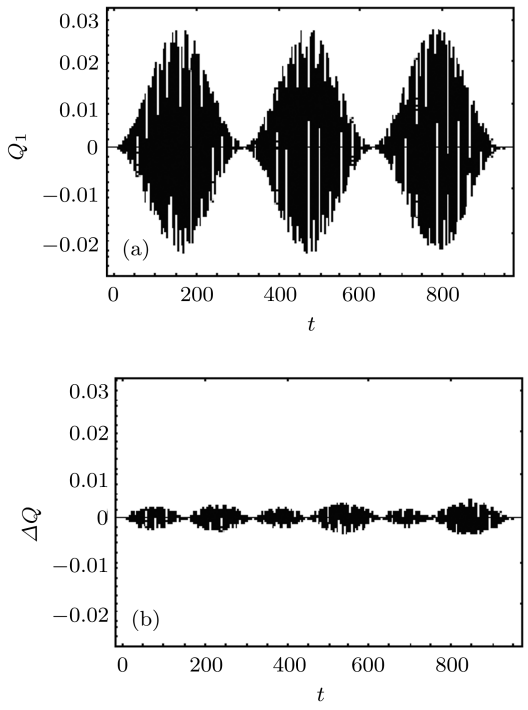


Fig.1. The time evolution of Q_1 with (a) $\omega = 1.0$, $g = 0.1$, $\eta = 0.1$; (b) $\omega = 1.4$, $g = 0.1$, $\eta = 0.1$; (c) $\omega = 1.0$, $g = 0.12$, $\eta = 0.1$; (d) $\omega = 1.0$, $g = 0.1$, $\eta = 0.2$.

From Fig.1 we can find that Q_1 varies periodically with time and the quadrature component U_1 is squeezed periodically without rotating-wave approximation. There are two modes of oscillation in Fig.1. One is quantum Rabi oscillation, which is generated from atomic frequent emission and absorption of photons. The other is the collapse and revival effect, which appears with the interaction between the optical field and atoms. The former changes quickly. From Figs.1(a) and 1(b), we can find that the larger the atomic eigenfrequency is, the bigger the quantum Rabi oscillation frequency turns, while the collapse and revival frequency is invariable. The collapse and revival frequency varies in the same direction with the interaction intensity g between the optical field and atoms from Figs.1 (a) and 1(c). By comparing Fig.1(d) with Fig.1(a), we can easily find that the bigger the distributing parameter of the binomial states field η is, the deeper the squeezing degree of U_1 turns.

In order to fully illuminate the influence of the virtual photon field on the squeezing properties of an atom laser in the system of the binomial states field interacting with atomic BEC, we get the counterpart Q_{RW1} of Q_1 with rotating-wave approximation by the same method of calculating Q_1 and obtain the time evolution curve of Q_1 and $\Delta Q\,(= Q_1 - Q_{\mathrm{RW1}})$ with the same parameter via numerical calculation, as shown in Fig.2.



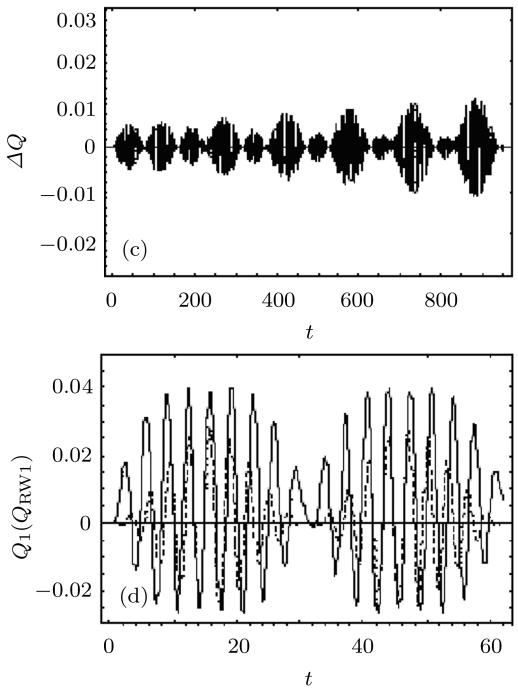


Fig.2. (a) The time evolution of Q_1 with $\omega = 1.0$, $g = 0.01$, $\eta = 0.1$. (b) The time evolution of ΔQ with $\omega = 1.4$, $g = 0.01$, $\eta = 0.1$. (c) The time evolution of ΔQ with $\omega = 1.0$, $g = 0.02$, $\eta = 0.1$. (d) The time evolution of Q_1 (solid line) and $Q_{\rm RW1}$ (dashed line) with $\omega = 1.0$, $g = 0.1$, $\eta = 0.1$.

By comparing Fig.2(b) with Fig.2(a), it can be found that the effect of the virtual photon field is a little oscillation superposing on the Rabi oscillation of $Q_{\rm RW1}$, which resembles collapse and revival oscillation. The reason is that the coherence of the virtual photon process and actual photon process in the system of the optical field interact with atomic BEC. By comparing Fig.2(b) with Fig.2(c), we can find that when the parameter g turns larger, the virtual photon effect becomes enhanced. So rotating-wave approximation cannot be applied in the system of strong optical field interacting with atomic BEC. It is shown that the degree of squeezing an atom laser strengthens without rotating-wave approximation in Fig.2(d).

