Vol 18 No 5, May 2009
1674-1056/2009/18(05)/1867-05

Chinese Physics B

© 2009 Chin. Phys. Soc.
and IOP Publishing Ltd

Scheme for implementing quantum logic gates for two atoms trapped in different cavities*

Lin Li-Hua(林丽华)†

Department of Electronic Science and Applied Physics, Fuzhou University, Fuzhou 350002, China

(Received 13 August 2008; revised manuscript received 3 September 2008)

A scheme is presented for realizing quantum logic gates for two atoms localized in two distant optical cavities. Our scheme works in a regime in which the atom–cavity coupling strength is smaller than the cavity decay rate. Thus the requirement on the quality factor of the cavities is greatly relaxed. Furthermore, the fidelity of our scheme is not affected by detection inefficiency and atomic decay. These advantages are important in view of experiment.

Keywords: quantum logic gate, cavity QED, atomic decay
PACC: 4250

1. Introduction

Recently, quantum computation has become a topical subject across many disciplines, such as physics, information, and mathematics. Quantum computers can solve some problems that are intractable with classical computers.[1] Furthermore, it has been shown that a search with a quantum computer for an item from a disordered system needs much less time than that with a classical computer, and the advantage becomes more and more obvious with the increase of the number of the items in the system.[2] The basic elements for construction of a quantum computer are the two-qubit logic gates.[3] The elementary gates have been experimentally demonstrated in the ion trap,[4] NMR,[5] and cavity QED systems.[6]

The cavity QED system is especially promising for quantum information processing. In such a system the atoms or ions are suitable for storing quantum information, while the photons are ideal for transferring quantum information. Schemes have been proposed for entanglement engineering and quantum state teleportation between two distant atoms by single-photon interference at photodetectors.[7−13] A scheme was proposed for the conditional production of quantum logic gates between two distant atoms by photon detection.[14] In the scheme the laser pulse is very weak so that the probability that both atoms are excited is negligible. Therefore, the success probability is very small. On the other hand, Zou and Mathis proposed a scheme for implementing a conditional quantum phase gate between two atoms trapped in different cavities via photon interference.[15] The scheme is insensitive to the imperfection of photodetectors. However, in the scheme each atomic state is first mapped to the respective cavity mode via Raman coupling induced by the cavity mode and a classical field. Thus, the scheme requires that the Raman coupling strength be much larger than the cavity decay rate. This puts a very stringent requirement on the cavity quality and thus is experimentally challenging.

In this paper, we propose an alternative scheme for conditional quantum phase gates between two atoms trapped in two separate cavities. The distinct advantage of our scheme is that it does not require complete mapping between the atomic state and the photonic state. Thus our scheme works in a regime where the atom–cavity coupling strength is smaller than the cavity decay rate, greatly relaxing the requirement on the cavity quality. The fidelity of the gate operation is insensitive to the atomic spontaneous emission and the detection imperfection.

The paper is organized as follows. In Section 2, we study the dynamics for a four-level atom resonantly interacting with two cavity modes. In Section 3, we propose a scheme for the implementation of quantum phase gates based on the combination of cavity QED and linear optics elements. Discussions on experimental issues appear in Section 4.

*Project supported by the Doctoral Foundation of the Ministry of Education of China (Grant No 20070386002).
†E-mail: lhlin11@yahoo.com.cn

