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Circuitry implementation of a novel four-dimensional nonautonomous hyperchaotic Liu system and its experimental studies on synchronization control*

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Based on the three-dimensional Liu chaotic system, this paper appends a feedback variable to construct a novel hyperchaotic Liu system. Then, a control signal is further added to construct a novel nonautonomous hyperchaotic Liu system. Through adjusting the frequency of the control signal, the chaotic property of the system can be controlled to show some different dynamic behaviors such as periodic, quasi-periodic, chaotic and hyperchaotic dynamic behaviours. By numerical simulations, the Lyapunov exponent spectrums, bifurcation diagrams and phase diagrams of the two new systems are studied, respectively. Furthermore, the synchronizing circuits of the nonautonomous hyperchaotic Liu system are designed via the synchronization control method of single variable coupling feedback. Finally, the hardware circuits are implemented, and the corresponding waves of chaos are observed by an oscillograph.

Keywords: Liu chaotic system，Liu hyperchaotic system, nonautonomous system, synchronization control
PACC: 0545，2960

1. Introduction

Since Lorenz found the first chaotic model in 1963,[1] chaotic dynamics has been greatly developed in the research field of nonlinearity. In recent years, chaotic dynamics has been widely applied in encrypted communications. However, the common 3-dimensional chaotic system has a relatively narrow bandwidth, which will cause the chaotic sequence to be easily filtered by digital filters. But, for hyperchaotic systems, the signal bandwidths of the chaotic sequences are very wide, so there is a broad application prospect to use hyperchaotic signals as encrypting signals.

Recently, a method has attracted more and more attention, in which the hyperchaotic can be obtained via appending a state feedback controller to a continuous chaotic system.[2−5] Using simple electronic elements, one can easily construct a hyperchaotic system based on the original chaotic systems.[6−11]

In 1989, Pecora and Carroll found that, in given conditions, some subsystems of a chaotic system can be synchronized to each other. Here, the synchronization between two systems means that the orbit of one system converges to that of the other in the same values, they always keep pace with each other, and the synchronization is structurally steady. After that, some effective control methods for chaotic synchronization have been proposed, such as coupled synchronization,[12] continuous variable feedback synchronization,[13−16,17] adaptively controlled synchronization,[18] etc.

In this paper, we introduced a feedback controlled variable to the second equation of the chaotic Liu system in [19], and appended a 1^{st}-order differential equation to construct a fourth-order autonomous hyperchaotic Liu system. Employing the method in [20], we then add a control signal to the hyperchaotic Liu

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system and further construct a novel four-dimensional nonautonomous hyperchaotic Liu system. The system characteristic is that we can control the evolving process via adjusting the frequency of the input signal.

The Lyapunov exponent spectrums, bifurcation diagrams and phase diagrams of the systems are analyzed via numerical simulations. Both the numerical simulation results and electrical experimental results are confirmed. Furthermore, we use the single variable coupled feedback controlling method[13,14] to adjust the coupling coefficient, and realize the synchronization of the nonautonomous hyperchaotic Liu systems. Then the hardware circuits are implemented, and the corresponding waves of chaos are observed by an oscillograph.

2. A novel hyperchaotic Liu system and characteristic analysis of its nonautonomous system

2.1. The construction of the novel hyperchaotic Liu system

The expression of the Liu chaotic system is shown as follows:

$$\begin{aligned}\dot{x} &= a(y-x),\\ \dot{y} &= bx - hxz,\\ \dot{z} &= cx^2 - dz.\end{aligned} \tag{1}$$

In this paper, we introduce a control variable w into the second equation of the system (1), and let $\dot{w} = -ny$, then we can obtain a novel system (2) as follows:

$$\begin{aligned}\dot{x} &= a(y-x),\\ \dot{y} &= bx - hxz + \lambda w,\\ \dot{z} &= cx^2 - dz,\\ \dot{w} &= -ny.\end{aligned} \tag{2}$$

Let $a = 15$, $b = 35$, $c = 4$, $d = 2.5$, $h = 1$, $\lambda = 1$, and n be a constant to be determined. Choosing n as a variable, we can obtain the Lyapunov exponent spectrums and bifurcation diagrams of the novel system via simulations, which are shown in figure 1. We can see that, when $n \in (0, 36]$, the system has two positive Lyapunov exponents, so it is in the hyperchaotic state; when $n \in (38, 44]$ or $n \in (54, 63)$, the system has only one positive Lyapunov exponent, so it is in the chaotic state; when $n \in (44, 54]$ or $n \in (63, 75]$, the largest Lyapunov exponent is zeros, so the system is in the periodic state; when $n \in (75, 80]$, the system has no positive Lyapunov exponent, and the largest two Lyapunov exponents are zero, so the system is in the quasi-periodic state.

