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Transverse effects in photorefractive two-wave mixing*

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In a biased photorefractive crystal, the process of two one-dimensional waves mixing, i.e., the dynamical evolution of both pump beam and signal beam, is traced by numerically solving the coupled-wave equation. Direct simulations show that the propagation and stability of the two beams are completely determined by the system parameters, such as the external bias field, the intensity and the beam waist of the pump beam. By adjusting these parameters, one can control the state of two Gaussian waves mixing. The numerical results are helpful for performing a two-wave mixing experiment.

Keywords: two-wave mixing, transverse effects, Gaussian distribution, photorefractive crystal
PACC: 4265S, 4265J, 7820W, 4240D

1. Introduction

The process of photorefractive (PR) two-wave mixing (TWM) shows much promise in the field of optical processing. TWM geometry is used as a building block for more complex optical applications, including optical image processing, optical amplification based on wave mixing, holographic data storage, dissipative holographic solitons,[1−6] etc. This has impelled a great variety of studies on the TWM phenomenon in order to learn about it and to enhance its response under different external stimuli.[7−13] In these studies, the effects of transverse distribution of optical beams on the process of two-wave mixing, except for the studies of modulation instability (MI) and spatial solitons,[6] have not yet been investigated sufficiently. However, the optical beams used in an actual experiment are almost of Gaussian distribution transversely. So we can naturally raise the problem of how this kind of transverse distribution will affect the process of two-wave mixing and affect further the optical amplification, holographic data storage and dissipative holographic solitons.

In order to solve the above problems, in this paper, we consider the process of two-wave mixing and investigate the variations of the peak intensities and beam width (intensity FWHM) of both signal beam and pump beam with propagation distance and the effects of system parameters on them, such as the bias field, the intensity and the beam waist of the pump beam. By using the model described in Ref.[6], many new results and predictions are set forth in this paper. Our numerical results indicate that, under some suitable conditions, the signal beam can be absorbed or amplified and the pump beam exhibits common decaying behaviour without modulation instability (MI). These results are helpful for performing a two-wave mixing experiment in PR crystals. It is worth noting that both one- and two-dimensional Gaussian beams appear in real experiments and our model can reflect the one-dimensional case more exactly than the two-dimensional case.

This paper is organized as follows. In Section 2, a model to describe two-wave mixing in a biased PR crystal is constructed. In Section 3, direct numerical simulations of the two beams are performed. We draw some conclusions in Section 4.

2. Theoretical model

The schematic setup of an imaginary experiment to allow the observation of two-wave mixing is shown

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in Fig.1. The PR crystal considered here is a strontium barium niobate (SBN) crystal with its optical c-axis oriented along the x-coordinate. Both signal and pump beams are ordinarily polarized. They are made to propagate in the crystal and each beam makes a small angle (θ) of its k vector with the z-axis. The initial signal beam and pump beam have a one-dimensional Gaussian spatial distribution which is confined in the x direction, and extended (non-diffracting) in the y direction. In this configuration, the electric field should be applied along the y direction to ensure that there are no focusing effects induced by the PR screening nonlinearity in the x direction. The two beams interact through the holographic focusing mechanism coming from the index grating that they form.

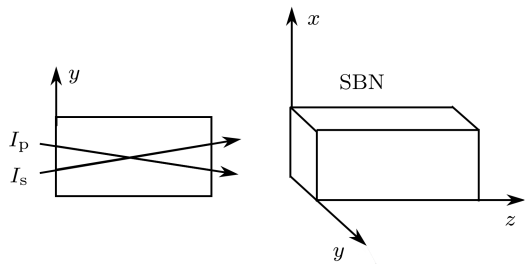


Fig.1. Schematic configuration to observe two-wave mixing in a PR crystal. The signal and pump beams are coupled through the holographic focusing mechanism.

