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Effects of dust size distribution on nonlinear waves in a dusty plasma

Chen Jian-Hong(陈建宏)†

Department of Physics, Lanzhou City University, Lanzhou 730070, China

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For two-dimensional unmagnetized dusty plasmas with many different dust grain species, a Kadomtsev–Petviashvili (KP) equation, a modified KP (mKP) equation and a coupled KP(cKP) equation for small, but finite amplitude dust-acoustic solitary waves are obtained for different physical conditions respectively. The influence of an arbitrary dust size distribution described by a polynomial expressed function on the properties of dust-acoustic solitary waves is investigated numerically. How dust size distribution affects the sign and the magnitude of nonlinear coefficient A (D) of KP (mKP) equation is also discussed in detail. It is noted that whether a compressive or a rarefactive solitary wave exists depends on the dust size distribution in some dusty plasmas.

Keywords: dusty plasmas, solitary waves, dust size distribution
PACC: 0340K, 5235

1. Introduction

Dusty plasma has been the focus of intense research in the past decades.[1−5] Dusty plasma is an ionized gas containing small particles of solid matter which acquire a large electric charge by collecting electrons and ions from the plasma, which has been observed widely in planetary rings, asteroid zones, cometary tails as well as the lower part of the earth's atmosphere.[6−12] In the laboratory the dust grains exist as impurities and significantly influence the behavior of the surrounding plasma.[13] Since the masses of the dust grains are much greater than that of both electrons and the ions in the dusty plasma, the dust grains nearly have no consequence on high frequency oscillations except on the damping factor. Rao *et al*[14] firstly predicted the existence of dust-acoustic waves (DAWs) in the unmagnetized dusty plasma. Shukla and Silin have investigated the dust ion acoustic waves (DIAWs) at higher frequency.[15] Their results are confirmed by subsequent experimental observations.[16,17] However, most research work on dust-acoustic waves only considers the dusty plasma for mono-sized dust grains because it is easier to treat.[11,14,15] In a real case, the dust grains have many different size distributions both in the space plasmas and in the laboratory experiments.[18−22] The dust size distribution can be often described by a power law distribution (PLD) in space plasmas (e.g. the F-ring and the G-ring of Saturn)[19,21] and by a Gaussian distribution in plasmas experiments (e.g. dusty crystals and charging experiments).[22] By using the two different dust size distribution for different cases, the effect of dust size distribution on the dust plasma frequency, the properties of the dust-acoustic wave and the instability of waves have been extensively studied.[23−27] In fact, two distribution functions above are only two roughly approximated descriptions compared to the real size distribution of dust grains. In some cases, the dust size distribution function can be the other one. For example, the size distribution of large dust clusters has been found to be well described by the lognormal distribution using the scanning electron microscopic method and the polarization-sensitive laser light scattering method.[28,29] Therefore the dust size distribution can be an arbitrary function in the laboratory plasmas or even in space plasmas, which depends on the environmental and the experimental conditions. It is important to investigate the role of an arbitrary dust size distribution on the behavior of dusty plasmas, because the research results can be applied to the dusty plasmas considering dust size distribution both in laboratory and in space plasmas. Recently, a few authors have paid their attention to the effect of an arbitrary dust size distribution on the properties of both linear and nonlinear wave modes in dusty plasmas.[30−32] Following the method of Ref.[31], we assume a poly-

†Corresponding author. E-mail: chenyuwen1982@163.com

nomial expressed function to describe the arbitrary dust size distribution, that is, for charged dust grains with radii r in a given range $[r_1, r_2]$, the differential distribution is of the form

$$n(r)\,\mathrm{d}r = \left[a_0 + a_1 r + a_2 r^2 + \ldots\right]\,\mathrm{d}r, \qquad (1)$$

where r is the radii of dust grains and $a_0, a_1, a_2 \ldots$ are all constants which should satisfy the following equation

$$N_{\mathrm{tot}} = \int_{r_1}^{r_2} n(r)\mathrm{d}r, \qquad (2)$$

N_{tot} is the total number density of dust grains. r_1 and r_2 represent the minimum and maximum radii of dust grains, respectively. Outside the limits $r < r_1$ and $r > r_2$, we use $n(r) = 0$.

In this paper, we study the small but finite amplitude dust-acoustic solitary waves in an unmagnetized and collisionless cold dusty plasma considering the dust size distribution. By using the reductive perturbation method, we obtain a Kadomtsev–Petviashvill (KP) equation, a modified KP (mKP) equation, and a coupled KP (cKP) equation, respectively, for different physical conditions. Moreover, we numerically study how the arbitrary dust size distribution affects the characters of the dust-acoustic solitary waves in detail.