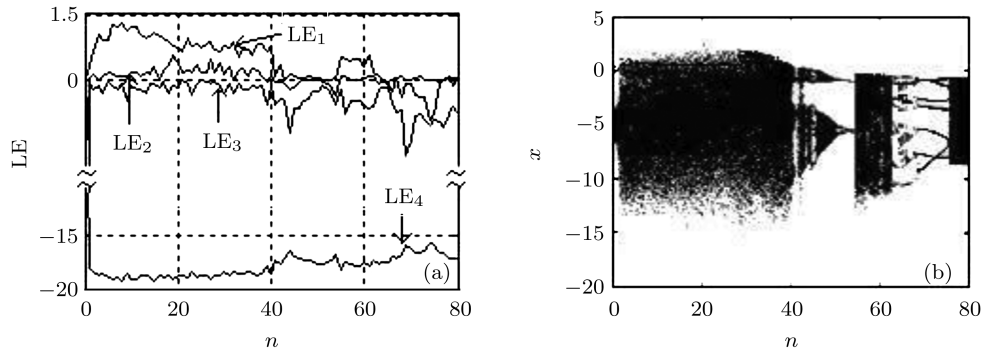


Fig.1. The Lyapunov exponent spectrum of system (2) and its bifurcation diagram about variable x.

2.2. Construction of the nonautonomous hyperchaotic Liu system

In order to overcome the limit that the potentiometer cannot be adjusted accurately, in this paper, we add a control signal $k\sin(\omega t)$ into the fourth equation of the system (2) and construct a novel nonautonomous Liu system，which is shown as

$$\begin{aligned}\dot{x} &= a(y-x),\\ \dot{y} &= bx - xz + \lambda w,\\ \dot{z} &= cx^2 - dz,\\ \dot{w} &= -ny - k\sin(\omega t).\end{aligned} \tag{3}$$

Let $n = 18.5$, we will change the frequency of the control signal to control the dynamic property of the system. Here, k is the amplitude of the input signal, and ω is its angular frequency. For convenience, we express the system (3) as a five-dimensional autonomous system including a sine signal, which is shown as

$$\begin{aligned}\dot{x} &= 15(y-x),\\ \dot{y} &= 35x - xz + \lambda w,\\ \dot{z} &= 4x^2 - 2.5z,\\ \dot{w} &= -ny - k\sin v,\\ \dot{v} &= \omega.\end{aligned} \tag{4}$$

Through simulations, we find that: when the amplitude k is less than 200, while changing the frequency of the control signal, system (4) will always stay in the hyperchaotic state; when k is between 200 and 500, while changing the frequency of control signal, the periodic behavior in system (4) is not very obvious; when k is larger than 500, while changing the frequency of the control signal, all the behaviors in the evolving process (such as periodic, quasi-periodic, chaotic and hyperchaotic) can appear.

So we choose the system parameter $n = 18.5$ and $k = 700$ to construct system (4). Treating the angular frequency of the input driving signal as a variable, we can obtain the Lyapunov exponent spectrum and bifurcation diagram of system (4), as is shown in figure 2. We can see that: when $\omega \in (16.5, 35]$, there are two positive Lyapunov exponents, so the system is in a hyperchaotic state; when $\omega \in (13, 16.5)$, the system has only one positive Lyapunov exponent, so it is in a chaotic state; when $\omega \in (0, 4.6)$, (5.2, 9) or (10, 13), the largest Lyapunov exponent is 0, so the system is in a periodic state; when $\omega = 4.6$, 4.8, 5.2, 9, 9.8 or 10, the system has two Lyapunov exponents equal to 0, so it is in a quasi-periodic state.