In the slowly varying approximation, we find ϕ_p and ϕ_s, the slowly varying envelopes of the electric field component of the pump beam and the signal beam respectively, satisfy the following paraxial coupled-wave equations:[6]

$$\frac{1}{2k}\frac{\partial^2\phi_p}{\partial x^2}+\mathrm{i}\cos\theta\frac{\partial\phi_p}{\partial z}+\left(\Gamma_0+\mathrm{i}\frac{1}{2}\Gamma\right)\frac{|\phi_s|^2}{|\phi_s|^2+|\phi_p|^2}\phi_p+\mathrm{i}\frac{1}{2}\alpha_0\phi_p=0, \tag{1a}$$

$$\frac{1}{2k}\frac{\partial^2\phi_s}{\partial x^2}+\mathrm{i}\cos\theta\frac{\partial\phi_s}{\partial z}+\left(\Gamma_0-\mathrm{i}\frac{1}{2}\Gamma\right)\frac{|\phi_p|^2}{|\phi_s|^2+|\phi_p|^2}\phi_s+\mathrm{i}\frac{1}{2}\alpha_0\phi_s=0, \tag{1b}$$

where $k=n_0k_0$ ($k_0=2\pi/\lambda_0$ with λ_0 being the free space wavelength of the light wave employed, and n_0 is the ordinary refractive index); Γ and Γ_0 are the intensity and phase coupling coefficients, respectively; α_0 is the absorption coefficient of the crystal. From PR theory,[3,5] $\Gamma/2+\mathrm{i}\Gamma_0=\delta\tilde{E}E_s/(\tilde{E}+E_s)$, where $\tilde{E}=E_d+\mathrm{i}E_0$, with E_0 being the bias field and $E_d=4\pi k_BT\sin\theta/\lambda_0e$ being the diffusion field (k_B is the Boltzmann constant, e is the electron charge, and T is the absolute temperature); $E_s=eN_A\lambda_0/(4\pi\varepsilon\varepsilon_0\sin\theta)$ is the saturation field, with N_A being the acceptor density and ε and ε_0 being the effective dielectric constant and the permittivity of the vacuum, respectively; and $\delta=\pi n_e^3\gamma_{eff}/\lambda_0$. For this configuration the effective electro-optic coefficient γ_{eff} can be expressed as[5,14] $\gamma_{eff}=\bar{n}^3\gamma_{33}\cos^2\theta-\bar{n}^{-1}\gamma_{13}\sin^2\theta$, where $\bar{n}=n_e/n_0$ and n_e is the extraordinary index of refraction.

We now rewrite the coupled-wave equation in a dimensionless form by letting $s=x/x_0$, $\xi=z/(z_0\cos\theta)$, $z_0=kx_0^2$, $g_0=z_0\Gamma_0$, $g=z\Gamma_0/2$, $\alpha=z_0\alpha_0/2$, $U_s=I_0^{-1/2}\phi_s$ and $U_p=I_0^{-1/2}\phi_p$, where x_0 is an arbitrary width and I_0 is an arbitrary intensity. The Eqs.(1) can be expressed in terms of U_s and U_p as the following:

$$\mathrm{i}\frac{\partial U_p}{\partial\xi}+\frac{1}{2}\frac{\partial^2U_p}{\partial s^2}+(g_0+\mathrm{i}g)\frac{|U_s|^2}{|U_p|^2+|U_s|^2}U_p+\mathrm{i}\alpha U_p=0, \tag{2a}$$

$$\mathrm{i}\frac{\partial U_s}{\partial\xi}+\frac{1}{2}\frac{\partial^2U_s}{\partial s^2}+(g_0-\mathrm{i}g)\frac{|U_p|^2}{|U_p|^2+|U_s|^2}U_s+\mathrm{i}\alpha U_s=0, \tag{2b}$$

where g represents the coefficient of gain, which characterizes the energy transformation from the pump beam to the signal beam, and g_0 represents the phase coupling coefficient.

In what follows, we search for the dynamic evolution of the two Gaussian beams in this system. To do so, the initial signal and pump beams are assumed as:

$$U_{\rm s}(s,\xi=0)=I_{\rm s}^{1/2}\exp[-(s^2/W_{\rm s}^2)], \qquad (3a)$$

$$U_{\rm p}(s,\xi=0)=I_{\rm p}^{1/2}\exp[-(s^2/W_{\rm p}^2)], \qquad (3b)$$

where $I_{\rm s}$ and $I_{\rm p}$ denote the peak intensities normalized by I_0; $W_{\rm s}$ and $W_{\rm p}$ are the parameters related to the widths normalized by x_0. The actual width (intensity FWHM) of the signal and pump beams are $\Delta x_{\rm s}=x_0\Delta s=2x_0W_{\rm s}\sqrt{\ln\sqrt{2}}$ and $\Delta x_{\rm p}=x_0\Delta s=2x_0W_{\rm p}\sqrt{\ln\sqrt{2}}$, respectively.