2. Basic equations

Now we study the nonlinear DAWs in a dusty plasma which satisfies an arbitrary dust size distribution. We consider a three-component dusty plasma consisting of dust grains which are extremely massive and usually negatively charged, inertialess Boltzmann distributed electrons and ions. The dust grains are much larger and more massive than ions or electrons, but the size of dust grains is different. We assume that there are N different species of dust grains whose masses are m_{dj} $(j = 1, 2, 3, \cdots, N)$ with different dust sizes. Charge neutrality at equilibrium reads: $n_{\mathrm{i}0} = \sum_{j=1}^{N} Z_{\mathrm{d}0j} n_{d0j} + n_{e0}$, where $n_{\mathrm{i}0}$, n_{e0} and $n_{\mathrm{d}0j}$ are the equilibrium number densities of ion, electron and the j-th dust grain, respectively. $Z_{\mathrm{d}0j}$ is the equilibrium number of charges residing on the j-th dust grain measured in the unit of the electron charge.

For two-dimensional low-frequency dust-acoustic motion, we have the following dimensionless equations:

$$\frac{\partial n_{dj}}{\partial t} + \nabla\cdot\left(n_{dj}\boldsymbol{u}_{dj}\right) = 0,$$

$$\frac{\partial \boldsymbol{u}_{dj}}{\partial t} + \left(\boldsymbol{u}_{dj}\cdot\nabla\right)\boldsymbol{u}_{dj} = \frac{Z_{dj}}{m_{dj}}\nabla\phi,$$

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = \sum_{j=1}^{N} Z_{dj} n_{dj} + n_e - n_{\mathrm{i}}, \qquad (3)$$

where $\boldsymbol{u}_{dj} = (u_{dj}, v_{dj})$, $\nabla = (\partial/\partial x, \partial/\partial y)$. n_{dj}, m_{dj}, u_{dj} and v_{dj} refer to the normalized number density, the normalized mass, the normalized velocity components in the x and y directions of the j-th dust grain, respectively. Z_{dj} is the charge number of the j-th dust grain. ϕ is the normalized electrostatic potential. n_{e} (n_{i}) is the normalized electron (ion) number density: $n_{\mathrm{e}} = \nu\exp(s\beta\phi)$, $n_{\mathrm{i}} = \mu\exp(-s\phi)$, with $\beta = T_{\mathrm{i}}/T_{\mathrm{e}}$ and $s = 1/(\mu + \nu\beta)$, where μ and ν are the normalized ion and electron number densities which can be determined by $\nu = n_{\mathrm{e}0}/(\overline{Z_{\mathrm{d}0}}N_{\mathrm{tot}})$, $\mu = n_{\mathrm{i}0}/(\overline{Z_{\mathrm{d}0}}N_{\mathrm{tot}})$.

The normalized quantities are defined as follows. The effective temperature T_{eff} is given by $T_{\mathrm{eff}} = (T_{\mathrm{i}}T_{\mathrm{e}})/(\mu T_e + \nu T_i)$, where $T_e(T_i)$ is electron (ion) temperature. N_{tot} is the total number density of dust grains, which is written as $N\mathrm{tot} = \sum_{j=1}^{N} n_{\mathrm{d}0j}$. $\overline{Z_{\mathrm{d}0}}$ is the average number of charges residing on dust grains, which is determined by the equation $\overline{Z_{\mathrm{d}0}}N_{\mathrm{tot}} = \sum_{j=1}^{N} n_{\mathrm{d}0j}Z_{\mathrm{d}0j}$. $\overline{r}$ is the average radii of dust size given by $\overline{r} = \sum_{j=1}^{N} r_j n_{\mathrm{d}0j}/N_{\mathrm{tot}}$, where r_j is the radii of the j-th dust grain. The dust density is normalized by N_{tot}, $Z_{\mathrm{d}0j}$ is normalized by $\overline{Z_{\mathrm{d}0}}$. The time variable t is normalized by the dust plasma period $\omega_{\mathrm{pd}}^{-1} = (\overline{m_d}/4\pi e^2\overline{Z_{\mathrm{d}0}}^2 N_{\mathrm{tot}})^{1/2}$ and the space variables (x, y) are normalized by the Debye length $\lambda_{\mathrm{Dd}} = (T_{\mathrm{eff}}/4\pi e^2\overline{Z_{\mathrm{d}0}}N_{\mathrm{tot}})^{1/2}$. The velocity (u_{dj}, v_{dj}) and the electrostatic potential ϕ are normalized by the effective dust-acoustic speed $C_{\mathrm{d}} = (\overline{Z_{\mathrm{d}0}}T_{\mathrm{eff}}/\overline{m_{\mathrm{d}}})^{1/2}$ and T_{eff}/e, respectively, where $\overline{m_{\mathrm{d}}}$ is the average mass of dust grains determined by the equation of $\overline{m_{\mathrm{d}}} = \sum_{j=1}^{N} n_{\mathrm{d}0j}m_{dj}/N_{\mathrm{tot}}$, m_{dj} is normalized by $\overline{m_{\mathrm{d}}}$.

