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Simulating train movement in railway traffic using a car-following model*

Li Ke-Ping(李克平) and Guan Li-Jia(管立加)

State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing 100044, *China*

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Based on a car-following model, in this paper, we propose a new traffic model for simulating train movement in railway traffic. In the proposed model, some realistic characteristics of train movement are considered, such as the distance headway and the safety stopping distance. Using the proposed traffic model, we analyse the space-time diagram of traffic flow, the trajectory of train movement, etc. Simulation results demonstrate that the proposed model can be successfully used for simulating the train movement. Some complex phenomena can be reproduced, such as the complex acceleration and deceleration of trains and the propagation of train delay.

Keywords: train movement, railway traffic, car-following model
PACC: 0550

1. Introduction

Train movement calculation is used to calculate the velocity and distance profile as a train travels from one station to another. When the theoretical control algorithm of train movement is analysed and evaluated, a computer-based simulation model is necessary. An important problem is how to establish and solve this simulation model. Recently, a number of studies have been performed in this field.[1−5] However, many characteristic behaviours of train movement remain unknown. What are the mechanisms by which the train delays emerge? How do the tracked trains affect each other?

The car-following model is one of the important traffic models in studying traffic flow, and it is used mainly to describe the behaviour of an individual driver. Several car-following models have been proposed, including the early car-following models,[6−8] and some recent improved models.[9−14] Since in the car-following model realistic driver behaviour and detailed vehicle characters are included, it can be used to simulate various traffic flows which are observed in real traffic.[15,16] In addition, by using the car-following model, some results can be derived analytically.

In 1995, Bando *et al* [9] proposed an improved car-following model, which is called the optimal velocity model. Such an improved model attracts a lot of attention because of its distinctive features in representing real traffic flow. Lately, research has been carried out on extending the optimal velocity model. A well-known extended model, called the "general optimal velocity model", was proposed by Nagatani[16] in 1999 and Sawada[17] in 2001 separately. But most of them focused on analyzing the characters of density wave from the viewpoint of physics.

In the present work, based on the optimal velocity car-following model, we propose a new model for simulating train movement, where the influence of tracked trains is considered. Using the proposed simulation model, we study and discuss some characteristic behaviours of train movement in railway traffic. To our knowledge, this work explicitly shows this effect for the car-following model for the first time. The remainder of the present paper is organized as follows. In Section 2, we introduce the optimal velocity model. The principle of the train control system is introduced in Section 3. In Section 4, we outline the proposed model. The numerical and the analytical results are presented in Section 5. Finally, the conclusions obtained by using this approach are presented in Section 6.

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2. The optimal velocity model

The car-following model describes the motion of a vehicle following its leader vehicle without making a lane change. In the car-following model, a follower vehicle tries to maintain a space gap behind its leader vehicle. A classical car-following model was proposed by Pipes.[6] In order to account for the time lag, Chandler *et al* [7] suggested an improved car-following model,

$$\ddot{x}_n(t+T) = \lambda[\dot{x}_{n+1}(t) - \dot{x}_n(t)], \tag{1}$$

where $x_n(t)$ is the position of vehicle n at time t, T is a response time lag, and λ is the sensitive coefficient. For a more realistic description, Newell[8] presented the optimal velocity car-following model,

$$\dot{x}_n(t+T) = V^{\rm opt}(\Delta x_n(t)), \tag{2}$$

where $V^{\rm opt}$ is called the optimal velocity function, and $\Delta x_n(t) = x_{n+1}(t) - x_n(t)$ is the headway of vehicle n at time t. More than 30 years later, Bando *et al* [9] suggested a well-known optimal velocity model,

$$\ddot{x}_n(t) = \frac{V^{\rm opt}(\Delta x_n(t)) - \dot{x}_n(t)}{\tau}, \tag{3}$$

where τ is the relaxation time.