2. The dynamics of the atom–cavity system

The atoms have two degenerate excited states $|e_\mathrm{l}\rangle$ and $|e_\mathrm{r}\rangle$ and two degenerate ground states $|g_\mathrm{l}\rangle$ and $|g_\mathrm{r}\rangle$, as shown in Fig.1. The transition $|g_\mathrm{l}\rangle \rightarrow |e_\mathrm{l}\rangle$ is coupled to the left-circularly polarized cavity mode, and $|g_\mathrm{r}\rangle \rightarrow |e_\mathrm{r}\rangle$ is coupled to the right-circularly polarized cavity mode. The setup is shown in Fig.2. Two distant atoms are trapped in two two-mode optical cavities, respectively. Photons leaking from the first cavity transmit through a quarter-wave plate and a half-wave plate. On the other hand, photons leaking from the second cavity transmit through a quarter-wave plate. Then the photons transmit through a polarization beam splitter, which destroys the information on which cavity the photons comes from. Next, each photon transmits through a half-wave plate and a polarization beam splitter. Finally, the photons are detected by the photodetectors. We here require that the cavities are one-sided so that photon leakage only occurs through the sides of the cavities facing the polarization beam splitter.

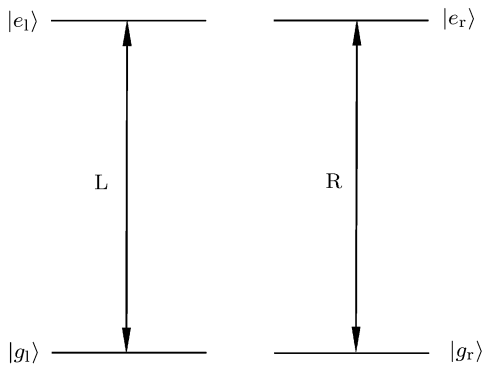


Fig.1. The level configuration of the atoms. The transitions $|g_\mathrm{l}\rangle \rightarrow |e_\mathrm{l}\rangle$ and $|g_\mathrm{r}\rangle \rightarrow |e_\mathrm{r}\rangle$ are coupled to the left-circularly and right-circularly polarized cavity modes, respectively.

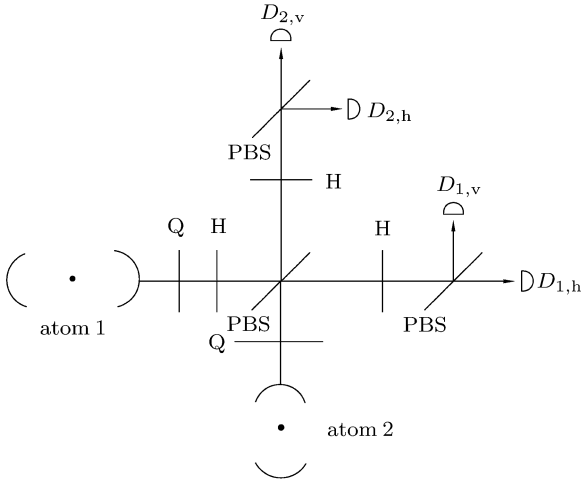


Fig.2. The experimental setup to implement the quantum phase gate for two atoms trapped in two separate cavities. Q, H, and PBS represent the quarter-wave plate, half-wave plate, and polarization beam splitter, respectively. $D_{h,1}$, $D_{v,1}$, $D_{h,2}$, and $D_{v,2}$ are photodetectors.

The quantum information is encoded on the two ground states $|g_\mathrm{l}\rangle$ and $|g_\mathrm{r}\rangle$. We first perform the transformation $|g_\mathrm{l}\rangle \rightarrow |e_\mathrm{l}\rangle$ and $|g_\mathrm{r}\rangle \rightarrow |e_\mathrm{r}\rangle$. Then the atoms resonantly couple with the respective cavity modes. In the frame rotating at the frequency of the cavity modes, the Hamiltonian in the kth (k=1, 2) cavity is

$$H_k = g(a_{\mathrm{l,k}}S^+_{\mathrm{l,k}} + a_{\mathrm{r,k}}S^+_{\mathrm{r,k}} + a^+_{\mathrm{l,k}}S^-_{\mathrm{l,k}} + a^+_{\mathrm{r,k}}S^-_{\mathrm{r,k}}), \quad (1)$$

