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A solvable operator motion equation of an excited-atom coupled leaky cavity in single-photon generation

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A model for an excited-atom coupled leaky cavity in single-photon generation is proposed based on universal modes. Solvable motion equations of the atomic operators are obtained under the single-photon condition by adopting the Lorentzian line type of the universal modes.

Keywords: atomic operator evolution, leaky cavity, universal mode, single photon
PACC: 4250, 3280, 0367

1. Introduction

Single-photon generation means that only one photon is emitted at a predetermined time within a short time interval, that is, if the single-photon source is switched on, then with a high degree of certainty just one photon emerges at a known, user-specified time. Such an optical source exhibits non-classical behaviour; it is impossible to generate single photons with even very weak laser pulses, for which the photon number obeys Poissonian statistics. A single-photon source could find applications in quantum information processing,[1] quantum cryptography[2] and certain quantum computation problems.[3] Single photons can be generated by a variety of schemes, including single atom in cavity,[4] single molecule,[5] single quantum dot,[6] photonic crystals[7] and parametric processes.[8]

Several authors have made a theoretical analysis of the generation of single photons from a coupled atom–cavity system. In Ref.[9] a high efficiency scheme for generating a single-photon state transmitted out of an optical cavity was reported. The controlled generation of single photons from a strongly coupled atom–cavity system was discussed in Ref.[10]. In Ref.[11], the emission spectra and quantum efficiency of single-photon sources in the cavity-QED strong-couple regime were investigated. In this paper we shall use another method, called the universal modes, to go further into the theoretical problem. Firstly we put forward the model of an excited-atom coupled leaky cavity in single-photon generation based on the universal modes. Then we give the motion equations of the atomic operators in the Heisenberg picture and make a theoretical analysis of the equations.

2. Model

In our model, a Λ-type three level atom is coupled to an ideal cavity and a leaky cavity, as shown as Fig.1. The transitions $|3\rangle - |1\rangle$ and $|3\rangle - |2\rangle$ are allowed and the transition $|2\rangle - |1\rangle$ is forbidden, and an independent repumping scheme brings the system back to the initial state. We consider atom–ideal cavity coupling in order to treat the spontaneous emission process between atomic levels $|3\rangle$ and $|1\rangle$. Like the models of Refs.[9] and [10], the bad cavity regime has been adopted to minimize loss due to the spontaneous decay process. We describe the leaky cavity by the so called universal modes. In the universal mode picture, the leaky cavity is a part of the whole cavity with one cavity mirror at infinity, and the field is rigorously quantized in a larger cavity (the 'universe'). For example, using the universal mode, the authors of Refs.[12] and [13] have developed the one-dimensional output coupling laser theory in which coupling loss can be naturally considered, derived automatically and no loss term is explicitly added to the field equation.

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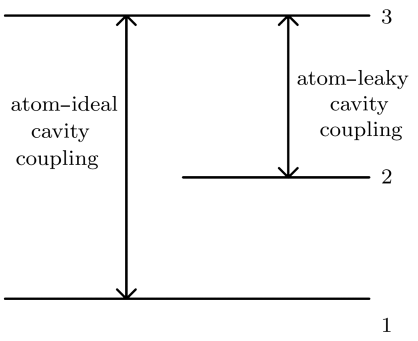


Fig.1. Sketch map of a Λ-type three level atom coupled to an ideal cavity and a leaky cavity

Due to the universal mode used in our model, the photon leakage rate need not be introduced; instead the atom and leaky cavity coupling becomes the atom and the universal mode coupling. In comparison, in the models of Refs.[9] and [10], the photon leakage rate was explicitly added to the master equation for the density matrix operator. For the problem of operator evolution of the exited-atom coupled leaky cavity in single-photon generation, we shall obtain operator equations that are solvable based on the universal modes.

In addition, the lifetimes of atomic levels $|1\rangle$ and $|3\rangle$ are assumed long so that their relaxation is ignored for all models mentioned above.