In the optimal velocity model, the deceleration and the acceleration of the vehicle are described by a simple differential equation. According to such a differential equation, the behaviour of the driver is dependent mainly on the headway (the distance between two successive vehicles). This means that the driver is assumed to be only looking at the leader vehicle. But, in more realistic situations, the driver considers more information about vehicles around him/her. Apart from this, the relaxation time τ in Eq.(3) is an important parameter, which must be reasonably selected to avoid the collision between two successive vehicles.

In order to reproduce traffic as realistically as possible, some complex models are proposed, but they are at the cost of introducing a large number of parameters, or, losing their realistic features in the deterministic limit. For example, the "generalized-force model" with a generalized optimal velocity function was proposed in Ref.[10]. This model could successfully reproduce the time-dependent gaps and velocities. However, the acceleration time and deceleration time in this model are still unrealistically small. As a result, a great effort has been made to improve the optimal velocity car-following model and to make it suitable for simulating various realistic traffics.

3. Principle of the train control system

The train control system plays a central role in railway traffic. Usually, it is used to decide how a driver operates under safety restrictions and other considered constraints. Two types of the train control systems have been developed, i.e. the fixed-block system and the moving block system. The fixed-block system has been widely used in modern railway traffic for more than one century. However, as signaling technology is developed, the moving block system becomes of considerable importance and will probably replace the fixed-block system some day.

With the moving block system, electronic communications between the control centre and trains continuously control the trains, and make them maintain the safety stopping distance. In a moving block equipped system, using the information about the velocities and the locations of all the trains in its area, controllers can optimize system performance and respond to events quickly and effectively. The advantages of the moving block system are that the line capacity can be increased and the traffic fluidity and the energy efficiency can be improved.

In the moving block system, between two successive trains, the following train needs to adjust its velocity continuously. If the distance between two successive trains is smaller than the safety stopping distance, the following train will be forced to brake to a lower velocity or stop at a site on the track line. Several types of moving block systems have been developed, i.e. the moving space block, the moving time block and the pure moving block.[18] In the present paper, we employ the moving space block as the studied block system. Some approaches can be easily extended to other block systems.

4. Our model

The behaviour of train movement in railway traffic differs from that of vehicle movement in road traffic. The deceleration and the acceleration of the train are limited to a range between –2 m/s^2 and 2 m/s^2. But such a range in road traffic is between –3 m/s^2 and 4 m/s^2. The velocity relaxation time is larger than that in city traffic and freeway traffic (realistic velocity relaxation time is of the order of 10 s in city traffic and 40 s in freeway traffic). In addition, the

safety stopping distance is strongly related to the maximum velocity and the deceleration rate of the train. Usually, the safety stopping distance x_c is defined as $x_c = v_{\max}^2/(2b) + sm$, where sm is called the safety margin distance, $v_{\max}$ is the maximum velocity of the train and b is the deceleration rate of the train. In order to use the car-following model to simulate the train movement in railway traffic, we here improve the optimal velocity model, and make it suitable for describing train movement in railway traffic.

The car-following model proposed in Ref.[9] is a well-known optimal velocity model, where the optimal velocity function $V^{\rm opt}(\Delta x_n(t))$ is taken as $V^{\rm opt}(\Delta x_n(t)) = (v_{\max}/2)\{\tanh[\Delta x_n(t) - x_c] + \tanh[x_c]\}$. Based on such a car-following model, we propose a new improved model for simulating train movement. The proposed model is as follows:

$$\ddot{x}_n(t) = c_1[1 - \exp(-c_2\tau)]\frac{V^{\rm opt}(\Delta x_n(t)) - \dot{x}_n(t)}{\tau}. \quad (4)$$

In the above equation, we have introduced a function $c_1[1 - \exp(-c_2\tau)]$. This function ensures that the improved equation model meets the following two conditions: (i) the deceleration and the acceleration of the train are limited to a given range; (ii) a crash can be avoided when the velocity relaxation time τ is of a high order. Here c_1 and c_2 are adjustable parameters. In simulation, we need to select reasonable values of c_1 and c_2.

