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Entanglement and coherence of a three-level atom in Λ configuration interacting with two fields*

Zhang Jian-Song(张建松) and Xu Jing-Bo(许晶波)†

Zhejiang Institute of Modern Physics and Department of Physics, Zhejiang University, Hangzhou 310027, *China*

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We investigate the entanglement of a three-level atom in Λ configuration interacting with two quantized field modes by using logarithmic negativity. Then, we study the relationship of the atomic coherence and the entanglement between two fields which are initially prepared in vacuum or thermal states. We find that if the two fields are prepared in thermal states, the atomic coherence can induce the entanglement between two thermal fields. However, there is no coherence-induced entanglement between two vacuum fields.

Keywords: quantum entanglement, coherence, cavity QED
PACC: 4250

1. Introduction

Entanglement, one of the remarkable features of quantum mechanics, has been considered as a key resource of quantum information processing.[1−4] Entanglement can exhibit the nature of a nonlocal correlation between quantum systems that have no classical interpretation. In recent years, quantum information processing by utilizing cavity quantum electrodynamics (QED) has attracted much attention.[5−19] A cavity QED system, where atoms interacting with a quantized electromagnetic field inside a cavity, can be used to create entanglement between atoms in cavities and establish quantum communications between different optical cavities. Many efforts have been devoted to the study of the manipulation of quantum entanglement with atoms and photons in cavities. It has been shown that the entanglement can arise in the interaction of an atom with a cavity field in a thermal state[6,7] and two atoms with a cavity field.[13,14]

Recently, much attention has been focused on the study of N-level quantum systems to implement quantum information processing. For example, key distribution protocols based on entangled three-level quantum systems are proposed.[20] It was pointed out that quantum key distribution schemes using three-level systems are more secure against symmetric attacks than protocols using two-level systems.[21] Three-level systems are also used to implement quantum phase gates and controlled-NOT gates.[22,23]

On the other hand, atomic coherence leads to many quantum optical effects, such as electromagnetically induced transparency and lasing without inversion.[24] It has been shown that atomic coherence and entanglement are closely related for a system consisting of one three-level atom in V configuration interacting with two fields.[25] Further, the authors in Ref.[25] pointed out that atomic coherence, no matter how small it is, can lead to entanglement between two vacuum or thermal fields.

In this paper, we first study the entanglement of a three-level atom in Λ configuration interacting with two fields by adopting logarithmic negativity.[26−28] It is found that logarithmic negativity is a periodic function of time. Their periods and maximal values depend on the detuning and the initial state of the atom. We calculate the logarithmic negativity of the atom and two thermal fields. The influence of the mean photon numbers of the two thermal fields on the logarithmic negativity is also investigated. Then, we study the relationship of the atomic coherence and the entanglement between two thermal or vacuum fields interacting with a three-level atom in Λ configuration. We find that if the two fields are prepared in thermal states, the atomic coherence can lead to entangle-

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†Corresponding author. E-mail: xujb@zju.edu.cn

ment between two thermal fields. However, there is no coherence-induced entanglement between two vacuum fields.

2. Entanglement of a three-level atom interacting with two vacuum fields

Now we consider a three-level atom in the Λ configuration interacting with two quantized fields f and f'. Under the rotating-wave approximation, the Hamiltonian of the system reads ($\hbar = 1$)[29]

$$H = \sum_{i=1}^{3} \epsilon_i \sigma_{ii} + \sum_{i=1}^{2} \omega_i a_i^\dagger a_i + g(a_1\sigma_{31} + a_1^\dagger \sigma_{13} + a_2\sigma_{32} + a_2^\dagger \sigma_{23}), \quad (1)$$

where $a_1(a_2)$ and $a_1^\dagger(a_2^\dagger)$ are annihilation and creation operators of field 1 (field 2), ϵ_i is the atomic frequency of level i, ω_i is the frequency of mode i, g is the coupling constant and σ_{ij} $(i, j = 1, 2, 3)$ are atomic operators given by $\sigma_{ij} = |i\rangle\langle j|$.