3. KP equation

In order to study nonlinear DAWs, we use the results of He and Duan[31] to treat the problem at hand. Thus the following Kadomtsev–Petviashvili (KP) equation governing the behavior of small but finite amplitude nonlinear dust-acoustic waves under the transverse perturbation is obtained as follows:

$$\frac{\partial}{\partial\xi}\left[\frac{\partial\Phi_1}{\partial\tau} + A\Phi_1\frac{\partial\Phi_1}{\partial\xi} + B\frac{\partial^3\Phi_1}{\partial\xi^3}\right] + C\frac{\partial^2\Phi_1}{\partial\eta^2} = 0, \quad (4)$$

where

$$A = -\frac{3}{2v_0^3}\sum_{j=1}^{N}\frac{n_{\mathrm{d}0j}Z_{dj}^3}{m_{dj}^2} - \frac{v_0}{2}s^2(\nu\beta^2-\mu),$$
$$B = \frac{v_0}{2},$$
$$C = \frac{v_0}{2}. \tag{5}$$

The linear phase velocity v_0 of waves is given by $v_0^2 = \sum_{j=1}^{N} n_{\mathrm{d}0j}Z_{dj}^2/m_{dj}$. B and C are well known to be the coefficients of dispersion and dissipation terms, respectively. It is noted that the signs of coefficients B and C depend on v_0. Equation (4) is a KPII equation, which pertains to the system with a negative dispersion relation. However, the sign of nonlinear term A is not so easily determined, because it seems from Eq.(5) that A can be seen as a complex function of many variables. In the following discussion, we are interested in how the dust size distribution affects the sign and magnitude of the nonlinear term A.

When we assume that all dust grain sizes $r_j \ll \lambda_{\mathrm{Dd}}$, we can express the mass and charge of dust grains as follows:[33,34] $m_{dj} = k_m r_j^3$, $Z_{dj} = k_z r_j$, where $k_m \approx 4\pi\rho_{\mathrm{d}}/3$, $k_z \approx 4\pi\varepsilon_0 V_0/e$ are approximately constant. ρ_{d} is the mass density of the dust grains, V_0 is the electric surface potential at equilibrium, ε_0 is the vacuum permittivity. For the continuous case, from the arbitrary dust size distribution of Eqs.(1) and (2) we have

$$\sum_{i=1}^{N} n_{\mathrm{d}0j} = \int_{r_1}^{r_2} n(r)\mathrm{d}r = \int_{r_1}^{r_2}[a_0 + a_1 r + a_2 r + \ldots]\mathrm{d}r. \tag{6}$$

For a typical dusty plasma, the second term of A can be neglected compared with the first term in Eq.(5). Substituting Eq.(6) into Eqs.(4) and (5), we have

$$A = -\frac{3k_z^3}{2v_0^3k_m^2}\Big[-\frac{a_0}{2}(r_2^{-2}-r_1^{-2}) - a_1(r_2^{-1}-r_1^{-1}) + a_2(\ln r_2 - \ln r_1) + \ldots\Big], \tag{7}$$

where

$$v_0^2 = \frac{k_z^2}{k_m}\Big[a_0(\ln r_2 - \ln r_1) + a_1(r_2 - r_1) + \frac{a_2}{2}(r_2^2 - r_1^2) + \ldots\Big]. \tag{8}$$