The dynamic behaviour of train movement near a station is more complex. When train n prepares to travel into a station, if the station in front of train n is occupied by other trains, train n must keep away from the station, otherwise, train n can travel into the station directly. In the former case, train n maintains a safety stopping distance from the train in front of it, but in the latter case, the safety stopping distance between train n and the station is neglected. In the former case, the optimal velocity function for train n is $V^{\rm opt}(\Delta x_n(t)) = (v_{\max}/2)\{\tanh[\Delta x_n(t) - x_c] + \tanh[x_c]\}$, and in the latter case, the optimal velocity function for train n can be simplified into $V^{\rm opt}(\Delta x_n(t)) = (v_{\max}/2)\{\tanh[\Delta x_n(t)]\}$.

The boundary condition used in this paper is open. Considering a single track line with a length of L, the boundary condition is as follows: (i) when the section from site 1 to site L_s is empty, a train with the velocity $v_{\max}$ is created. This train immediately travels according to the equation model (4). Here the parameter L_s is called the departure interval, and it must be larger than or equal to the safety stopping distance x_c. (ii) At site L, trains simply move out of the system. In order to compare simulation results with field measurements, one iteration roughly corresponds to 1 s, and the length of a unit is about 1 m. This means, for example, that $x_c = 10$ units/update corresponds to $x_c = 36$ km/h.

5. Simulation

We use the proposed model to simulate the train movement in single line railway traffic. The train control system employs the moving space block system. The dynamic equation of train movement is described by Eq.(4). The proposed equation model is suitable for computer programming. To start with, by changing the time derivative, the proposed equation model is simplified into a discrete model. Then we iterate the discrete equation model under the open boundary condition. The basic program is that at each time step, for all trains, we use the current velocities and positions of trains to calculate their velocities and positions at the next time step.

In simulation, a system with length $L = 10000$ is considered ($L = 10000$ units correspond to $L = 10000$ m). The number of the iteration time steps is $T_s = 2000$. One station is designed in the middle of the system, i.e. the station is at the site $l = 5000$. As a train arrives at a station, it needs to stop for a time (say, T_d) and then leaves the station. T_d is called the station dwell time. The values of c_1 and c_2 are related mainly to the maximum velocity $v_{\max}$. When $v_{\max}$ is larger, c_1 and c_2 should take smaller values. On the contrary, when $v_{\max}$ is smaller, c_1 and c_2 should take larger values. For a given $v_{\max}$, when c_1 is larger, c_2 should take a smaller value, and when c_1 is smaller, c_2 should take a larger value. Because the deceleration rate b is a variant, the safety stopping distance x_c is also a variant. But, in the process of train operation, x_c usually is considered as a constant. So b in the formula of x_c is set at a middle value of the decelerations. The velocity relaxation time τ and the safety margin distance sm are respectively set to be $\tau = 100$ and $sm = 10$. After sufficient transient time, we begin to record the data of traffic flow.

Simulation results demonstrate that when the departure interval L_s is large, trains can travel without any disturbance, and when the departure interval L_s is small, train delays begin to emerge. In order to study the characteristic behaviour of train movement,

we investigate the space-time diagram of the railway traffic flow. Figure 1 shows part of the space-time diagram of railway traffic flow for $x_c = 40$. Here we plot 10000 sites in 1000 consecutive time steps. The horizontal direction indicates the direction in which trains travel forwards. The vertical direction indicates the evolution time step. The parameters c_1 and c_2 are respectively set to be $c_1 = 50$ and $c_2 = 0.001$. In Fig.1, the positions of trains are indicated by dots. From Fig.1, we can find that the traffic flow shown in Fig.1(a) is free, where the traffic flux can be increased by reducing the departure interval L_s, and figure 1(b) presents complex traffic flow where train delays move backwards.