3. Theoretical analysis

For the system under consideration, the three-level excited-atom coupled leaky cavity (universal modes) and coupled ideal cavity (single mode) can be described by a Hamiltonian of the form of the electric dipole approximation

$$\begin{aligned} H = & \sum_{j=1}^{3} \hbar\Omega_j R_{jj} + \hbar\omega_1 a_1^+ a_1 + \sum_s \hbar\omega_s a_s^+ a_s \\ & -\mathrm{i}\hbar g_1 (a_1 R_{31} - a_1^+ R_{13}) \\ & -\mathrm{i}\hbar \left(\sum_s g_s a_s R_{32} - \sum_s g_s a_s^+ R_{23} \right). \end{aligned} \tag{1}$$

Here the first term is the energy of a free three-level atom, and the operator R_{jj} describes the occupation of the jth level with energy $\hbar\Omega_j$. The second term and third term present the energy of the mode of a free electromagnetic field, $a_1^+(a_1)$ is the creation (annihilation) operator for the ideal cavity mode and $a_s^+(a_s)$ is the creation (annihilation) operator for a photon of the universal mode, where $\omega_1 = \Omega_3 - \Omega_1$, ω_s is the frequency of the s mode. The fourth and fifth terms are the energy of the atom–field interaction in the dipole approximation, where g_1 and g_s are the coupling constant (assuming g_1 and g_s are real) and the operator R_{ij} describes the transition from state $|j\rangle$ to $|i\rangle$ $(i \neq j)$.

If both cavities are considered as ideal cavities, the symbol $\sum$ in the fifth term of Eq.(1) does not appear, and Eq.(1) can only be used to describe the case of the two-photon process. The authors of Ref.[14] have studied the dynamical behaviour of level filling and occupation numbers of photon mode in the two-photon process in a three-level system. If only one leaky cavity is considered, there are no terms including $a(a_1^+)$ in Eq.(1), which corresponds to the Janes–Cumming (J–C) mode in a leaky cavity. The authors of Ref.[15] have presented an exact pure-state approach for the J–C model in a leaky optical cavity based on the universal mode. In fact our model is the extension of the two models mentioned above and therefore we can combine the analysis methods mentioned above to make a theoretical analysis in what follows.

According to the commutative law of atomic operators, and the equation of motion of any observable in the Heisenberg picture, we obtain the Heisenberg equation of motion for the atomic operators $R_{jj}(t)$:

$$\dot{R}_{11}(t) = g_1[a_1^+(t)R_{13} + a_1(t)R_{31}(t)], \tag{2}$$

$$\dot{R}_{22}(t) = \sum_s g_s a_s^+(t) R_{23}(t) + \sum_s g_s a_s R_{32}(t) \tag{3}$$

and $\dot{R}_{33}(t)$ can be got by the relation

$$\sum_j R_{jj} = 1. \tag{4}$$

We define the occupation number operators for the ideal cavity mode and the s mode of the universal modes,

$$N_1(t) = a_1^+ a_1, \tag{5}$$

$$N_s(t) = a_s^+ a_s. \tag{6}$$

Then we obtain by the Heisenberg motion equations for N_1 and N_s,

$$\dot{N}_1(t) = \dot{R}_{11}(t), \tag{7}$$

$$\sum_s \dot{N}_s(t) = \dot{R}_{22}(t). \tag{8}$$

That means,

$$N_1(t) - R_{11}(t) = M_1, \tag{9}$$

$$\sum_s N_s(t) - R_{22}(t) = M_2, \tag{10}$$

where the operators M_1 and M_2 are constants independent of time t.

Using the Heisenberg motion equations of $\dot{N}_1(t)$, from Eqs.(2) and (7) we obtain

$$\begin{aligned}\ddot{R}_1(t) = {} & g_1^2[2a_1^+a_1(R_{33}-R_{11})+2R_{33}] \\ & -g_1\left(\sum_s g_s a_s^+ a_1 R_{21}+\sum_s g_s a_1^+ a_s R_{12}\right). \end{aligned} \tag{11}$$

Combining Eqs.(4), (5), (9) and (11), we find

$$\begin{aligned}\ddot{R}_1(t) = {} & 2g_1^2(M_1+1)(1-2R_{11}-R_{22}) \\ & -g_1\left(\sum_s g_s a_s^+ a_1 R_{21}+\sum_s g_s a_1^+ a_s R_{12}\right). \end{aligned} \tag{12}$$