where $S^+_{\mathrm{l,k}} = |e_{\mathrm{l,k}}\rangle\langle g_{\mathrm{l,k}}|$, $S^+_{\mathrm{r,k}} = |e_{\mathrm{r,k}}\rangle\langle g_{\mathrm{r,k}}|$, $S^-_{\mathrm{l,k}} = |g_{\mathrm{l,k}}\rangle\langle e_{\mathrm{l,k}}|$, $S^-_{\mathrm{r,k}} = |g_{\mathrm{r,k}}\rangle\langle e_{\mathrm{r,k}}|$ are the atomic flipping operators, $a_{\mathrm{l,k}}$ ($a_{\mathrm{r,k}}$) and $a^+_{\mathrm{l,k}}$ ($a^+_{\mathrm{r,k}}$) are the annihilation and creation operators for the left-circularly (right-circularly) polarized cavity mode in the kth cavity, and g is the atom–cavity coupling strength.

We now consider cavity decay. Under the condition that no detection of photon has been registered the kth atom–cavity system is governed by the non-Hermitian Hamiltonian

$$H_{\mathrm{c,k}} = H_\mathrm{k} - \mathrm{i}\Gamma(a^+_{\mathrm{l,k}}a_{\mathrm{l,k}} + a^+_{\mathrm{r,k}}a_{\mathrm{r,k}}), \quad (2)$$

where Γ is the cavity decay rate. We first assume that the kth atom is initially in the state $|g_{\mathrm{l,k}}\rangle$ and the cavity modes are initially in the vacuum state $|0_{\mathrm{l,k}}\rangle|0_{\mathrm{r,k}}\rangle$. After the transformation $|g_{\mathrm{l,k}}\rangle \rightarrow |e_{\mathrm{l,k}}\rangle$, the kth atom undergoes a transition in the kth cavity. Under the condition that $g < \Gamma/2$ the time evolution of the kth atom–cavity system is

$$|\phi_{\mathrm{l,k}}(t)\rangle = \mathrm{e}^{-\Gamma t/2}\Bigg[\left(\frac{\mathrm{e}^{\alpha t/2}+\mathrm{e}^{-\alpha t/2}}{2} - \frac{\Gamma}{\alpha}\frac{\mathrm{e}^{-\alpha t/2}-\mathrm{e}^{\alpha t/2}}{2}\right)|e_{\mathrm{l,k}}\rangle|0_{\mathrm{l,k}}\rangle|0_{\mathrm{r,k}}\rangle$$
$$- \frac{2g}{\alpha}\frac{\mathrm{e}^{-\alpha t/2}-\mathrm{e}^{\alpha t/2}}{2\mathrm{i}}|g_{\mathrm{l,k}}\rangle|1_{\mathrm{l,k}}\rangle|0_{\mathrm{r,k}}\rangle\Bigg], \quad (3)$$

where

$$\alpha = \sqrt{-4g^2 + \Gamma^2}. \tag{4}$$

On the other hand, if the kth atom is initially in the state $|g_{\mathrm{r,k}}\rangle$, after the transformation $|g_{\mathrm{r,k}}\rangle \to |e_{\mathrm{r,k}}\rangle$ the time evolution of the kth atom–cavity system is

$$\begin{aligned}|\phi_{\mathrm{r,k}}(t)\rangle = \mathrm{e}^{-\Gamma t/2}\Bigg[&\left(\frac{\mathrm{e}^{\alpha t/2}+\mathrm{e}^{-\alpha t/2}}{2} - \frac{\Gamma}{\alpha}\frac{\mathrm{e}^{-\alpha t/2}-\mathrm{e}^{\alpha t/2}}{2}\right)|e_{\mathrm{r,k}}\rangle\,|0_{\mathrm{l,k}}\rangle\,|0_{\mathrm{r,k}}\rangle \\ &- \frac{2g}{\alpha}\frac{\mathrm{e}^{-\alpha t/2}-\mathrm{e}^{\alpha t/2}}{2\mathrm{i}}\,|g_{\mathrm{r,k}}\rangle\,|0_{\mathrm{l,k}}\rangle\,|1_{\mathrm{r,k}}\rangle\Bigg].\end{aligned} \tag{5}$$