In this section we assume the fields are initially in vacuum states. We investigate the bipartite entanglement of the system by using logarithmic negativity. If the whole system is initially prepared in the state

$$|\psi(0)\rangle = \cos\theta|300\rangle + \sin\theta\sin\varphi|200\rangle + \sin\theta\cos\varphi|100\rangle, \quad (2)$$

it is easy to find that the state at time t is

$$|\psi(t)\rangle = \eta_1(t)|100\rangle + \eta_2(t)|200\rangle + \eta_3(t)|300\rangle + \eta_4(t)(|201\rangle + |110\rangle) \quad (3)$$

with

$$\begin{aligned} &\eta_1(t) = \sin\theta\cos\varphi, \quad \eta_2(t) = \sin\theta\sin\varphi, \\ &\eta_3(t) = \cos\theta\alpha(t), \quad \eta_4(t) = \cos\theta\beta(t), \\ &\alpha(t) = \cos(\Omega t), \quad \beta(t) = -\frac{\mathrm{i}}{\sqrt{2}}\sin(\Omega t), \\ &\Omega = \sqrt{\frac{\Delta^2}{4} + 2}, \\ &\Delta = \epsilon_3 - \epsilon_1 - \omega_1 = \epsilon_3 - \epsilon_2 - \omega_2. \end{aligned} \quad (4)$$

The reduced density matrix of atom A and field f is

$$\begin{aligned} \rho_{Af}(t) = {} & [\eta_1(t)|10\rangle + \eta_2(t)|20\rangle + \eta_3(t)|30\rangle][\eta_1^*(t)\langle 10| \\ & + \eta_2^*(t)\langle 20| + \eta_3^*(t)\langle 30|] + |\eta_4(t)|^2(|20\rangle\langle 20| \\ & + |11\rangle\langle 11|) + [\eta_4(t)|11\rangle(\eta_1(t)^*\langle 10| \\ & + \eta_2(t)^*\langle 20| + \eta_3(t)^*\langle 30|) + \text{h.c}], \end{aligned} \quad (5)$$

where h.c stands for Hermitian conjugate.

In this study, we investigate the entanglement of quantum systems by using logarithmic negativity. In recent years, many measures of entanglement including[26−28] entanglement of formation (concurrence)[30,31] and von Neuman entropy[32] have been introduced. Logarithmic negativity is a computable measure of entanglement. It was proved by Plenio that logarithmic negativity is an entanglement monotone both under local operations and classical cummunication (LOCC) and the more general setting of positive partial transpose preserving operation.[33]

Here, we first study the bipartite entanglement between the atom and the field f by employing logarithmic negativity. By definition, a quantum state ρ of two modes 1 and 2 is separable if, and only if, it can be expressed in the following form

$$\rho = \sum_i p_i \rho_{i1} \otimes \rho_{i2} \quad (6)$$

with $p_i > 0$, $\sum_i p_i = 1$ and ρ_{i1} (ρ_{i2}) are the density operators of mode 1 (mode 2). It is clear that for a separable state the partial transpose of the density matrix $\rho^{T_2} = \sum_i p_i \rho_{i1} \otimes (\rho_{i2})^T$ has only non-negative eigenvalues. If there are negative eigenvalues of the partial transposed matrix of a state, then the state must be entangled. For a bipartite system ρ, the logarithmic negativity is

$$E(\rho) \equiv \log_2(1 + 2N) = \log_2 ||\rho^T||, \quad (7)$$

where ρ^T is the partial transpose of ρ , $||\rho^T||$ stands for the trace norm of ρ^T and N is negativity which is defined as[26]

$$N \equiv \frac{||\rho^T|| - 1}{2}, \quad (8)$$

which is just the absolute value of the sum of the negative eigenvalues of ρ^T.