In order to give an actual example, the parameters of a SNB:60 crystal at $\lambda_0=0.5$ μm and $T=300$ K are taken as $n_{\rm e}=2.35$, $n_0=2.37$, $\gamma_{13}=47$ pm/V, $\gamma_{33}=235$ pm/V and $\varepsilon=880$. The other parameters are taken as $N_{\rm A}=5\times10^{17}$ cm^{-1}, $x_0=25$ μm, $2\theta=0.1^\circ$ and $\alpha=0.415$.

3. Numerical results and discussion

3.1. Effect of the bias field on the propagation of the signal and pump beams

We investigate the dynamical evolution of the signal and pump beams in the PR crystal by numerically solving Eqs.(2) for different values of E_0, which will result in different values of system parameters g and g_0.

Firstly, we consider the case when the intensity and beam waist of the signal beam are equal to those of the pump beam, such as $I_{\rm s}=I_{\rm p}=0.1$, $W_{\rm s}=W_{\rm p}=1.7$ ($\Delta x_{\rm s}=\Delta x_{\rm p}=50.04$ μm). The evolution of the peak intensities and the intensities FWHM of the two beams are analysed for $E_0=-200$ V/cm, $E_0=-1700$ V/cm, and $E_0=-3000$ V/cm; the corresponding values of g are 0.391589, 0.433514, 0.523395 and g_0 are -6.89591, -58.6152, -103.4930, respectively, which are depicted in Fig.2 (calculated to $\xi=0.5$, i.e., $z\approx0.93$ cm).