3. Quantum phase gate

Assume that the two atoms are initially in the state

$$\begin{aligned}|\psi(0)\rangle = (&\lambda\,|g_{\mathrm{l},1}\rangle\,|g_{\mathrm{l},2}\rangle + \mu\,|g_{\mathrm{l},1}\rangle\,|g_{\mathrm{r},2}\rangle \\ &+ \nu\,|g_{\mathrm{r},1}\rangle\,|g_{\mathrm{l},2}\rangle + \xi\,|g_{\mathrm{r},1}\rangle\,|g_{\mathrm{r},2}\rangle),\end{aligned} \tag{6}$$

where λ, μ, ν, and ξ are the coefficients for the initial state of the two atoms. After the transformation $|g_{j,k}\rangle \to |e_{j,k}\rangle$ ($j = \mathrm{l}, \mathrm{r}$) and atom–cavity coupling the whole system evolves to

$$\begin{aligned}|\psi(t)\rangle = &\lambda\,|\phi_{\mathrm{l},1}(t)\rangle\,|\phi_{\mathrm{l},2}(t)\rangle + \mu\,|\phi_{\mathrm{l},1}(t)\rangle\,|\phi_{\mathrm{r},2}(t)\rangle \\ &+ \nu\,|\phi_{\mathrm{r},1}(t)\rangle\,|\phi_{\mathrm{l},2}(t)\rangle + \xi\,|\phi_{\mathrm{r},1}(t)\rangle\,|\phi_{\mathrm{r},2}(t)\rangle\,.\end{aligned} \tag{7}$$

The quarter-wave plates transform left-circularly polarized and right-circularly polarized photons into horizontally and vertically polarized photons, respectively. The half-wave plates perform the transformation

$$\begin{aligned}|1_{\mathrm{h}}\rangle &\longrightarrow \frac{1}{\sqrt{2}}(|1_{\mathrm{h}}\rangle + |1_{\mathrm{v}}\rangle), \\ |1_{\mathrm{v}}\rangle &\longrightarrow \frac{1}{\sqrt{2}}(|1_{\mathrm{h}}\rangle - |1_{\mathrm{v}}\rangle),\end{aligned} \tag{8}$$

where $|1_{\mathrm{h}}\rangle$ and $|1_{\mathrm{v}}\rangle$ represent horizontally and vertically polarized photons, respectively. The quarter-wave plates, half-wave plates, and polarization beam splitters perform the transformations

$$\begin{aligned}|1_{\mathrm{l},1}\rangle &\longrightarrow \frac{1}{2}(|1_{h,1}\rangle + |1_{v,1}\rangle + |1_{h,2}\rangle - |1_{v,2}\rangle), \\ |1_{\mathrm{r},1}\rangle &\longrightarrow \frac{1}{2}(|1_{h,1}\rangle + |1_{v,1}\rangle - |1_{h,2}\rangle + |1_{v,2}\rangle), \\ |1_{\mathrm{l},2}\rangle &\longrightarrow \frac{1}{\sqrt{2}}(|1_{h,2}\rangle + |1_{v,2}\rangle), \\ |1_{\mathrm{r},2}\rangle &\longrightarrow \frac{1}{\sqrt{2}}(|1_{h,1}\rangle - |1_{v,1}\rangle),\end{aligned} \tag{9}$$

where $|1_{h,k}\rangle$ and $|1_{v,k}\rangle$ represent horizontally and vertically polarized photons transmitting through the kth path, respectively. If one photon is detected, one can not distinguish from which cavity it leaks due to the photon interference. Suppose that $D_{1,\mathrm{h}}$ clicks at time t_1. Then the system collapses to

$$|\psi_{\mathrm{c}}(t_1)\rangle = \left[\frac{1}{2}(a_{\mathrm{l},1} + a_{\mathrm{r},1}) + \frac{1}{\sqrt{2}}a_{\mathrm{r},2}\right]|\psi(t_1)\rangle\,. \tag{10}$$