In Fig.1, we plot the logarithmic negativity as a function of t for several values of θ and φ. From this figure, it is not difficult to find that the logarithmic negativity is a periodic function of time. The period of the logarithmic negativity depends on the detuning parameter. When the atom is initially prepared in the excited state ($|3\rangle$) ($\theta = \varphi = 0$), the maximal value of logarithmic negativity is about 0.6. For the initial states $1/\sqrt{2}(|3\rangle + |1\rangle)$ $(\theta = \pi/4, \varphi = 0)$ and $1/\sqrt{2}(|3\rangle + |2\rangle)$ $(\theta = \pi/4, \varphi = \pi/2)$, the maximal logarithmic negativity are about 0.5 and 0.45, respectively. We also plot the logarithmic negativity as a function

of time when the atom is initially prepared in state $(1/\sqrt{3})(|3\rangle + |2\rangle + |1\rangle)$ ($\theta = \arccos(1/\sqrt{3}), \varphi = \pi/4$). Compared with the other three initial states of the atom, we see that if the atom is initially prepared in the excited state $|3\rangle$, the period of logarithmic negativity is the smallest.

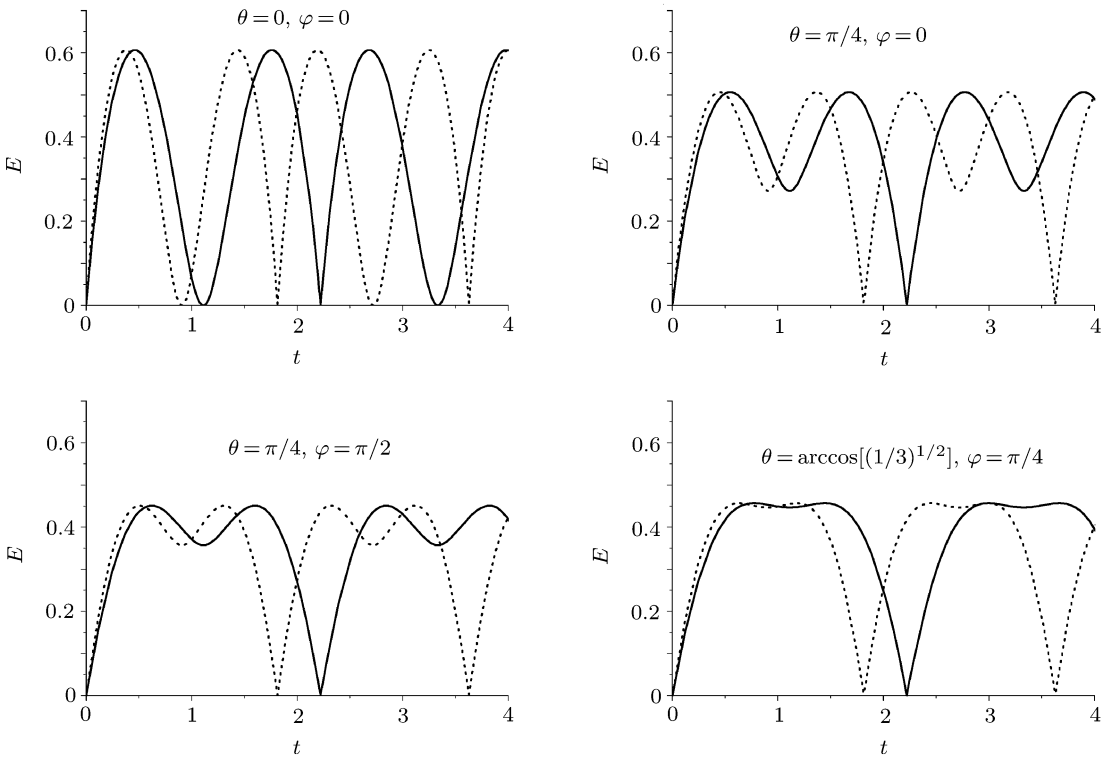


Fig.1. The logarithmic negativity E_{Af} is plotted as a function of t with $g = 1$ for $\Delta = 0$ (solid line) and $\Delta = 2$ (dotted line).