Also by the Heisenberg motion equation of $\dot{N}_s(t)$, we obtain from Eqs.(3) and (8),

$$\begin{aligned}\sum_s \ddot{N}_s(t) = {} & -g_1\left[a_1^+\sum_s g_s a_s R_{12}+\sum_s g_s a_s^+ a_1 R_{21}\right] \\ & +2\sum_s g_s^2 a_s^+ a_s(R_{33}-R_{22}) \\ & +2\sum_s g_s^2 R_{33}. \end{aligned} \tag{13}$$

Assuming the modes are independent, we have the following expression from Eq.(10)

$$\sum_s g_s^2 N_s(t) = \beta[R_{22}(t)+M_2], \tag{14}$$

where β is defined as

$$\beta = \sum_s g_s^4 \Big/ \sum_s g_s^2. \tag{15}$$

Substituting Eq.(14) into Eq.(13), we have

$$\begin{aligned}\sum \ddot{N}_s(t) = {} & -g_1\left[a_1^+\sum_s g_s a_s R_{12}+\sum_s g_s a_s^+ a_1 R_{21}\right] \\ & -2\beta(1+M_2)R_{22} \\ & +2(\alpha+\beta M_2)(1-R_{11}-R_{22}), \end{aligned} \tag{16}$$

where

$$\alpha = \sum_s g_s^2. \tag{17}$$

We note the same part in Eqs.(12) and (16), and define the part as B,

$$B(t) = \sum_s g_s a_s^+ a_1 R_{21}+\sum_s g_s a_s a_1^+ R_{12}. \tag{18}$$

Now we derive $\dot{B}(t)$ by the Heisenberg motion equation $\mathrm{i}\hbar\dot{B} = [B, H]$, and obtain

$$\begin{aligned}\mathrm{i}\hbar\dot{B} = {} & R_{21}a_1\sum_s g_s a_s^+\hbar(\Omega_1-\Omega_2+\omega_1-\omega_s) \\ & +\mathrm{i}\hbar g_1\sum_s g_s a_s^+ a_1 a_1^+ R_{23} \\ & +\mathrm{i}\hbar\left(\sum_s g_s a_s\right)\left(\sum_s g_s a_s^+\right)R_{31}a_1 \\ & -\mathrm{h.c.}, \end{aligned} \tag{19}$$

where h.c. is the conjugating term.

The atom–field coupling g_s^2 (assuming g_s is real) is proportional to a Lorentzian,[15] i.e.,

$$g_s^2 = \frac{\eta^2}{(\omega_s-\Omega)^2+\kappa^2}, \tag{20}$$

where $\Omega = \Omega_3-\Omega_2$, η depends on the atomic dipole and κ is the decay rate.

Due to the Lorentzian of g_s^2, there is a symmetry about the centre frequency Ω, and the first tem of Eq.(19) and the corresponding term of h.c. can be neglected. Then combining Eqs.(2) and (3), we obtain from Eq.(19)

$$\begin{aligned}\dot{B}(t) = {} & g_1 a_1 a_1^+\dot{R}_{22} \\ & +\left(\sum_s g_s a_s\right)\left(\sum_s g_s a_s^+\right)\dot{R}_{11}/g_1. \end{aligned} \tag{21}$$

In Eq.(21), the second term involves the factor $\sum_s g_s a_s \sum_s g_s a_s^+$, now we make the time derivation of the factor using the Heisenberg motion equation of the factor. We find

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\sum_s g_s a_s\sum_s g_s a_s^+\right) = \sum_s g_s^2\dot{R}_{22}. \tag{22}$$

Then we have

$$\sum_s g_s a_s\sum_s g_s a_s^+ = \sum_s g_s^2 R_{22}+\alpha M_3, \tag{23}$$

where the operator M_3 is a constant independent of time.