Using Eqs.(7) and (10), we obtain the state of the system

$$\begin{aligned}|\psi_{\mathrm{c}}(t_1)\rangle = &\lambda\,|g_{\mathrm{l},1}\rangle\,|0_{\mathrm{l},1}\rangle\,|0_{\mathrm{r},1}\rangle\,|\phi_{\mathrm{l},2}(t_1)\rangle \\ &+ \mu\,|g_{\mathrm{l},1}\rangle\,|0_{\mathrm{l},1}\rangle\,|0_{\mathrm{r},1}\rangle\,|\phi_{\mathrm{r},2}(t_1)\rangle \\ &+ \mu\sqrt{2}\,|\phi_{\mathrm{l},1}(t_1)\rangle\,|g_{\mathrm{r},2}\rangle\,|0_{\mathrm{l},2}\rangle\,|0_{\mathrm{r},2}\rangle \\ &+ \nu\,|g_{\mathrm{r},1}\rangle\,|0_{\mathrm{l},1}\rangle\,|0_{\mathrm{r},1}\rangle\,|\phi_{\mathrm{l},2}(t_1)\rangle \\ &+ \xi\,|g_{\mathrm{r},1}\rangle\,|0_{\mathrm{l},1}\rangle\,|0_{\mathrm{r},1}\rangle\,|\phi_{\mathrm{r},2}(t_1)\rangle \\ &+ \xi\sqrt{2}\,|\phi_{\mathrm{r},1}(t_1)\rangle\,|g_{\mathrm{r},2}\rangle\,|0_{\mathrm{l},2}\rangle\,|0_{\mathrm{r},2}\rangle\,.\end{aligned} \tag{11}$$

Suppose that no photon is detected during the time interval $[t_1, t_2]$. Then the time evolution of the system during this time interval is

$$\begin{aligned}\left|\psi'(t)\right\rangle = &\lambda\,|g_{\mathrm{l},1}\rangle\,|0_{\mathrm{l},1}\rangle\,|0_{\mathrm{r},1}\rangle\,|\phi_{\mathrm{l},2}(t)\rangle \\ &+ \mu\,|g_{\mathrm{l},1}\rangle\,|0_{\mathrm{l},1}\rangle\,|0_{\mathrm{r},1}\rangle\,|\phi_{\mathrm{r},2}(t)\rangle \\ &+ \mu\sqrt{2}\,|\phi_{\mathrm{l},1}(t)\rangle\,|g_{\mathrm{r},2}\rangle\,|0_{\mathrm{l},2}\rangle\,|0_{\mathrm{r},2}\rangle \\ &+ \nu\,|g_{\mathrm{r},1}\rangle\,|0_{\mathrm{l},1}\rangle\,|0_{\mathrm{r},1}\rangle\,|\phi_{\mathrm{l},2}(t)\rangle \\ &+ \xi\,|g_{\mathrm{r},1}\rangle\,|0_{\mathrm{l},1}\rangle\,|0_{\mathrm{r},1}\rangle\,|\phi_{\mathrm{r},2}(t)\rangle \\ &+ \xi\sqrt{2}\,|\phi_{\mathrm{r},1}(t)\rangle\,|g_{\mathrm{r},2}\rangle\,|0_{\mathrm{l},2}\rangle\,|0_{\mathrm{r},2}\rangle\,.\end{aligned} \tag{12}$$

If $D_{2,\mathrm{v}}$ clicks at time t_2 the system collapses to

$$\left|\psi'_{\mathrm{c}}(t_2)\right\rangle = \left[\frac{1}{2}(-a_{\mathrm{l},1} + a_{\mathrm{r},1}) + \frac{1}{\sqrt{2}}a_{\mathrm{l},2}\right]\left|\psi'(t_2)\right\rangle. \tag{13}$$