For simplicity, we let $n(r) = a_0$. In this case, the number density of all dust grains with different dust sizes is same. Therefore we call this distribution the unform distribution of the dust size. For typical dusty plasmas:[20] $n_{\mathrm{i}0} \sim 10^5 - 10^{10}$ cm^{-3}, $T_{\mathrm{i}} \sim T_{\mathrm{e}} \sim 0.1$ eV, $n_{\mathrm{d}0} \sim 10^5$ cm^{-3}, $Z_{\mathrm{d}} \sim 10^4 - 10^5$, $m_{\mathrm{d}} \sim 10^{-15} - 10^{-12}$ $g \sim 10^9 - 10^{12} m_{\mathrm{i}}$, $\rho_{\mathrm{d}} \sim 1$ g cm^{-3}, $a \sim 0.1 - 1$ μm. It seems from Eqs.(7) and (8) that nonlinear coefficient A is the function of a_0. The variation of A with respect to a_0 for different c is plotted in Fig.1(a), where c is the ratio of maximum dust size to minimum dust size, i.e., $c = r_2/r_1$. It seems that the sign of A is positive and the quantity of A decreases as a_0 increases but increases as c increases. From Eqs.(1) and (2) it is easy to find that a_0 stands for the magnitude of the total number density of dust grains, therefore it should always be positive. This suggests that, in the uniform distribution of the dust size, the magnitude of the nonlinear coefficient A decreases as the total number density of dust grains increases but increases as the ratio of maximum dust size to minimum dust size increases.

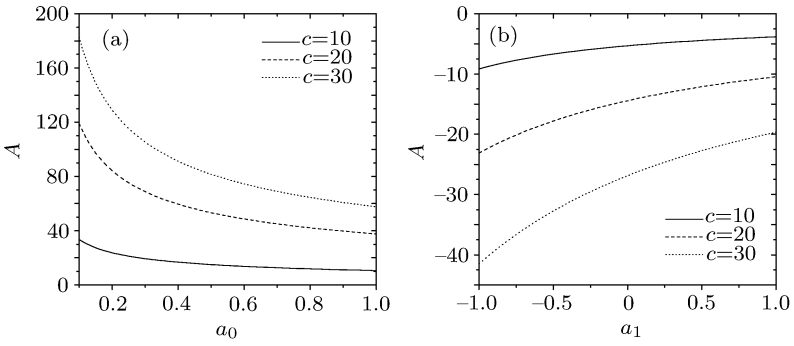


Fig.1. The variation of nonlinear coefficient A of the KP equation (4) with respect to a_0 and a_1 for $c = 10$, 20, 30 respectively.

Now we consider a more complex case: $n(r) = a_0 + a_1 r$. By letting $a_0 = 1$, we obtain the numerical results which are plotted in Fig.1(b). It seems that the quantity of A decreases as a_1 increases but increases as c increases. As suggested from Ref.[31], if $a_1 > 0$ the number density of larger dust grains is greater than that of smaller dust grains. However, if $a_1 < 0$ the number density of larger dust grains is greater than that of smaller dust grains. As is well known, in many space plasmas the dust size distribution can be described by a PLD distribution which just satisfies the case of $a_1 < 0$. So, for the case of $a_1 < 0$, we can describe the real behavior of dusty plasma in space. We also find from Fig.1(b) that for real dusty plasma of $a_1 < 0$, the sign of A is negative and the quantity of A is greater than that of the unusual case $a_1 > 0$.

There is a stationary solitary wave solution for the KP equation of Eq.(4) as follows:

$$\phi_1 = \phi_m \mathrm{sech}^2 \left(\frac{k\xi + l\eta - u_0 \tau}{\omega} \right), \qquad (9)$$

where

$$\phi_m = 3(u_0 k - C l^2)/A k^2$$

and $\omega = 2k^2 \sqrt{B/(u_0 k - C l^2)}$ are the amplitude and width of solitary waves, respectively. u_0 is an arbitrary parameter which is generally assumed to be a constant. k and l are the wave numbers along the x and y directions, respectively. For continuous distribution of the dust size, when we consider uniform distribution of the dust size, it seems from Eqs.(7)–(9) that the expressions of both the amplitude ϕ_m and the width ω of solitary waves are the functions of a_0. By using the same typical dusty plasma parameters which have been given above, we can obtain the numerical results of ϕ_m and ω. It is noted from Figs.2(a) and 2(b) that ϕ_m and ω increase as a_0 increases. Since a_0 represents the magnitude of the total number density of dust grains, as the total number density of dust grains increases the amplitude of solitary waves decreases but the width of solitary waves increases. This is the same as that of Ref.[31]. Figures 2(a) and 2(b) also show that as c increases ϕ_m decreases but ω increases. This suggests that as the ratio of maximum dust size to minimum dust size increases the amplitude and the width of solitary waves increases.