In the present paper, we focus on the case where the departure interval L_s is small. Under such a condition, we can observe how trains change their velocities, and maintain the safety stopping distance between them. In Fig.1(b), all trains start from the departure site, i.e. the site $l = 1$, and then arrive at the station, i.e. the middle site $l = 5000$. After the station dwell time T_d, they leave the station. When they arrive at the arrival site, i.e. the site $l = 10000$, they leave the system immediately. Before the station, the train delays form and propagate backwards. During the train delays propagating backwards, their time intervals stopping at track increase. This is the characteristic behaviour of train movement when the departure interval L_s is small.

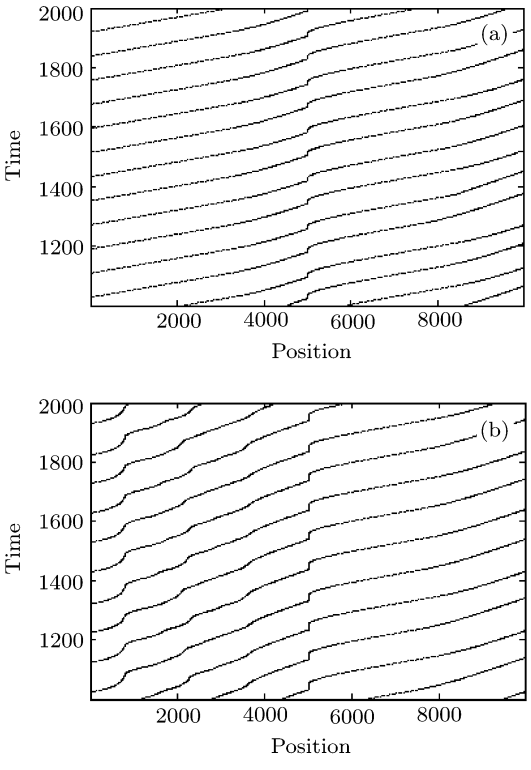


Fig.1. Local space-time diagram of railway traffic flow with (a) $L_s = 2x_c$ and $T_d = 10$, and (b) $L_s = x_c$ and $T_d = 20$.

Complex dynamic behaviour of train movement occurs near a station. When a train passes through a station, it needs to stop to enable passengers to board and alight. In this process, it needs to accelerate and decelerate continuously. If the station dwell time T_d is large, the train delays possibly form and propagate backwards. Figure 2 shows the local space-time diagram, which displays the positions and velocities of the trains near a station. Here numbers represent the velocities of trains. The values of c_1 and c_2 are set to be $c_1 = 100$ and $c_2 = 0.001$. From Fig.2, we can clearly see that as train C stops at the station, train D that is directly behind train C needs to decelerate, and then stops at the site in front of train E. As time goes on, a number of trains before train C are delayed. These results demonstrate that the proposed model can successfully capture the expected delays in railway traffic.

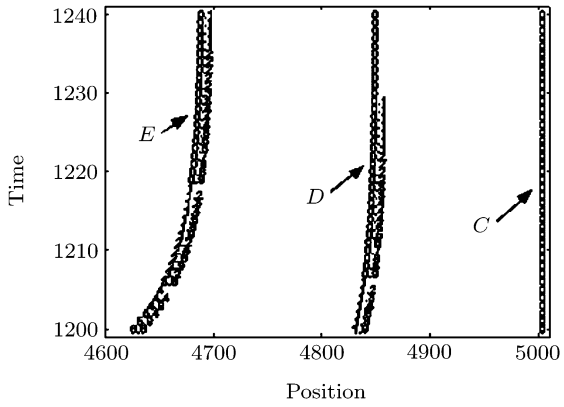


Fig.2. A diagram displaying the positions and velocities of trains for $x_c = 15$, $L_s = x_c$ and $T_d = 100$.