3. Entanglement of a three-level atom interacting with two thermal fields

Recently, it was pointed out that entanglement can be generated by the interaction of a large system in any mixed state with a qubit in a pure state.[6] In Ref.[29], they have illustrated this feature by employing the Jaynes–Cummings model. In this section, we study the entanglement of a three-level atom interacting with two thermal fields. For the sake of simplicity, we set $g = 1$ and $\Delta = \epsilon_3 - \epsilon_1 - \omega_1 = \epsilon_3 - \epsilon_2 - \omega_2 = 0$. We obtain a solution of the system using the dynamic algebraic approach[34]

$$\begin{aligned}
|1,n,m\rangle \to\ & \alpha_1(n,m,t)|1,n,m\rangle \\
& + \alpha_2(n,m,t)|2,n-1,m+1\rangle \\
& + \alpha_3(n,m,t)|3,n-1,m\rangle, \\
|2,n,m\rangle \to\ & \alpha_4(n,m,t)|2,n,m\rangle \\
& + \alpha_5(n,m,t)|1,n+1,m-1\rangle \\
& + \alpha_6(n,m,t)|3,n,m-1\rangle, \\
|3,n,m\rangle \to\ & \alpha_7(n,m,t)|3,n,m\rangle \\
& + \alpha_8(n,m,t)|1,n+1,m\rangle \\
& + \alpha_9(n,m,t)|2,n,m+1\rangle \qquad (9)
\end{aligned}$$

with

$$\begin{aligned}
\alpha_1(n,m,t) &= \mathrm{e}^{-\mathrm{i}\varphi_1 t}(\sin^2\theta_1 + \cos^2\theta_1 \cos\Omega_1 t), \\
\alpha_2(n,m,t) &= \mathrm{e}^{-\mathrm{i}\varphi_1 t}\sin\theta_1\cos\theta_1(\cos\Omega_1 t - 1), \\
\alpha_3(n,m,t) &= -\mathrm{i}\mathrm{e}^{-\mathrm{i}\varphi_1 t}\cos\theta_1 \sin\Omega_1 t, \\
\alpha_4(n,m,t) &= \mathrm{e}^{-\mathrm{i}\varphi_2 t}(\cos^2\theta_2 + \sin^2\theta_2 \cos\Omega_2 t), \\
\alpha_5(n,m,t) &= \mathrm{e}^{-\mathrm{i}\varphi_2 t}\sin\theta_2\cos\theta_2(\cos\Omega_2 t - 1), \\
\alpha_6(n,m,t) &= -\mathrm{i}\mathrm{e}^{-\mathrm{i}\varphi_2 t}\sin\theta_2 \sin\Omega_2 t, \\
\alpha_7(n,m,t) &= \mathrm{e}^{-\mathrm{i}\varphi_3 t}\cos\Omega_3 t, \\
\alpha_8(n,m,t) &= -\mathrm{i}\mathrm{e}^{-\mathrm{i}\varphi_3 t}\cos\theta_3 \sin\Omega_3 t,
\end{aligned}$$

$$\alpha_9(n,m,t) = -\mathrm{i}\mathrm{e}^{-\mathrm{i}\varphi_3}\sin\theta_3 t\sin\Omega_3 t,$$
$$\varphi_1 = \epsilon_3 + (n-1)\omega_1 + m\omega_2,$$
$$\varphi_2 = \epsilon_3 + n\omega_1 + (m-1)\omega_2,$$
$$\varphi_3 = \epsilon_3 + n\omega_1 + m\omega_2,$$
$$\theta_1 = \arctan\sqrt{\frac{m+1}{n}},$$
$$\theta_2 = \arctan\sqrt{\frac{m}{n}},$$
$$\theta_3 = \arctan\sqrt{\frac{m+1}{n+1}},$$
$$\Omega_1 = \Omega_2 = \sqrt{n+m+1},$$
$$\Omega_3 = \sqrt{n+m+2}, \tag{10}$$

where $|n\rangle$ is the Fock state. We assume the initial state of the whole system is

$$\rho(0) = |3\rangle\langle 3| \otimes \left(\sum_{n=0}^{\infty} p_n |n\rangle\langle n|\right) \otimes \left(\sum_{m=0}^{\infty} p'_m |m\rangle\langle m|\right) \tag{11}$$

with $p_n = \dfrac{\overline{n}^n}{(1+\overline{n})^{n+1}}$ and $\overline{n} = (\mathrm{e}^{\beta\omega}-1)^{-1}$ is the mean photon number at the inverse temperature β and ω is the frequency of the optical mode. The reduced density matrix of A and f is obtained by tracing out field f'