It is shown that, when the control signal is added, the dynamic property of system (4) will change. While increasing the angular frequency, the system property will evolve from periodic to quasi-periodic, chaotic and hyperchaotic successively. The results shown in the bifurcation diagram are consistent with the Lyapunov exponent spectrum.

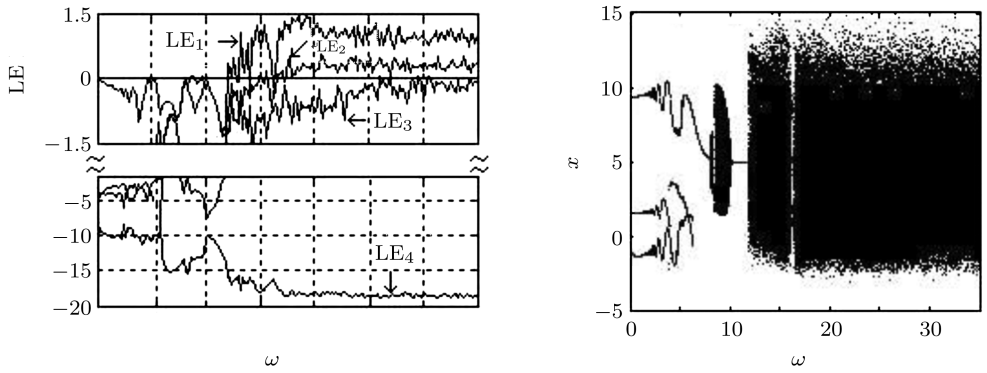


Fig.2. The Lyapunov exponent spectrum of system (3) and its bifurcation diagram about variable x.

3. Numerical simulations and the chaotic diagram

System (2) and system (4) are numerically simulated, and their phase diagrams are obtained, as is shown in figure 3. From figure 3 we can see that, while increasing n (or ω), all the states including hyperchaotic, chaotic, periodic and quasi-periodic states can appear in both systems (2) and (4), which shows the evolving process of chaos.

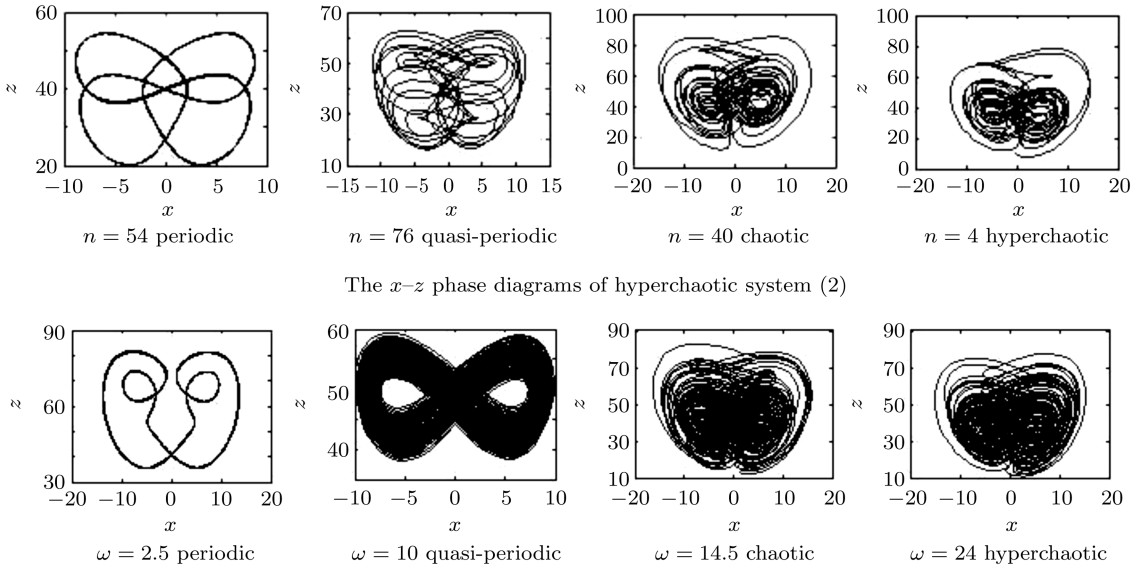


Fig.3. The x–z phase diagrams of system (2) and system (3).