Using Eqs.(12) and (13), we obtain the state of the system

$$|\psi_{\rm c}'(t_2)\rangle = \lambda\,|g_{\rm l,1}\rangle\,|g_{\rm l,2}\rangle - \mu\,|g_{\rm l,1}\rangle\,|g_{\rm r,2}\rangle + \nu\,|g_{\rm r,1}\rangle\,|g_{\rm l,2}\rangle + \xi\,|g_{\rm r,1}\rangle\,|g_{\rm r,2}\rangle]\,|0_{\rm l,1}\rangle\,|0_{\rm r,1}\rangle\,|0_{\rm l,2}\rangle\,|0_{\rm r,2}\rangle\,. \tag{14}$$

Therefore, the phase of the qubit system is shifted by π if and only if the system is initially in the state $|g_{\rm l,1}\rangle\,|g_{\rm r,2}\rangle$, which corresponds to a conditional phase gate. If $D_{2,\rm v}$ detects a vertically polarized photon at time $\rm t_1$ and $D_{1,\rm h}$ detects a horizontally polarized photon at time $\rm t_2$ the system also collapses to the state of Eq.(14). If $D_{1,\rm v}$ and $D_{2,\rm h}$ click the system again collapses to the state of Eq.(14). If $D_{1,\rm h}$ and $D_{2,\rm h}$ or $D_{1,\rm v}$ and $D_{2,\rm v}$ click the system collapses to

$$|\psi_{\rm c}'(t_2)\rangle = \lambda\,|g_{\rm l,1}\rangle\,|g_{\rm l,2}\rangle + \mu\,|g_{\rm l,1}\rangle\,|g_{\rm r,2}\rangle + \nu\,|g_{\rm r,1}\rangle\,|g_{\rm l,2}\rangle - \xi\,|g_{\rm r,1}\rangle\,|g_{\rm r,2}\rangle]\,|0_{\rm l,1}\rangle\,|0_{\rm r,1}\rangle\,|0_{\rm l,2}\rangle\,|0_{\rm r,2}\rangle\,. \tag{15}$$

In this case the phase of the qubit system is shifted by π if and only if the system is initially in the state $|g_{\rm r,1}\rangle\,|g_{\rm r,2}\rangle$, which also corresponds to a conditional phase gate. Suppose that we wish to wait for a time $t_{\rm d}$. The probability of success is

$$P = \frac{1}{2}\left\{1 - {\rm e}^{-\Gamma t_{\rm d}}\left[\frac{1}{4}{\rm e}^{\alpha t_{\rm d}}\left(1+\frac{\Gamma}{\alpha}\right)^2 + \frac{1}{4}{\rm e}^{-\alpha t_{\rm d}}\left(1-\frac{\Gamma}{\alpha}\right)^2 + \frac{1}{2}\left(1-\frac{\Gamma^2}{\alpha^2}\right) + \frac{g^2}{\alpha^2}({\rm e}^{\alpha t_{\rm d}} + {\rm e}^{-\alpha t_{\rm d}} - 2)\right]\right\}^2. \tag{16}$$

If $t_{\rm d}$ is long enough so that ${\rm e}^{-(\Gamma-\alpha)t_{\rm d}} \ll 1$ the probability is about 1/2.