$$\rho_{Af}(t) = \sum_{n=0}^{\infty}[(a_n(t)|3\rangle\langle 3| + b_n(t)|2\rangle\langle 2| + c_n(t)|1\rangle\langle 1|) \otimes |n\rangle\langle n| + (\mathrm{i}d_n(t)|3\rangle\langle 1| \otimes |n\rangle\langle n+1| + \mathrm{h.c})],$$
$$a_n(t) = p_n \sum_{m=0}^{\infty} p'_m |\alpha_7(n,m,t)|^2,$$
$$b_n(t) = p_n \sum_{m=0}^{\infty} p'_m |\alpha_9(n,m,t)|^2,$$
$$c_n(t) = p_{n-1} \sum_{m=0}^{\infty} p'_m |\alpha_8(n-1,m,t)|^2,$$
$$d_n(t) = p_n \sum_{m=0}^{\infty} p'_m \alpha_7(n,m,t)\alpha_8(n,m,t)^*, \tag{12}$$

where we have used Eqs.(9). This reduced density matrix is defined in an infinite dimensional Hilbert space. In general, it is very difficult to measure entanglement of a general continuous variable state. Here, we study the entanglement of the above state by using the logarithmic negativity. It is easy to show that the logarithmic negativity is

$$E_{Af}(t) = \log_2\left[1 + \sum_{n=0}^{\infty}(|\lambda_n^-(t)| - \lambda_n^-(t))\right], \tag{13}$$

where

$$\lambda_n^-(t) = \frac{1}{2}[a_{n+1}(t) + b_{n+1}(t) + c_n(t) - \sqrt{[a_{n+1}(t) + b_{n+1}(t) - c_n(t)]^2 + 4d_n(t)^2}]. \tag{14}$$

In Fig.2, we show the logarithmic negativity as a function of time t for thermal states with different mean photon numbers. From Fig.2, one can clearly realize that the logarithmic negativity firstly increase with time t and reaches its maximal value, then decreases with time. Another feature is that the logarithmic negativity relies heavily on the mean photon numbers of the fields. It decreases with the mean photon numbers of fields. To show this feature more clearly, we plot the logarithmic negativity as a function of $\overline{n}$ and $\overline{m}$ at time $t = 0.5$ in Fig.3. Here, $\overline{n}$ and $\overline{m}$ are the mean photon numbers of fields f and f', respectively. It is easy to see that the logarithmic negativity reaches its maximal value $E = 0.6$ when the fields are in vacuum states, i.e., $\overline{n} = \overline{m} = 0$, which is consistent with the previous results (see with $\theta = \varphi = 0$). Our results show that high temperatures reduce entanglement between the atom and the field. In fact if the temperature is high enough all the entanglement vanishes as was pointed out by several authors.[35]

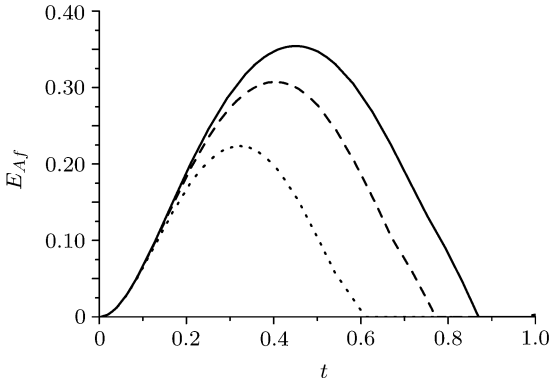


Fig.2. The logarithmic negativity E_{Af} is plotted as a function of t with $\overline{n} = 0.5$, $g = 1$ for $\overline{m} = 0$ (solid line), $\overline{m} = 0.5$ (dashed line), and $\overline{m} = 2$ (dotted line). Here, $\overline{n}$ and $\overline{m}$ are the mean photon numbers of fields f and f', respectively.