4. Circuitry implementation of the hyperchaotic Liu system and its nonautonomous system

For circuit design, some problems should be noticed. The type of operation amplifier chosen by us is LM741, and the type of the multiplier is AD633JN. From the phase diagrams in figure 3 obtained by Matlab simulations, it is shown that the highest level of signal in a chaotic state is about 90 voltages. In order to ensure the circuit works normally, we use the transform method of proportionally compressing to re-adjust the equations of the original system, and change equations (2) and (4) as follows:

$$\begin{aligned} 0.1\dot{x} &= 1.5(y-x),\\ 0.1\dot{y} &= 3.5x - xz + 0.1w,\\ 0.1\dot{z} &= 4x^2 - 0.25z,\\ 0.1\dot{w} &= -0.1ny, \end{aligned} \tag{5}$$

$$\begin{aligned} 0.1\dot{x} &= 1.5(y-x),\\ 0.1\dot{y} &= 3.5x - xz + 0.1w,\\ 0.1\dot{z} &= 4x^2 - 0.25z,\\ 0.1\dot{w} &= -0.1ny - 0.01k\sin\omega t. \end{aligned} \tag{6}$$

The adjusted systems (5) and (6) are equivalent to the original systems (2) and (4) respectively, and the physical property of the systems is not changed.

Because the gain of the multiplier AD633JN is 0.1, from figure 4 we can obtain the circuit equations as follows:

$$\begin{aligned} (R_0C)\frac{\mathrm{d}x}{\mathrm{d}t} &= -\frac{R}{R_1}x + \frac{R}{R_2}y,\\ (R_0C)\frac{\mathrm{d}y}{\mathrm{d}t} &= \frac{R}{R_5}x - \frac{R}{10R_3}xz + \frac{R}{R_4}w,\\ (R_0C)\frac{\mathrm{d}z}{\mathrm{d}t} &= \frac{R}{10R_7}x^2 - \frac{R}{R_6}z,\\ (R_0C)\frac{\mathrm{d}w}{\mathrm{d}t} &= -\frac{R}{R_8}y, \end{aligned} \tag{7}$$

$$\begin{aligned} (R_0C)\frac{\mathrm{d}x}{\mathrm{d}t} &= -\frac{R}{R_1}x + \frac{R}{R_2}y,\\ (R_0C)\frac{\mathrm{d}y}{\mathrm{d}t} &= \frac{R}{R_5}x - \frac{R}{10R_3}xz + \frac{R}{R_4}w,\\ (R_0C)\frac{\mathrm{d}z}{\mathrm{d}t} &= \frac{R}{10R_7}x^2 - \frac{R}{R_6}z,\\ (R_0C)\frac{\mathrm{d}w}{\mathrm{d}t} &= -\frac{R}{R_8}y - \frac{R}{R9}\sin(\omega t). \end{aligned} \tag{8}$$

According to equations (5) and (6) we have

$$\begin{aligned} &a = \frac{10R}{R_1} = \frac{10R}{R_2} = 15, \quad b = \frac{10R}{R_5} = 35,\\ &h = \frac{R}{10R_3} = 1, \quad \lambda = \frac{10R}{R_4} = 1,\\ &c = \frac{R}{10R_7} = 4, \quad d = \frac{10R}{R_6} = 2.5,\\ &n = \frac{10R}{R_8}, \quad k = \frac{10R}{R_9} = 700, \quad R_0C = 0.1, \end{aligned}$$

where n is determined by R_8, k is determined by R_9. Then we designed the circuits of the hyperchaotic Liu system and its nonautonomous system, as shown in figure 4. The difference between systems (7) and (8) is that system (8) has a control signal and potentiometer R_9.