In the derivation above, we have used the condition of the single photon of one atom, that is, for the single photon case, the state of the photon satisfies

$$|n_s\rangle = |0\rangle \quad \text{or} \quad |1\rangle\,. \tag{24}$$

Hence corresponding to Eq.(24), there is the following relation for the single photon case,

$$\begin{aligned}& \sum_s g_s a_s\hbar\omega_s\sum_s g_s a_s^+ \\ & -\sum_s g_s a_s\sum_s g_s a_s^+\hbar\omega_s = 0. \end{aligned} \tag{25}$$

From Eqs.(23) and (9), Eq.(21) can be written as

$$\dot{B}(t) = g_1(1+M_1+R_{11})\dot{R}_{22} + \left(\sum_s g_s^2 R_{22} + M_3\right)\dot{R}_{11}/g_1. \quad (26)$$

Substituting Eqs.(2) and (3), the expressions of $\dot{R}_{22}$ and $\dot{R}_{11}$, into Eq.(26), we note commutative relations among $R_{ij}(i,j=1,2,3)$, such as $R_{11}R_{23}=0$, $R_{11}R_{32}=0$, $R_{22}R_{31}=0$, $R_{22}R_{13}=0$, hence in Eq.(26), $R_{11}\dot{R}_{22}=0$, $\sum_s g_s^2 R_{22}\dot{R}_{11}=0$, we obtain from Eq.(26)

$$\dot{B}(t) = g_1(1+M_1)\dot{R}_{22} + M_3\dot{R}_{11}/g_1. \quad (27)$$

By integrating both sides of Eq.(27), we obtain

$$B(t) = g_1(1+M_1)R_{22} + M_3R_{11}/g_1 + g_1^2K, \quad (28)$$

where K is a constant independent of time. Now combining Eqs.(9), (12), (18) and (28), we obtain the second order derivative equation of R_{11},

$$\ddot{R}_{11} = -3g_1^2(1+M_1)R_{22} - [\alpha M_3 + 4g_1^2(1+M_1)]R_{11} + 2g_1^2(1+M_1) - g_1^2K. \quad (29)$$

Similarly, combining Eqs.(10), (16), (18), (28), we obtain the second order derivative equation of operator R_{22}

$$\ddot{R}_{22} = -\left[g_1^2(1+M_1) + 2(\alpha+\beta+2\beta M)\right]R_{22} - \left[2(\alpha+\beta M_2) + \alpha M_3\right]R_{11} + 2(\alpha+\beta M_2) - g_1^2K. \quad (30)$$

According to the initial condition $\dot{R}_{11}(0)$, $R_{11}(0), \dot{R}_{22}(0)$ and $R_{22}(0)$, we can easily obtain the solution of the derivative equations Eqs.(29) and (30), as shown in the Appendix.

4. Physical meaning of equations

Equations (29) and (30) present the dynamic behaviour of atomic level population operators, in which M_1, M_2, and M_3 are constants, expressed by Eqs.(9), (10) and (23). From Eqs.(9) and (10), we can obtain the statistical moment operators of the photon numbers. Thus when we know the dynamic behaviour of the level populations, that is, the solutions of Eqs.(29) and (30), we can introduce the characteristic function operators and obtain information about photon statistical properties.

5. Summary

In this paper we have proposed a model for an excited-atom coupled leaky cavity in single-photon generation based on universal modes. In the model there are two cavities, an ideal cavity and a leaky cavity. The leaky cavity is described by the universal mode. In the Heisenberg picture we established the motion questions of the atomic operators. We here adopted the Lorentzian line type of the universal mode and used the single-photon condition, and we have made a theoretical analysis of the equations of atomic operators. We have finally obtained solvable equations of the atomic operators . The equations can be used to further treat the problems of single-photon generation in the future.