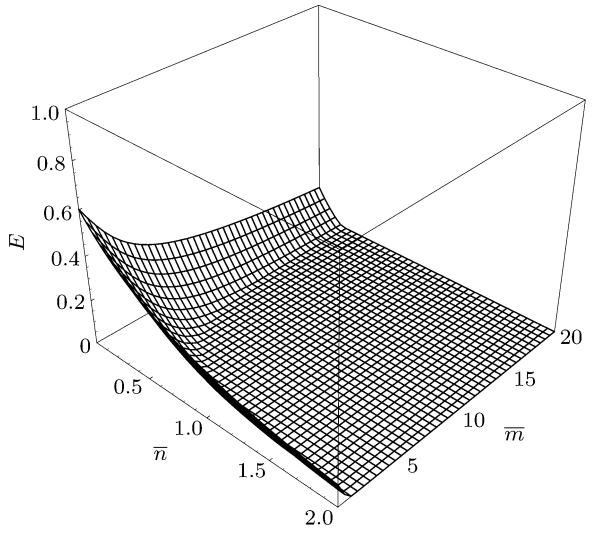


Fig.3. The logarithmic negativity E_{Af} is plotted as a function of the mean photon number of field f and f' with $t = 0.5$, $g = 1$.

4. Coherence-induced entanglement

In this section, we study the relationship of the atomic coherence and the entanglement of two thermal or vacuum fields. It has already been found that the atomic coherence and entanglement are closely related for a system consisting of one three-level atom in V configuration interacting with two fields.[25] The authors of Ref.[25] pointed out that the atomic coherence, no matter how small it is, can lead to entanglement between two vacuum or thermal fields. In the present paper, we show that two thermal fields can be entangled by interacting with a three-level atom with coherence in Λ configuration. However, two vacuum fields cannot be entangled via atomic coherence and interaction between the three-level atom and the fields which is different from Ref.[25].

The initial state of the atom-field system is

$$\begin{aligned}\rho(0) = {} & (l_{33}|3\rangle\langle 3| + l_{22}|2\rangle\langle 2| + l_{11}|1\rangle\langle 1| + l_{21}|2\rangle\langle 1| \\ & + l_{21}^{*}|1\rangle\langle 2|) \otimes \left(\sum_{n=0}^{\infty} p_n|n\rangle\langle n|\right) \\ & \otimes \left(\sum_{m=0}^{\infty} p'_m|m\rangle\langle m|\right). \end{aligned} \tag{15}$$

Inserting Eq.(9) into Eq.(15) and tracing over the degrees of the freedom of the atom leads to the reduced density matrix of fields

$$\begin{aligned}\rho_{ff'}(t) = {} & \sum_{n,m=0}^{\infty} [(h_{n,m}|n+1,m\rangle\langle n,m+1| + \text{h.c}) \\ & + e_{nm}|n,m\rangle\langle n,m|], \end{aligned} \tag{16}$$

where

$$\begin{aligned} e_{n,m} = {} & p_n p_m\{l_{33}[|\alpha_7(n,m)|^2 + |\alpha_8(n-1,m)|^2 \\ & + |\alpha_9(n,m-1)|^2] + l_{22}[|\alpha_4(n,m)|^2 \\ & + |\alpha_5(n-1,m+1)|^2 + |\alpha_6(n,m+1)|^2] \\ & + l_{11}[|\alpha_1(n,m)|^2 + |\alpha_2(n+1,m-1)|^2 \\ & + |\alpha_3(n+1,m)|^2]\}, \\ h_{n>0,m>0} = {} & p_n p_m l_{21}[\alpha_4(n+1,m)\alpha_2^{*}(n+1,m) \\ & + \alpha_5(n,m+1)\alpha_1^{*}(n,m+1) \\ & + \alpha_6(n+1,m+1)\alpha_3^{*}(n+1,m+1)], \\ h_{0,1} = {} & p_0 p'_1 l_{21} c_5(0,1)c_1^{*}(0,1), \\ h_{1,0} = {} & p_1 p'_0 l_{21} c_4(1,0)c_2^{*}(1,0), h_{0,0} = 0. \end{aligned} \tag{17}$$