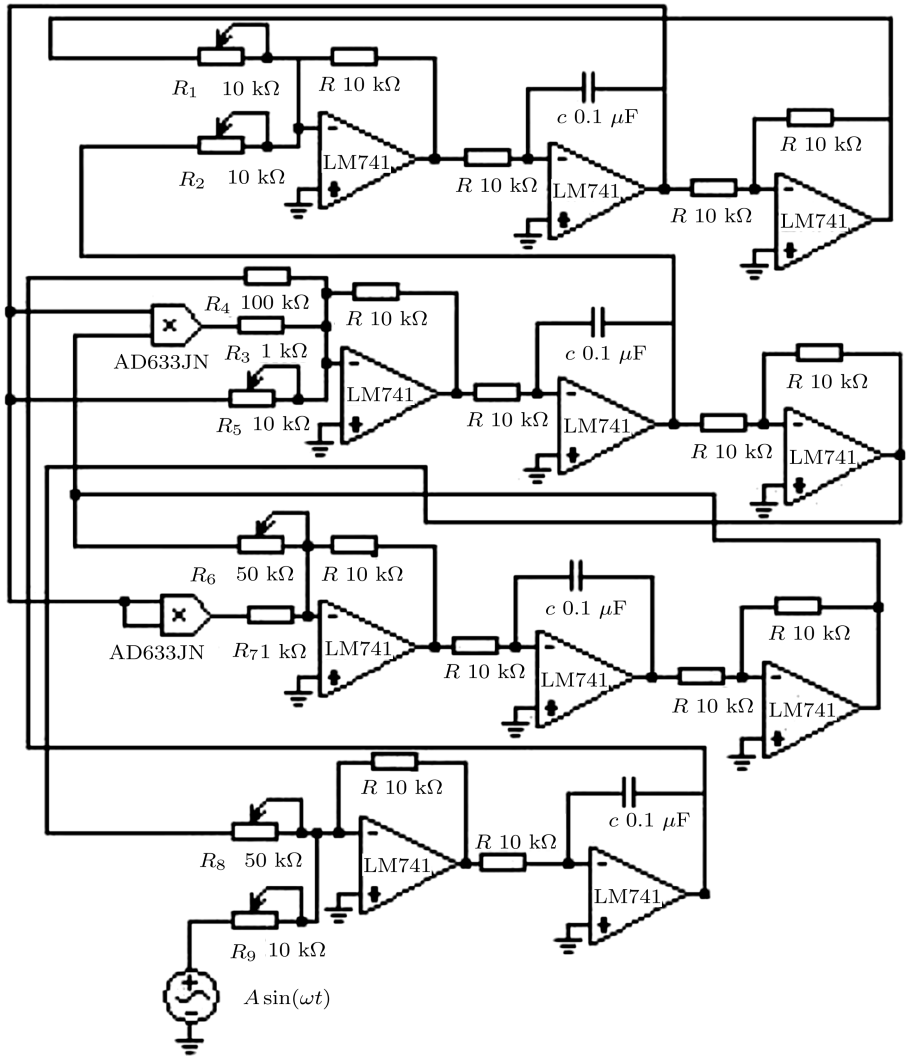
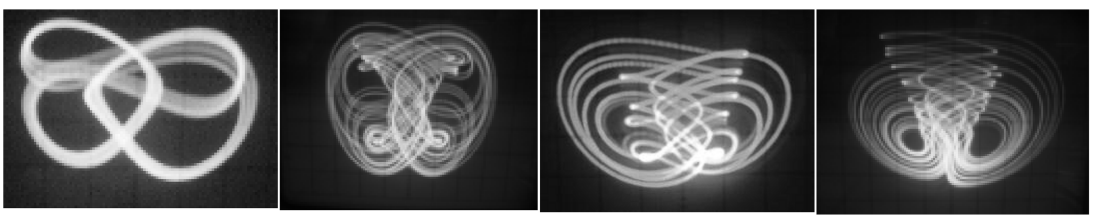


Fig.4. The circuit diagrams of the nonautonomous hyperchaotic Liu system.

In order to capture the wave easily, we let $R_0C = 10^{-4}$ to increase the frequency of the input signal for 1000 times, and let $C = 10$ nF, $R_0 = R = 10$ kΩ. Experiments show that we can obtain ideal output waves in this condition. If R_0C is too small, high frequency interferences are easy to be introduced, which will cause the distortion of the observing results.