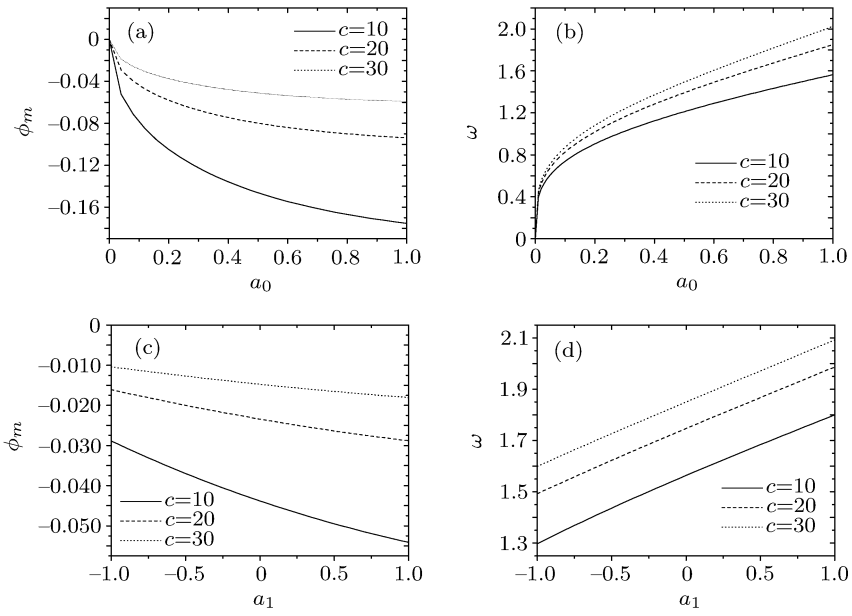


Fig.2. The variation of ϕ_m and ω from Eq.(9) with respect to a_0 and a_1 for $c = 10$, 20 , 30 respectively.

For a more complex case of $n(r) = a_0 + a_1 r$, letting $k = l = 1$, $a_0 = 1$, it is noted that both amplitude ϕ_m and width ω of solitary waves are the functions of a_1. Figures 2(c) and 2(d) show that ϕ_m and ω increase as a_1 increases. As mentioned above, $a_1 < 0$ means that the number density of larger dust grains is smaller than that of smaller dust grains, which is the usual case of dusty plasma in space. So, we can conclude that for real dusty plasma of $a_1 < 0$ the solitary wave amplitude ϕ_m and width ω are smaller than

that of unusual case $a_1 > 0$. This conclusion is similar to that of Ref.[31]. It is also noted that as c increases ϕ_m decreases but ω increases.

4. Modified KP equation

From Eq.(9) it seems that the amplitude of solitary waves depends highly on coefficient A which is not only a function of parameters μ, β and ν but also depends on the dust size distribution. For simplicity, we consider the uniform distribution case, i.e., $n(r) = a_0$, the nonlinear coefficient A of the KP equation(4) is

$$A = -\frac{v_0}{2}\left[p + \frac{\nu\beta^2 - \mu}{(\mu+\nu\beta)^2}\right], \tag{10}$$

where $p = 3k_z^3 a_0(r_1^{-2} - r_2^{-2})/(2v_0^4 k_m^2)$. It can be seen that p is always positive. If β and ν are all small enough, A may equal zero. The KP equation of Eq.(4) fails to describe the nonlinear evolution of DAWs. In this case we use the same stretched coordinates transform in Section 3 but use the following perturbation expansions:

$$\begin{aligned} n_{dj} &= n_{d0j} + \varepsilon n_{d1j} + \varepsilon^2 n_{d2j} + \varepsilon^3 n_{d3j} + \dots \\ u_{dj} &= \varepsilon u_{d1j} + \varepsilon^2 u_{d2j} + \varepsilon^3 u_{d23j} + \dots \\ v_{dj} &= \varepsilon^2 v_{d1j} + \varepsilon^3 v_{d2j} + \dots \\ \Phi &= \varepsilon\Phi_1 + \varepsilon^2\Phi_2 + \varepsilon^3\Phi_3 \dots . \end{aligned} \tag{11}$$