4. Experimental issues

We now briefly address the experimental feasibility of the proposed scheme. The required atomic level configuration can be achieved in $^{40}{\rm Ca}^{+}$. The hyperfine levels $|F=3/2, m=-1/2\rangle$ and $|F=3/2, m=1/2\rangle$ of $3^2D_{1/2}$ can act as the states $|e_{\rm l}\rangle$ and $|e_{\rm r}\rangle$, respectively, while the hyperfine levels $|F=1/2,\ m=1/2\rangle$ and $|F=1/2,\ m=-1/2\rangle$ of $4^2S_{1/2}$ can act as $|g_{\rm l}\rangle$ and $|g_{\rm r}\rangle$, respectively. The transitions $|g_{\rm l}\rangle \rightarrow |e_{\rm l}\rangle$ ($|e_{\rm r}\rangle \rightarrow |e_{\rm r}\rangle$) can be achieved by applying a resonant left-circularly (right-circularly) polarized classical field. As long as the classical pulses are strong enough the atom–cavity coupling is negligible during the application of the classical fields. The proposed gate works in the Lamb–Dicke regime, i.e., the spatial extension of the atomic wavefunction should be much smaller than the wave length of the cavity mode. Recent advances in the combination of ion trapping and cavity QED techniques allow achievement of the Lamb–Dicke limit.[16] In the experiment of Ref.[16], single $^{40}{\rm Ca}^{+}$ ions are localized at a fixed position in the cavity with a high precision for indefinite times, so the spatial variation of the coupling strength can be neglected.

Suppose that the photodetectors are imperfect, then there is a probability that two photons may have arrived at two different photodetectors but only one click or no click is registered. These events are discarded. Thus, the probability of success is given by $P^{'} = \eta^2 P$. However, the fidelity of the gate operation is not affected. We now consider the effect of the atomic spontaneous emission. The photons from the spontaneous emissions run in random directions and thus cannot be detected by the photodetectors $D_{1,\rm h}$, $D_{1,\rm v}$, $D_{2,\rm h}$, and $D_{2,\rm v}$. Therefore, the atomic decay also reduces the success probability, but does not affect the fidelity.

In summary, we have proposed a scheme for implementing quantum phase gates between two atoms trapped in two distant cavities. Our scheme is based on the combination of cavity QED and linear optics elements. Our scheme does not require each atomic state to be mapped to the corresponding cavity mode. The conditional phase gate can be realized when the atom–cavity coupling strength is smaller than the cavity decay rate. The fidelity of the gate operation is not affected by atomic spontaneous emission and detection imperfection.

References

[1] Shor P W 1994 *Proceedings of the 35th Annual Symposium on Foundations of Computer Science* (Los Alamitos CA: IEEE Computer Society Press) p116

[2] Grover L K 1997 *Phys. Rev. Lett.* **79** 325

[3] Sleator T and Weinfurter H 1995 *Phys. Rev. Lett.* **74** 4087

[4] Schmidt-Kaler F, Haffner H, Riebe M, Gulde S, Lancaster G P T, Deuschle T, Becher C, Roos C F, Eschner J and Blatt R 2003 *Nature* **422** 408
[5] Gershenfeld N A and Chuang I L 1997 *Science* **275** 350
[6] Rauschenbeutel A, Nogues G, Osnaghi S, Bertet P, Brune M, Raimond J M and Haroche S 1999 *Phys. Rev. Lett.* **83** 5166
[7] Cabrillo C, Cirac J, Garcia-Fernandez P and Zoller P 1999 *Phys. Rev.* A **59** 1025
[8] Bose S, Knight P L, Plenio M B and Vedral V 1999 *Phys. Rev. Lett.* **83** 5158
[9] Feng X L, Zhang Z M, Li X D, Gong S Q and Xu Z Z 2003 *Phys. Rev. Lett.* **90** 217902
[10] Duan L M and Kimble H J 2003 *Phys. Rev. Lett.* **90** 253601
[11] Browne D E, Plenio M B and Huelga S F 2003 *Phys. Rev. Lett.* **91** 067901
[12] Yu B, Zhou Z W, Zhang Y, Xiang G Y and Guo G C 2004 *Phys. Rev.* A **70** 014302
[13] Ye S Y 2007 *Chin. Phys.* **16** 2968
[14] Protsenko I E, Reymond G, Schlosser N and Grangier P 2002 *Phys. Rev.* A **66** 062306
[15] Zou X B and Mathis W 2005 *Phys. Rev.* A **71** 042334
[16] Keller M, Lange B, Hayasaka K, Lange W and Walther H 2004 *New J. Phys.* **6** 95