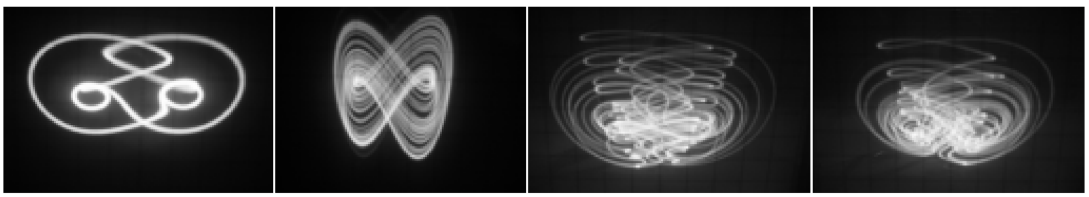
Changing R_8, the x–z phase diagrams for system (2) observed by the oscillograph are shown in figure 5.



$R_8 = 1.68$ kΩ periodic $R_8 = 1.27$ kΩ quasi-periodic $R_8 = 2.18$ kΩ chaotic $R_8 = 25.7$ kΩ hyperchaotic

Fig.5. The x–z phase diagrams of system (2) (observed by an oscillograph).

As shown in equation (6), after proportional compressing transformation, the term $k\sin(\omega t)$ is adjusted as $0.01\,k\sin(\omega t)$. Let the amplitude A of the input signal be equal to 1 V, the coefficient $0.01k$ is determined by R/R_9. Since k is 700, for approximation, in actual circuits we adjust R_9 to be 1.4 kΩ. Through changing the frequency of the control signal, the x–z phase diagrams of system (4) are observed by an oscillograph, as is shown in figure 6. We can see that, the practical observed results in figures 5 and 6 are consistent with the simulation results in figure 3.



$f = 400$ Hz periodic $f = 1.59$ kHz quasi-periodic $f = 2.3$ kHz chaotic $f = 3.82$ kHz hyperchaotic

Fig.6. The x–z phase diagrams of system (3) (observed by an oscillograph).

5. Implementation of the synchronization circuit for the nonautonomous hyperchaotic Liu system

There are many methods to control the synchronization of the chaotic systems, such as coupled synchronization,[13] continuous variable feedback synchronization,[14−17] adaptively controlled synchronization,[18] etc. Here, we use the method of single variable coupled feedback[14,15] to realize the synchronization of the two hyperchaotic nonautonomous systems.

Let the angular frequency ω of the control signal equal 24, simulations show that, only when the control term is introduced into the second equation of the hyperchaotic state equations, and the proper coupling factor m is chosen, could the systems have a good synchronizing effect.

Let the driving system (9) and responsive (controlled) system (10) be as follows respectively:

$$\begin{aligned}
\dot{x}_1 &= 15(y_1 - x_1),\\
\dot{y}_1 &= 35x_1 - x_1z_1 + w_1,\\
\dot{z}_1 &= 4x_1^2 - 2.5z_1,\\
\dot{w}_1 &= -18.5y_1 + 700 * \sin(\omega t),
\end{aligned} \tag{9}$$

$$\begin{aligned}
\dot{x}_2 &= 15(y_2 - x_2),\\
\dot{y}_2 &= 35x_2 - x_2z_2 + w_2 + m(y_1 - y_2),\\
\dot{z}_2 &= 4x_2^2 - 2.5z_2,\\
\dot{w}_2 &= -18.5y_2.
\end{aligned} \tag{10}$$

Figure 7 shows that, when $m = 200$, the systems already have a good synchronizing effect; when $m = 2000$, the synchronizing effect is the best.

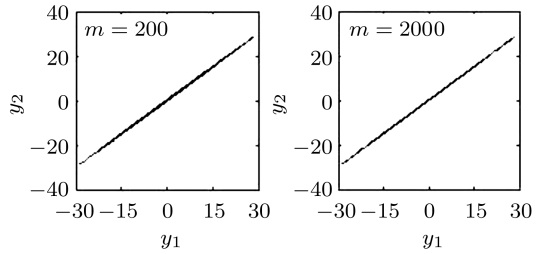


Fig.7. Synchronization simulations of the nonautonomous hyperchaotic Liu systems.