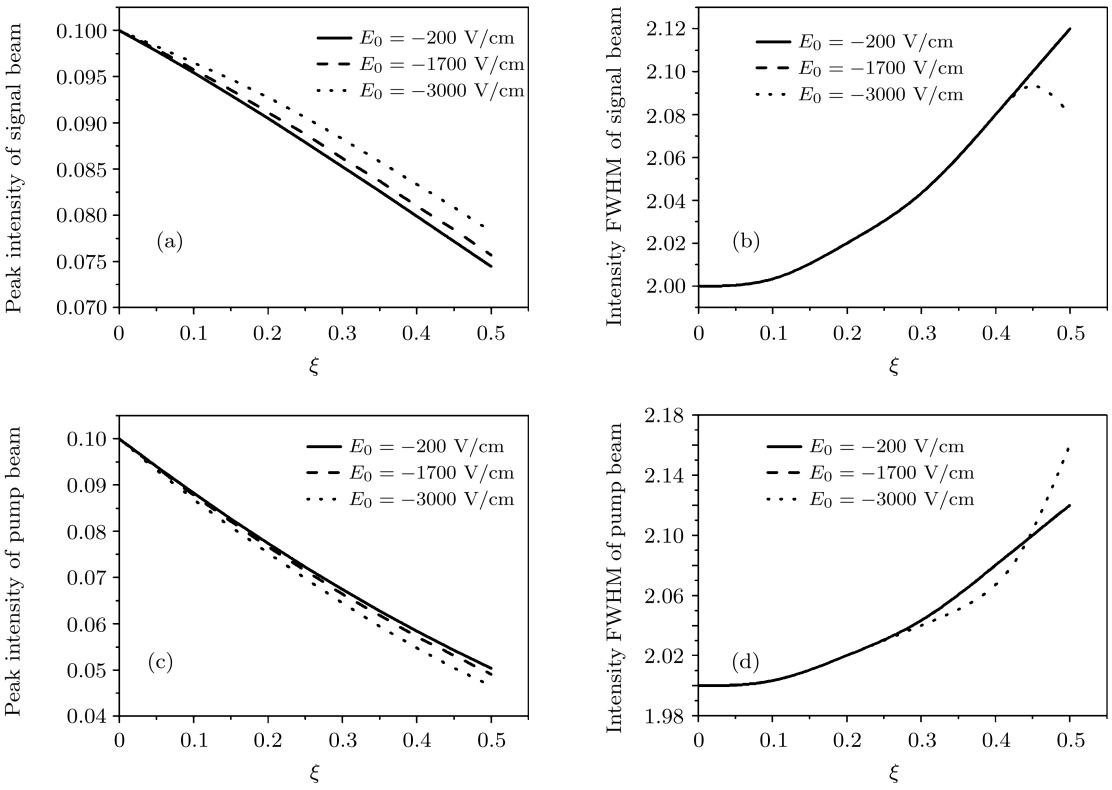


Fig.2. Effect of the bias field on the evolution of the peak intensity and the intensity FWHM of the signal beam and pump beam with $I_{\rm s}=I_{\rm p}=0.1$ and $W_{\rm s}=W_{\rm p}=1.7$. The solid curve stands for $E_0=-200$ V/cm, the dashed curve for $E_0=-1700$ V/cm and the dotted curve for $E_0=-3000$ V/cm. (a), (b) Signal beam, (c), (d) pump beam.

The peak intensities of the two beams decrease and the beam widths increase with propagation distance. The bigger the bias field E_0 is, the slower the peak intensity of the signal beam decreases and the faster that of the pump beam increases, as show in Figs.2(a) and 2(c). Also, when the bias field is small, the effect of it on the evolution of the beam widths is not obvious, while as E_0 increases to -3000 V/cm, the intensity FWHM of the signal beam decreases and the intensity FWHM of the pump beam increases faster in the end, as shown in Figs.2(b) and 2(d).

However, if the bias field is large enough, such as $E_0 = -5000$ V/cm, the corresponding value of g is 0.758761 and g_0 is -172.397, both signal beam and pump beam completely break up in the crystal, as shown in Fig.3.

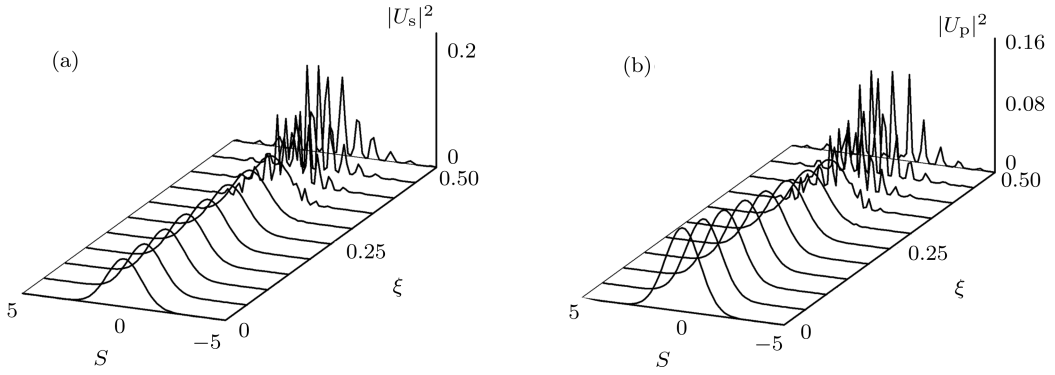


Fig.3. Evolution of the signal beam and the pump beam at $E_0 = -5000$ V/cm. (a) Signal beam, (b) pump beam.

Then, we consider the case when the intensity of the pump beam is larger than that of the signal beam and the beam waists are equal, such as $I_s = 0.1$, $I_p = 10$ and $W_s = W_p = 1.7$. Here the evolution of the peak intensities and the beam widths of the two beams are analysed for $E_0 = -1700$ V/cm, $E_0 = -3000$ V/cm and $E_0 = -5000$ V/cm. From Fig.4(a), we can find that the peak intensity of the signal beam varies from decreasing