The above state is a $2\times\infty$ quantum system whose entanglement is, in general, difficult to estimate. Here, we will follow the treatment of Refs.[16] and [25]. We first project the above reduced density matrix onto the subspace spanned by $\{|n\rangle, |n+1\rangle\} \otimes \{|m\rangle, |m+1\rangle\}$. This is a 2×2 quantum system. Then, we use the Peres–Horodecki criterion to check whether the system is entangled. The Peres–Horodecki criterion states that if there are negative eigenvalues of the partial transposed matrix of a state, then the state must be entangled. This inseparability criterion is a necessary and sufficient condition for (2×2) and (2×3)-dimensional systems, but ceases to be a sufficient condition for higher dimensional systems.[26−28]

Projecting the reduced density matrix $\rho_{ff'}$ on the subspace spanned by $\{|n\rangle, |n+1\rangle\}\otimes\{|m\rangle, |m+1\rangle\}$ and using the Peres–Horodecki criterion we find out that the reduced density matrix is entangled if

$$\eta_{n,m} = |h_{n,m}| - \sqrt{e_{n,m}e_{n+1,m+1}} > 0. \tag{18}$$

Note that in the case of $l_{21} = 0$, i.e., there is no atomic coherence, the reduced density matrix is $\rho_{ff'}(t) = \sum_{n,m} e_{n,m}|n,m\rangle\langle n,m|$. It is clear that the fields are separable. Thus, the two fields cannot be entangled without the atomic coherence. In Fig.4, we plot the parameter $\eta_{n,m}$ as a function of time. As we know, the projection is a local action which cannot increase the entanglement on average. Thus if the reduced density matrix is entangled in the subspace spanned by basis vectors

$\{|n\rangle, |n+1\rangle\} \otimes \{|m\rangle, |m+1\rangle\}$, i.e., $\eta_{n,m} > 0$, where n, m are arbitrary integers, the system is entangled in the whole Hilbert space. Bearing this in mind, one can easily see from Fig.4 that when the two fields are initially prepared in thermal states, they can be entangled due to atomic coherence and the interaction between the atom and fields. If the two fields are initially prepared in vacuum states, the reduced density matrix is a 2×2 system. Thus if $\eta_{00} \leq 0$, then the two fields are disentangled. In Ref.[25], the authors pointed out that for a three-level atom in V configuration interacting with two vacuum fields, there is coherence-induced entanglement between two fields no matter how small the atomic coherence is. However, as we can see from Eqs.(17) and (18), $\eta_{00} < 0$, i.e., for a three-level atom in Λ configuration the atomic coherence cannot lead to entanglement between two vacuum fields.

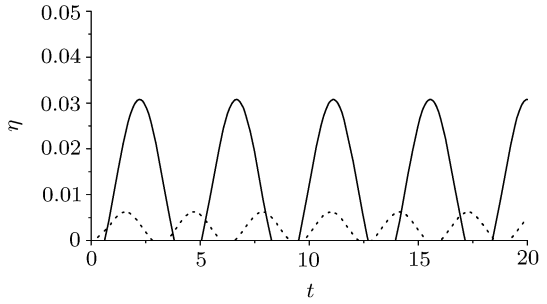


Fig.4. $\eta_{n,m}$ is plotted as a function of time t for $\overline{n} = \overline{m} = 0.1$ with $l_{33} = l_{22} = l_{11} = 1/3$, $l_{21} = 1/2$, $g = 1$. The solid and dotted lines are for η_{01} and η_{11}, respectively.

5. Conclusions

In summary, we first considered a system consisting of a three-level atom in the Λ configuration interacting with two vacuum fields. The entanglement between the atom and the two fields are calculated by using the logarithmic negativity. Our results show that the entanglements are periodic functions of time. The periods of them depend on the detuning and the initial state of the atom. In the case of two thermal fields, the influence of the mean photon numbers of the two fields is investigated. Then, we investigate the relation between atomic coherence and the entanglement of the two thermal or vacuum fields. We show that two thermal fields can be entangled by interacting with a three-level atom with coherence in Λ configuration. However, two vacuum fields cannot be entangled via atomic coherence and interaction between the three-level atom in Λ configuration and the two vacuum fields which is different from Ref.[25].

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