Figure 8 shows the synchronizing circuits of the nonautonomous systems.

Through adjusting the potentiometer R_x we found that, when $R_x = 480\ \Omega$, the systems have a good synchronizing effect; and when $R_x = 26\ \Omega$, the synchronizing effect is the best. Figure 9 shows the results observed by an oscillograph.

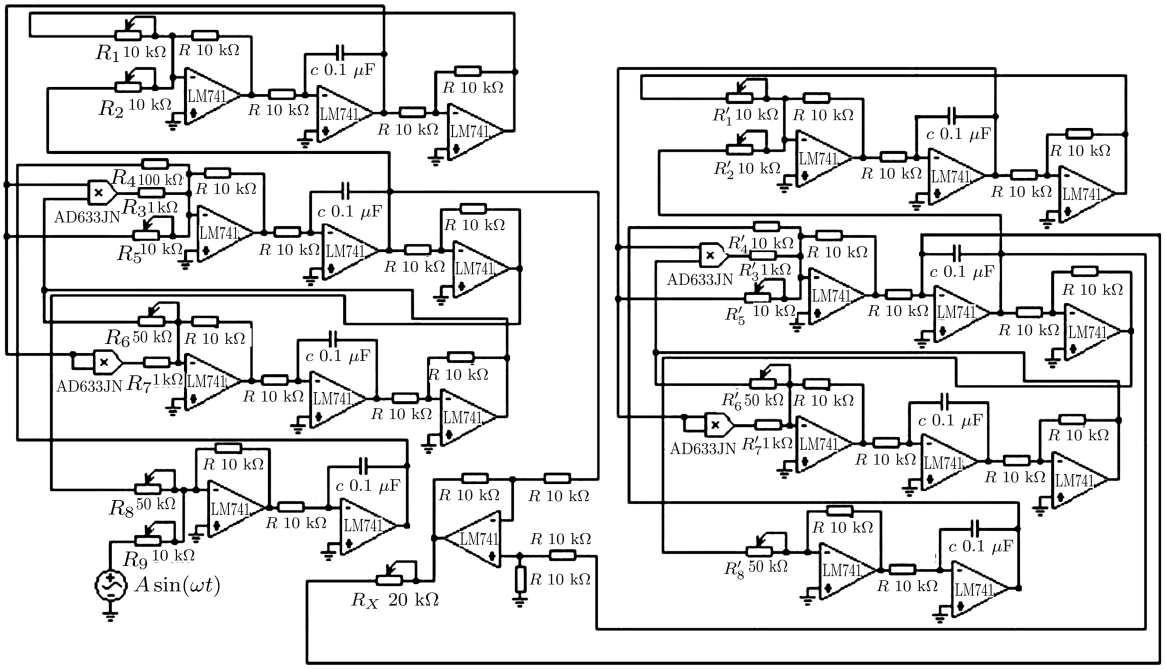


Fig.8. The control circuits for synchronization of the nonautonomous hyperchaotic Liu systems.

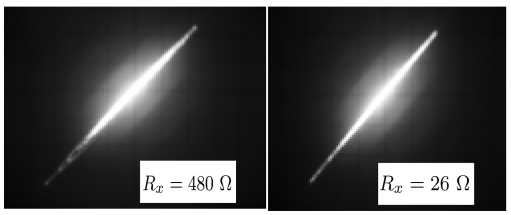


Fig.9. The synchronization phase diagrams (y_1–y_2) of the nonautonomous hyperchaotic Liu systems (observed by an oscillograph).

6. Conclusions

Through introducing a state feedback controller into the Liu chaotic system, this paper constructed a novel hyperchaotic Liu system, and further added a control signal into the novel system to construct a novel four-dimensional nonautonomous hyperchaotic Liu system. Through adjusting the frequency of the control signal, the chaotic property of the system can be controlled to show some different dynamic behaviors (such as periodic, quasi-periodic, chaotic, hyperchaotic).

The synchronization control for the four-dimensional nonautonomous hyperchaotic Liu system is studied experimentally. The hardware circuits are implemented, and the corresponding waves of chaos have been observed by an oscillograph. The hardware experimental results are consistent with the simulation results, which can provide some good references for engineering applications

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