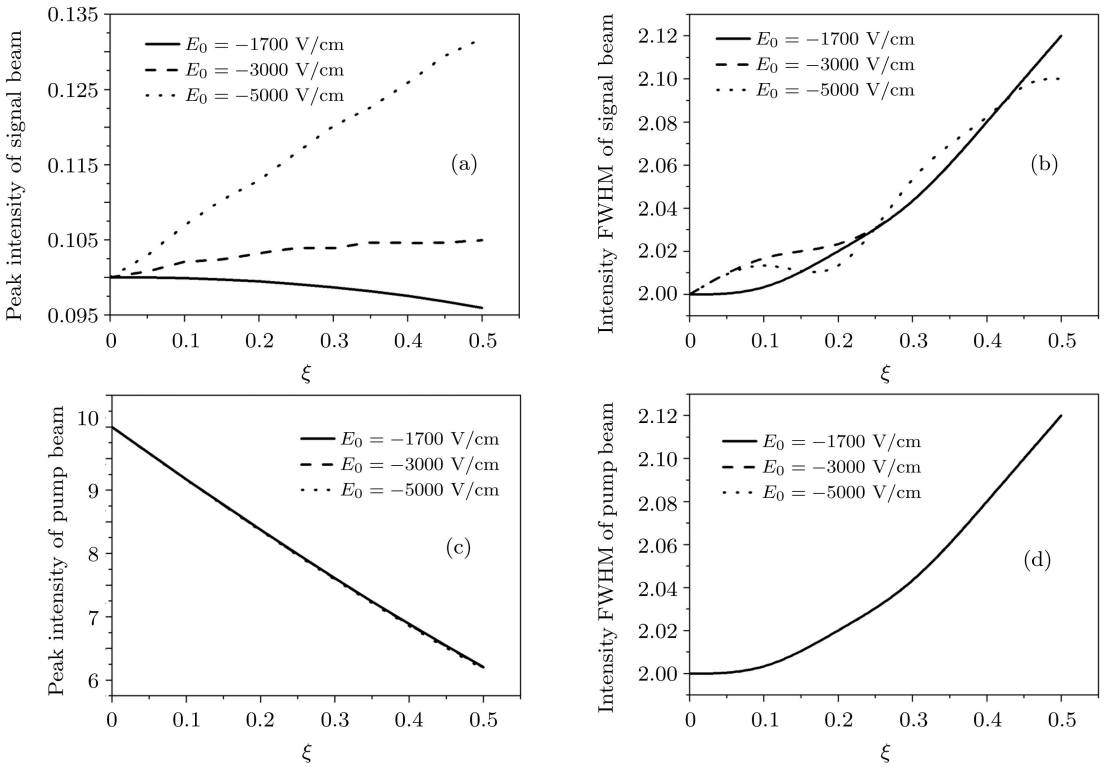


Fig.4. Effect of the bias field on the evolution of the peak intensity and the intensity FWHM of the signal beam and the pump beam with $I_s = 0.1$, $I_p = 10$ and $W_s = W_p = 1.7$. Solid curve for $E_0 = -1700$ V/cm, dashed curve for $E_0 = -3000$ V/cm and dotted curve for $E_0 = -5000$ V/cm.

to increasing as the bias field E_0 increases. The larger E_0 is, the more drastically the intensity FWHM of the signal beam varies, as shown in Fig.4(b). The peak intensity of the pump beam decreases and beam width increases with propagation distance, and the three evolution curves, corresponding to different values of E_0, overlap which means the variation of the bias field does not affect their evolution, as shown in Figs.4(c) and 4(d).

3.2. Effect of the intensity of the pump beam on the propagation of the signal beam

Here we investigate the influence of the intensity of the pump beam I_p on the propagation of the signal beam with $E_0 = -30$ V/cm and $E_0 = -3000$ V/cm; the corresponding values of g are 0.391014, 0.523395 and g_0 are -1.03439, -103.4930, respectively. Other parameters are the same as the above, such as $I_\mathrm{s} = 0.1$, $W_\mathrm{s} = W_\mathrm{p} = 1.7$.

When the bias field is small, such as $E_0 = -30$ V/cm, the gain is smaller than the loss, and the speed of the peak intensity of the signal beam decreasing becomes slower as I_p increases, as shown in Fig.5(a). From Fig.5(b), we can see the intensity FWHM of the signal beam increasing with propagation distance. Three evolution lines corresponding to different intensities of the pump beam overlap, which means the increase of I_p does not affect the evolution of the beam width of the signal beam when E_0 is small.