Substituting these expansions into Eq.(3) we obtained the following equations at the lowest order of ε:

$$\begin{aligned} n_{d2j} &= \frac{3n_{d0j}Z_{dj}}{v_0^4 m_{dj}^2}\Phi_1^2 - \frac{Z_{dj}}{v_0^2 m_{dj}}\Phi_2, \\ u_{d2j} &= \frac{Z_{dj}^2}{2v_0^3 m_{dj}^2}\Phi_1^2 - \frac{Z_{dj}}{v_0 m_{dj}}\Phi_2, \\ \frac{\partial v_{d1j}}{\partial\xi} &= -\frac{Z_{dj}}{v_0 m_{dj}}\frac{\partial\phi_1}{\partial\eta}, \end{aligned} \tag{12}$$

at higher order we obtained a modified Kadomtsev–Petviashvili (mKP) equation:

$$\frac{\partial}{\partial\xi}\left[\frac{\partial\Phi_1}{\partial\tau} + D\Phi_1^2\frac{\partial\Phi_1}{\partial\xi} + B\frac{\partial^3\Phi_1}{\partial\xi^3}\right] + C\frac{\partial^2\Phi_1}{\partial\eta^2} = 0, \tag{13}$$

where

$$\begin{aligned} D &= -\frac{15}{4}\sum_{j=1}^{N}\frac{n_{d0j}Z_{dj}^4}{v_0^5 m_{dj}^3} - \frac{v_0}{4}s^2(\nu\beta^3 + \mu), \\ B &= v_0/2, \qquad C = v_0/2. \end{aligned} \tag{14}$$

For the continuous dust size distribution:

$$D = -\frac{15k_z^4}{4v_0^5 k_m^3}\int_{r_1}^{r_2}\frac{n(r)}{r^5}\mathrm{d}a - \frac{v_0}{2}s^2(\nu\beta^3 + \mu). \tag{15}$$

By using the same method given in the last section, we investigate how the dust size distribution affects the sign and magnitude of nonlinear term D in Eq.(13). For the uniform distribution of $n(r) = a_0$, it seems that the expressions of both the amplitude ϕ_m and the width ω of solitary waves are the functions of a_0. By using the plasma parameters mentioned in the last section, we obtain the numerical results of D. Figure 3(a) shows that the magnitude of D decreases as a_0 increases but increases as c increases. For a more complex case of $n(r) = a_0 + a_1 r$, letting $a_0 = 1$, it is noted from Eq.(15) that D is the functions of a_1. The numerical results are given by Fig.3(b). From Fig.3(b) it is shown that D decreases as a_1 increases but increases as c increases. Those results are the same as that of nonlinear coefficient A of the KP equation discussed in Section 3. However, figures 3(a) and 3(b) show that the sign of nonlinear coefficient D of the mKP equation is always negative, which is different from that discussed in the last section.

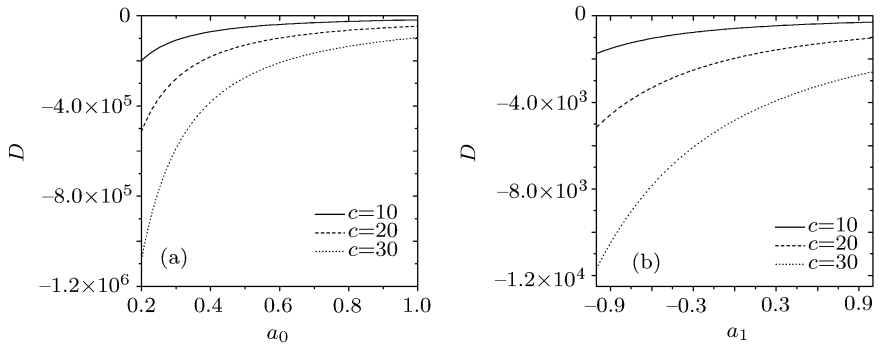


Fig.3. The variation of nonlinear coefficient D of the mKP equation Eq.(13) with respect to a_0 and a_1 for $c = 10$, 20, 30 respectively.

We assume that the stationary solitary wave solution of the mKP equation is: $\phi_1 = \phi_1(\theta)$, where $\theta = k\xi + l\eta - u_0\tau$. Substituting this expression into the mKP equation of Eq.(13), we obtain one solitary wave solution as follows:

$$\phi_1 = \phi_m \,\mathrm{sech}\left(\frac{k\xi + l\eta - u_0\tau}{\omega}\right), \tag{16}$$

where the amplitude of the solitary wave is $\phi_m = 6(u_0k - Cl^2/Dk^2)^{1/2}$ and the width of the solitary wave is $\omega = k^2[B/(u_0k - Cl^2)]^{1/2}$.