As mentioned in Section 4, when a train travels from one station to another, its deceleration and acceleration are limited to a range between –2 m/s^2 and 2 m/s^2. Using the proposed model, the deceleration and acceleration of trains can be made to enter into such a given range by controlling the values of c_1 and c_2. The relevant results are presented in Fig. 3. Here we record the decelerations and accelerations of all trains at all times. The parameters x_c, c_1 and c_2 are the same as those used in Fig.1. From Fig.3, we can see that the acceleration of trains is in the range between 0 and 2 m/s^2, and the deceleration of trains is in the range between –2 m/s^2 and 0 m/s^2. In general, the higher the values of c_1 and c_2 are, the larger the range limiting the deceleration and acceleration will be.

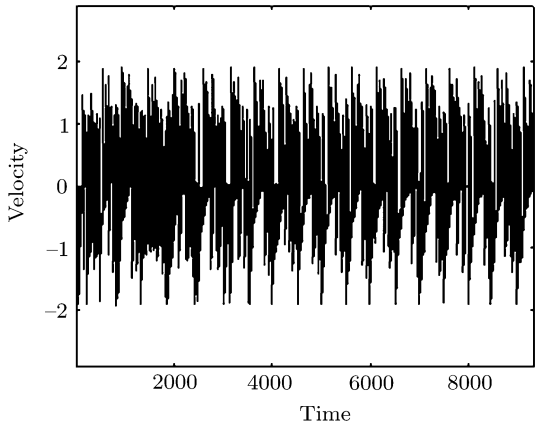


Fig.3. Acceleration and deceleration of trains for $L_s = x_c$ and $T_d = 20$.

In reality railway traffic, one of the important factors which affects train movement is the safety stopping distance. With the moving block system, the safety stopping distance between two successive trains must be maintained. For example, when the distance between two successive trains is smaller than the safety stopping distance, the following train must decelerate.

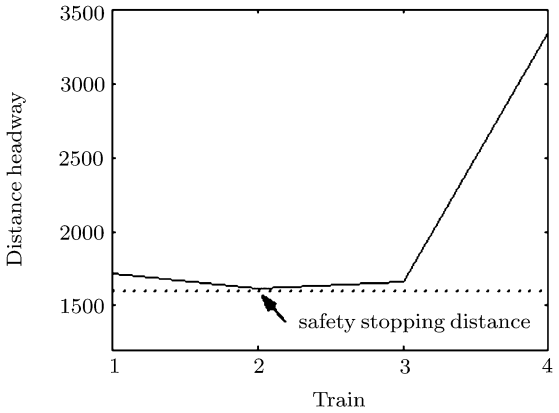


Fig.4. Distribution of distance headway for $x_c = 40$, $L_s = x_c$ and $T_d = 20$.

In order to further test the proposed model, we measure the distribution of the distance headway Δs_i at a given time, where Δs_i is the distance from train i to train $i + 1$. Figure 4 shows the distribution of the distance headway at time $t = 1000$, where the solid line denotes the measurement results, and the dotted line represents the safety stopping distance. In Fig.4, at time $t = 1000$, there are 4 trains traveling on the single line. From Fig.4, it is obvious that all the measurement results are larger than the safety stopping distance. The simulation results indicate that the proposed model is an effective tool for simulating train movement.

6. Conclusions

We proposed an improved car-following model for simulating the train movement in railway traffic. By iterating the proposed equation model, we simulate the train movement under the moving block system condition. The numerical simulation results indicate that the proposed model can describe well the train movement in railway traffic. Not only can the dynamic behaviour of train movement be described, but also some complex phenomena observed in real railway traffic, such as the train delays, can be reproduced.

In a practical train control system, train movement is restricted by several factors, such as the track geometry, traction equipment, train length, etc. In the proposed model, some factors have not been considered, thereby leading to the simplification of the train movement calculation. Although the proposed model is simplified, it can reproduce some characteristic behaviours of train movement, moreover it provides a new approach for the further analysis and evaluation of train control systems.

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