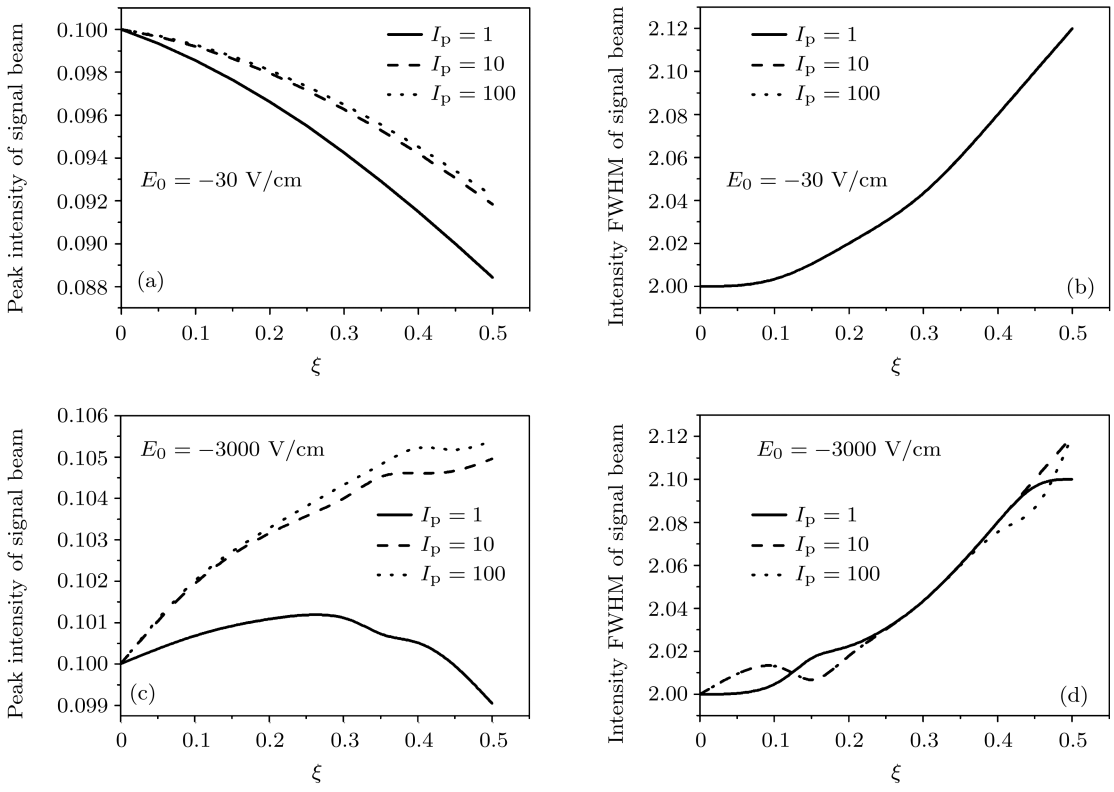


Fig.5. FWHM of the signal beam with $I_\mathrm{s} = 0.1$, $W_\mathrm{s} = W_\mathrm{p} = 1.7$. Solid curve for $I_\mathrm{p} = 1$, dashed curve for $I_\mathrm{p} = 10$ and dotted curve for $I_\mathrm{p} = 100$. (a), (b) $E_0 = -30$ V/cm, (c), (d) $E_0 = -3000$ V/cm.

When the bias field is large, such as $E_0 = -3000$ V/cm, gain is larger than loss, and the peak intensity of the signal beam varies from decreasing to increasing as I_p increases, as shown in Fig.5(c). On the other hand, the intensity FWHM also increases with propagation distance and the effect of I_p on its evolution is not obvious, as shown in Fig.5(d).