Appendix

Combining Eqs.(29) and (30), we obtain

$$\dddot{R}_{11} + [5g_1^2(1+M_1) + 2(\alpha+\beta+2M_2\beta) + \alpha M_3]\ddot{R}_{11} + \{2g_1^2(1+M_1)[\alpha(1-M_3) + \beta(4+5M_2) + 2g_1^2(1+M_1)] + 2[\alpha+\beta(1+2M_2)]\alpha M_3\}R_{11} + 3g_1^2(1+M_2)[2(\alpha+\beta M_2) - g_1^2K] = 0. \quad (A1)$$

We define

$$2r_\pm^2 = -[5g_1^2(1+M_1) + 2(\alpha+\beta+2\beta M_2) + \alpha M_3] \pm \left\{[5g_1^2(1+M_1) + 2(\alpha+\beta+2\beta M_2) + \alpha M_3]^2 - 8g_1^2(1+M_1)[\alpha(1-M_3) + \beta(4+5M_2) + 2g_1^2(1+M_1)] - 8[\alpha+\beta(1+2M_2)]\alpha M_3\right\}^{1/2},$$

$$r_1 = \frac{\sqrt{2}}{2}\sqrt{r_+^2}, \quad r_2 = -\frac{\sqrt{2}}{2}\sqrt{r_+^2}, \quad r_3 = \frac{\sqrt{2}}{2}\sqrt{r_-^2}, \quad r_4 = -\frac{\sqrt{2}}{2}\sqrt{r_-^2}, \quad (A2)$$

$$C_0 = \frac{3g_1^2(1+M_2)[2(\alpha+\beta M_2) - g_1^2K]}{2g_1^2(1+M_1)[\alpha(1-M_3) + \beta(4+5M_2) + 2g_1^2(1+M_1)] + 2[\alpha+\beta(1+2M_2)]\alpha M_3}. \quad (A3)$$

Then we have the general solutions of the Eq.(A1)

$$R_{11}(t) = C_1\exp(r_1 t) + C_2\exp(r_2 t) + C_3\exp(r_3 t) + C_4\exp(r_4 t) + C_0. \quad \text{(A4)}$$

From Eqs.(A4) and (29), the expression of $R_{22}(t)$ is easily written as follows

$$R_{22}(t) = \frac{1}{3g_1^2(1+M_1)}\{ -[C_1(r_1^2+q^2)\mathrm{e}^{r_1 t} + C_2(r_2^2+q^2)\mathrm{e}^{r_2 t} + C_3(r_3^2+q^2)\mathrm{e}^{r_3 t} + C_4(r_4^2+q^2)\mathrm{e}^{r_4 t}] + C_0 q^2 + [2g_1^2(1+M_1) - g_1^2 K]\}, \quad \text{(A5)}$$

where

$$q^2 = \alpha M_3 + 4g_1^2(1+M_1). \quad \text{(A6)}$$

The coefficients C_1, C_2, C_3 and C_4 can be derived according to the initial conditions $R_{11}(0)$, $R_{11}(0)$, $R_{22}(0)$ and $R_{22}(0)$,

$$\begin{aligned} C_1 &= (D_1 + D_2)/2r_1(r_3^2 - r_1^2), \\ C_2 &= (D_1 - D_2)/2r_1(r_3^2 - r_1^2), \\ C_3 &= (D_3 + D_4)/2r_3(r_1^2 - r_3^2), \\ C_4 &= (D_3 + D_4)/2r_3(r_1^2 - r_3^2), \end{aligned} \quad \text{(A7)}$$

where

$$\begin{aligned} \frac{D_1}{r_1} &= 3g_1^2(1+M_1)R_{22}(0) - C_0 q^2 - [2g_1^2(1+M_1) - g_1^2 K] + R_{11}(0)(r_3^2+q^2), \\ D_2 &= \dot{R}_{11}(0)(r_3^2+q^2) + 3g_1^2(1+M_1)\dot{R}_{22}(0), \\ \frac{D_3}{r_3} &= 3g_1^2(1+M_1)R_{22}(0) - C_0 q^2 - [2g_1^2(1+M_1) - g_1^2 K] + R_{11}(0)(r_1^2+q^2), \\ D_4 &= \dot{R}_{11}(0)(r_1^2+q^2) + 3g_1^2(1+M_1)\dot{R}_{22}(0). \end{aligned} \quad \text{(A8)}$$

From Eqs.(9) and (10), we can also obtain the expression of the occupation number operators $N_1(t)$ for the ideal cavity mode and $\sum_s N_s(t)$ for the universal modes.

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