Now we study the properties of the solitary wave solution of Eq.(13). For the uniform distribution of $n(r) = a_0$, we can obtain the numerical results of ϕ_m and ω, which are shown in Figs.4(a) and 4(b). It can be seen that ϕ_m and ω increase as a_0 increases. We also find from Figs.4(a) and 4(b) that as c increases ϕ_m decreases but ω increases. For a more complex case of $n(r) = a_0 + a_1 r$, letting $a_0 = 1$, it is noted from Eqs.(6) and (15) that both the solitary wave amplitude ϕ_m and width ω are functions of a_1. Figures 4(c) and 4(d) show that ϕ_m and ω increase as a_1 increases. It is also noted that as c increases ϕ_m decreases but ω increases. In comparison with the results in Section 3, we find that the amplitude is modified significantly but the width ω is not modified. Moreover, it is noted that the amplitude ϕ_m of solitary waves is always negative from Fig.2 but positive from Fig.4. In other words, the rarefactive solitary wave exists in the system described by the KP equation of Eq.(4) while the compressive solitary wave exists in the system described by the mKP equation of Eq.(13).

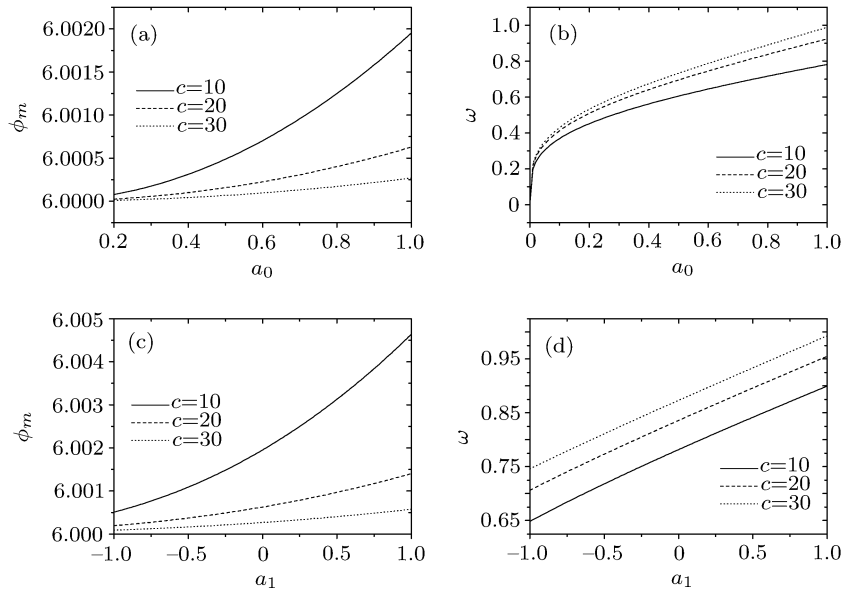


Fig.4. The variation of ϕ_m and ω from Eq.(16) with respect to a_0 and a_1 for $c = 10$, 20 , 30 respectively.

5. Coupled KP equation

If A is at the same order of ε, but not zero, we will derive a coupled Kadomtsev–Petviashvill (cKP) equation. By using the same stretched coordinates and same expansions as in Section 4, the coupled KP equation is obtained as follows:

$$\frac{\partial}{\partial\xi}\left[\frac{\partial\phi_1}{\partial\tau} + A\phi_1\frac{\partial\phi_1}{\partial\xi} + D\phi_1^2\frac{\partial\phi_1}{\partial\xi} + B\frac{\partial^3\phi_1}{\partial\xi^3}\right] + C\frac{\partial^2\phi_1}{\partial\eta^2} = 0, \tag{17}$$

where

$$\begin{aligned} A &= -\frac{3}{2v_0^3}\sum_{j=1}^{N}\frac{n_{d0j}Z_{dj}^3}{m_{dj}^2} - \frac{v_0}{2}s^2(\nu\beta^2 - \mu),\\ B &= \frac{v_0}{2},\\ C &= \frac{v_0}{2},\\ D &= -\frac{15}{4}\sum_{j=1}^{N}\frac{n_{d0j}Z_{dj}^4}{v_0^5m_{dj}^3} - \frac{v_0}{4}s^2(\nu\beta^3 + \mu). \end{aligned} \tag{18}$$

It is well known that equation (17) has a station-

ary solitary wave solution as follows:[34,35]

$$\phi_1 = \frac{1}{a_1 + a_2 \sinh^2\left(\dfrac{k\xi + l\eta - u_0\tau}{\omega}\right)}, \tag{19}$$

where

$$a_1 = \frac{k^2}{6(u_0k - Cl^2)}\left(A + \sqrt{A^2 + \frac{6D(u_0k - Cl^2)}{k^2}}\right),$$

$$a_2 = \frac{k^2}{3(u_0k - Cl^2)}\sqrt{A^2 + \frac{6D(u_0k - Cl^2)}{k^2}},$$

and

$$\omega = 2k^2\sqrt{\frac{B}{u_0k - Cl^2}},$$

where $1/a_1$ and ω refer to the amplitude and width of solitary waves, respectively.