3.3. Effect of the beam waist on the propagation of the signal and pump beams

In the above, all cases we have considered are that the beam waists of the two beams are equal to 1.7. A natural question to address here is what will happen if the beam waist of the pump beam is larger than that of the signal beam and the intensities of them are equal, such as $W_s = 1.7$, $W_p = 5$ and $I_s = I_p = 0.1$. From the simulation results, we find that when the bias field is small, such as $E_0 = -5$ V/cm, both the signal and the pump beams exhibit a common decaying process without MI, as shown in Figs.6(a) and 6(b); and when E_0 increases to $E_0 = -200$ V/cm, only the signal beam can propagate stably and the pump beam splits in the end, as shown in Figs.6(c) and 6(d); and when $E_0 = -1700$ V/cm the two beams completely break up, as shown in Figs.6(e) and 6(f).

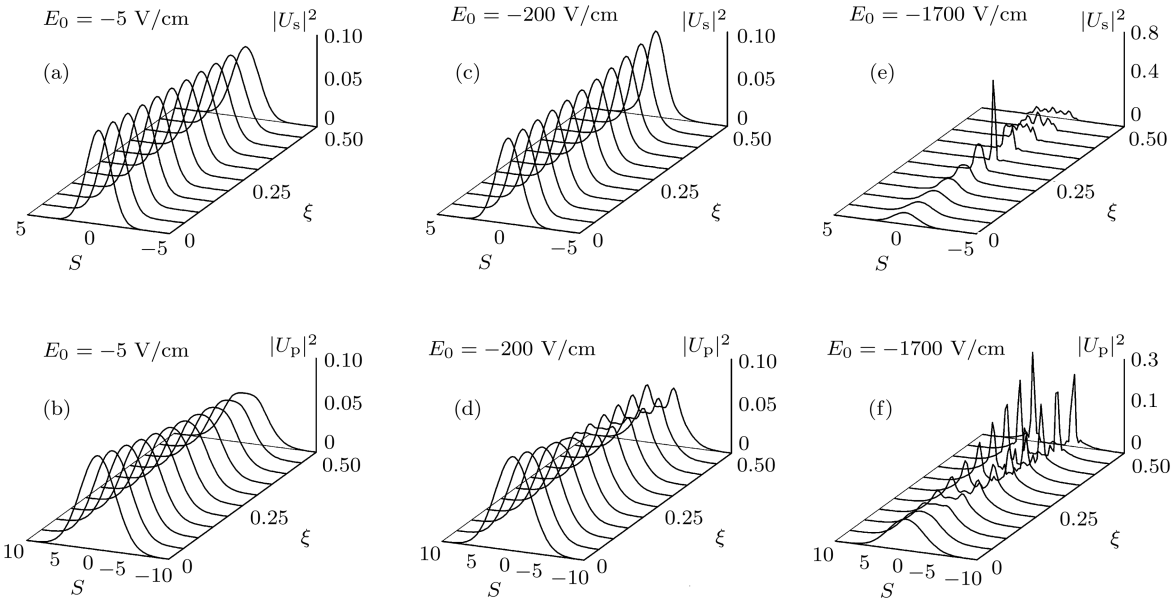


Fig.6. Evolution of the signal beam and pump beam with $I_s = I_p = 0.1$, $W_s = 1.7$ and $W_p = 5$. (a), (c), (e) Signal beam; (b), (d), (f) pump beam.

The results above demonstrate that, when the bias field is not small enough, the pump beam is apt to show MI that will result in the signal beam being extremely unstable. Therefore, the bias field being small enough is a necessary condition that both pump beam and signal beam can propagate stably in the crystal when the beam waist of the pump beam is larger than that of the signal beam and the intensities of them are equal.

4. Conclusion

The evolution and the transverse effects of a one-dimensional pump beam and signal beam in a biased PR crystal have been investigated. The results demonstrate that the evolution and the stability of the two beams are influenced by a number of system parameters, such as the bias field, the intensity of the pump beam and the beam waist. When the intensity and beam waist of the pump beam are equal to those of the signal beam, the bias field should not be made too large, otherwise both pump beam and signal beam will completely break up. When the intensity of pump beam is larger, the signal beam can be absorbed or amplified within a regime in which the pump beam is depleted slightly by adjusting the value of the bias field or the intensity of the pump beam. When the beam waist of the pump beam is larger, both pump beam and signal beam are apt to show MI if the bias field is not small enough. These results give potential indications for observing the evolution of two Gaussian waves mixing in a biased PR crystal.

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