4. Conclusion

In this paper, we study the squeezing properties of an atom laser by considering the virtual photon field in the system of the binomial states field interacting with two-level atomic BEC. The results show that two quadrature components of an atom laser can be squeezed periodically. The Rabi frequency of an atom laser is determined by atomic eigenfrequency, whereas the collapse and revival frequency of atom laser fluctuation are determined by the interaction intensity between the optical field and atoms. When the distributing parameter of the binomial states field turns bigger, the depth of squeezing an atom laser deepens distinctly. The effect of the virtual photon field is a little oscillation superposing on the Rabi oscillation of an atom laser with rotating-wave approximation, which deepens the depth of squeezing an atom laser.

References

[1] Anderson M H Jr, Ensher M R, Matthews, Wieman C E and Cornell E A 1995 *Science* **269** 198
[2] Davis K B, Mewes M O, Andrews M R, van Druten N J, Durfee D S, Kurn D M and Ketterle W 1995 *Phys. Rev. Lett.* **75** 3969
[3] Mewes M O, Andrews M R, Kurn D M, Durfee D S, Townsend C G and Ketterle W 1997 *Phys. Rev. Lett.* **78** 582
[4] Anderson B P and Kasevich M A 1998 *Science* **282** 1686
[5] Hagley E W, Deng L, Kozuma M, Wen J, Helmerson K, Rolston S L and Phillips W D 1999 *Science* **283** 1706
[6] Bloch I, Hansch T W and Esslinger T 1999 *Phys. Rev. Lett.* **82** 3008
[7] Kozuma M, Suzuki Y, Torii Y, Sugiura T, Kuga T, Hagley E W and Deng L 1999 *Science* **286** 2309
[8] Moy G M and Savage C M 1997 *Phys. Rev.* A **56** 1087
[9] Steck H, Naraschevski M and Wallis H 1998 *Phys. Rev. Lett.* **80** 1
[10] Moore M G, Zobay O and Meystre P 1999 *Phys. Rev.* A **60** 1491
[11] Wang H X and Yang S P 2008 *Acta Phys. Sin.* **57** 4700 (in Chinese)
[12] Lignier H, Sias C, Ciampini D, Singh Y, Zenessini A, Morsch O and Arimondo E 2007 *Phys. Rev. Lett.* **99** 220403
[13] You L, Lewenstein M and Cooper J 1995 *Phys. Rev.* A **51** 4712
[14] Sun C, Zhan H, Miao Y and Li J 1998 *Commun. Theor. Phys.* **29** 161
[15] Zheng X J, Cao S, Fang M F and Liao X P 2008 *Chin. Phys.* B **17** 431
[16] Wang R P and Zhang H R 2008 *Chin. Phys.* B **17** 194
[17] Zhou M, Fang J and Huang C 2003 *Acta Phys. Sin.* **52** 1916 (in Chinese)
[18] Zhang J M, Liu W M and Zhou D L 2008 *Phys. Rev.* A **77** 033620
[19] Kuang L M and Ouyang Z W 2000 *Phys. Rev.* A **61** 023604
[20] Wen L H, Liu M, Xiong H W and Zhan M S 2005 *Eur. Phys. J.* D **36** 89
[21] Huang C J, Fang J Y, He H Y, Kong F Z and Zhou M 2008 *Physica* A **387** 3449

[22] Yin J P, Gao W J, Hu J J and Wang Y Q 2002 *Opt. Commun.* **206** 99
[23] Wu Y and Yang X X 2004 *Opt. Lett.* **29** 839
[24] Ji A C, Liu W M, Song J L and Zhou F 2008 *Phys. Rev. Lett.* **101** 010402
[25] Haine S A, Olsen M K and Hope J J 2006 *Phys. Rev. Lett.* **96** 133601
[26] Yang W F, Gong S Q, Li R X and Xu Z Z 2006 *Phys. Rev.* A **74** 013407
[27] Wen L H, Wang J S, Feng J and Hu H Q 2008 *J. Phys.* B: *At. Mol. Opt. Phys.* **41** 135301
[28] Yu Z X, Jiao Z Y, Jin S and Wang J S 2007 *Chin. Phys. Lett.* **24** 330
[29] Fang Y C and Yang Z A 2008 *Acta Phys. Sin.* **57** 7438 (in Chinese)
[30] Li J P, Tan L, Zang X F and Yang K 2008 *Acta Phys. Sin.* **57** 7467 (in Chinese)
[31] Zhou Y X, Xia Q F and Sun C Y 2008 *J. At. Mol. Phys.* **25** 633 (in Chinese)
[32] Peng J S and Li G X 1993 *Acta Phys. Sin.* **42** 568 (in Chinese)
[33] Jing H, Chen J L and Ge M L 2001 *Phys. Rev.* A **63** 015601
[34] Stoler D, Saleh B E A and Teich M C 1985 *Opt. Acta* **32** 45