Now we study the characters of the solitary wave solution of Eq.(17). For the uniform distribution of $n(r) = a_0$, the expressions of both the solitary wave amplitude ϕ_m and width ω are functions of a_0. By using the typical dusty plasma parameters mentioned in Section 3, we can obtain the numerical results of ϕ_m and ω, which are shown in Fig.5. We find from Figs.5(a) and 5(b) that both ϕ_m and ω increase as a_0 increases. We also note that as c increases ϕ_m decreases but ω increases. For a more complex case of $n(r) = a_0 + a_1 r$, letting $a_0 = 1$, it is noted from Eqs.(6) and (7) that both the solitary wave amplitude ϕ_m and width ω are functions of a_1. We find from Fig.5 that both ϕ_m and ω increase as a_1 increases. We also note that as c increases ϕ_m decreases but ω increases. Moreover, it can be seen from Fig.5 that the amplitude ϕ_m of solitary waves is positive for the uniform distribution but positive for the more complex distribution. In other words, the rarefactive solitary wave exists for the uniform distribution of the dust size while the compressive solitary wave exists for a more complex distribution, which is quite different from the results obtained in Sections 3 and 4. This suggests that whether a compressive or a rarefactive solitary wave exists significantly depends on the dust size distribution in the dusty plasmas.

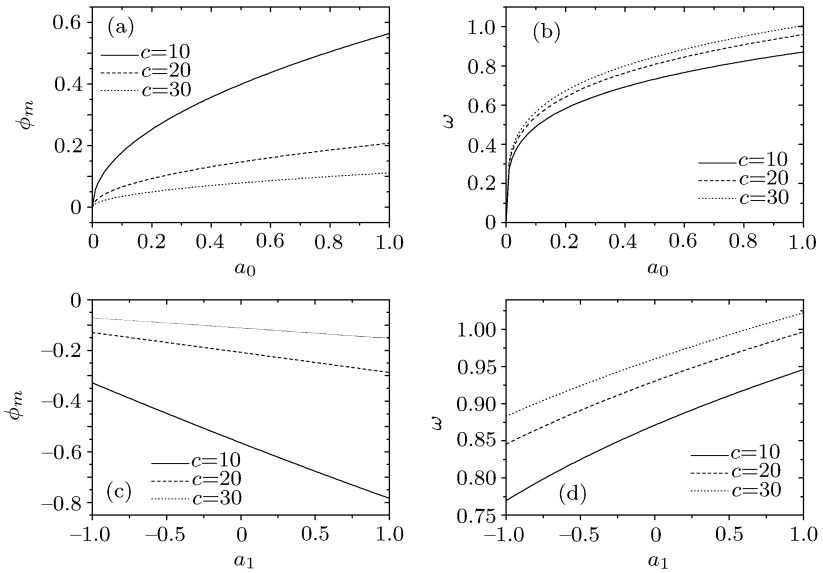


Fig.5. The variation of ϕ_m and ω from Eq.(19) with respect to a_0 and a_1 for $c = 10$, 20 , 30 respectively.

6. Conclusion

In summary, a series of KP equations (KP, mKP and cKP equations) governing two-dimensional dust-acoustic solitary waves in a three-component dusty plasma of Boltzmann distributed ions and electrons and charged dust grains of N different species are derived by using the standard reductive perturbation method respectively. For an arbitrary dust size distribution dusty plasma, we choose the polynomial expressed distribution function to investigate how the different dust size distribution influences the behavior

of the solitary waves. For the simplest case of the polynomial expressed distribution, i.e., the uniform distribution, we find that the amplitude and width of the nonlinear waves from the KP equation, the mKP equation and the cKP equation all increase as the total number density increases. It is also found that as the ratio c of the maximum dust size to the minimum dust size increases the amplitude all decrease but the width all increase. For a more complex case of the polynomial expressed distribution, it is noted that the solitary wave amplitude and width are greater for unusual dusty plasma than for the usual dusty plasma in space. When we analyze the solution of the cKP equation, numerical results show that a rarefactive solitary wave exists for the uniform distribution of dust size while a compressive solitary wave exists for the more complex distribution. This suggests that whether compressive or rarefactive solitary waves exist strongly depends on the dust size distribution in the dusty plasmas.

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