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Nonparaxial propagation of a vectorial apertured off-axis Lorentz beam*

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Based on the vectorial Rayleigh–Sommerfeld integral formula and the complex Gaussian expansion of the hard-edge aperture function, an analytical propagation expression for a nonparaxial vectorial off-axis Lorentz beam passing through a rectangular aperture is derived. The unapertured case, the far field expression and the scalar paraxial result are also presented as special cases of the general formulae, respectively. Some numerical examples are also given to show the propagation characteristics of a nonparaxial vectorial off-axis Lorentz beam through a rectangular aperture. It is indicated that the f parameter, the off-axis displacement and the truncation parameter all play an important role in determining nonparaxial propagation behaviour.

Keywords: vectorial Rayleigh–Sommerfeld integral, nonparaxial vectorial off-axis Lorentz beam, rectangular aperture

PACC: 4225B, 4260H

1. Introduction

As a new kind of realizable beam, the Lorentz beam was introduced by Gawhary and Severini.[1] With identical spatial extensions, the angular spreading of Lorentzian distribution is higher than that of Gaussian description.[2] Therefore, the Lorentz beam provides an appropriate model to describe some laser sources, e.g. double heterojunction $Ga_{1-x}Al_xAs$ lasers, which produce highly divergent fields.[2,3] Within the paraxial framework, the propagation of a Lorentz beam in the Fresnel approximation has been investigated in free space, and the corresponding closed-form scalar representation in a general transverse plane has been also presented.[1] However, the paraxial approximation does not hold for beams with a large divergent angle, which results in extending a Lorentz beam to the vectorial nonparaxial regime. By using the vector angular spectrum representation, the characteristic of a nonparaxial Lorentz beam in the far field has been examined from the vectorial structure, and the vectorial structure of a nonparaxial Lorentz beam has been compared with that of a nonparaxial Gaussian beam.[4] Based on the non-paraxial vectorial moment theory of beam propagation, the propagation properties of a nonparaxial Lorentz beam have been revealed.[5] In practice, the rectangular hard apertures always exist. Moreover, the general case of off-axis propagation is considered. Therefore, the propagation properties of a nonparaxial vectorial off-axis Lorentz beam passing through a rectangular aperture are investigated in the present paper.

The remainder of the present paper is organized as follows. Based on vectorial Rayleigh–Sommerfeld integral formula and the complex Gaussian expansion of the hard-edge aperture function, an analytical propagation equation of a nonparaxial vectorial apertured off-axis Lorentz beam is derived in Section 2. Numerical calculation results and analyses are presented in Section 3. Finally, the main conclusions drawn from the present study are summarized in Section 4.

2. Nonparaxial propagation of a vectorial apertured off-axis Lorentz beam

The vectorial off-axis Lorentz beam is considered to be linearly polarized in the x-direction, which is a familiar case. The z-axis is taken to be the propagation axis. The vectorial off-axis Lorentz beam in the source plane $z = 0$ is described by

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$$\begin{pmatrix} E_{0x}(x_0, y_0, 0) \\ E_{0y}(x_0, y_0, 0) \end{pmatrix} = \begin{pmatrix} \dfrac{E_0}{w_{0x} w_{0y}[1 + (x_0 - x_\mathrm{d})^2/w_{0x}^2][1 + (y_0 - y_\mathrm{d})^2/w_{0y}^2]} \\ 0 \end{pmatrix}, \tag{1}$$

where E_0 is a normalized constant, w_{0x} and w_{0y} are the parameters related to the beam widths in the x- and the y-directions, and x_d and y_d are the off-axis displacements in the x- and the y-directions, respectively. The time dependent factor $\exp(-\mathrm{i}\omega t)$ is omitted in Eq.(1); here ω is the circular frequency. A rectangular aperture is positioned in the source plane. The centre of the rectangular aperture coincides with the origin of the Cartesian coordinate system. The two transverse side-widths of the rectangular aperture are $2a$ and $2b$, which are respectively parallel to the x- and the y-axes. The optical field just behind the rectangular aperture is expressed as

$$\begin{pmatrix} E_x(x_0, y_0, 0) \\ E_y(x_0, y_0, 0) \end{pmatrix} = \begin{pmatrix} \dfrac{E_0}{w_{0x} w_{0y}[1 + (x_0 - x_\mathrm{d})^2/w_{0x}^2][1 + (y_0 - y_\mathrm{d})^2/w_{0y}^2]} \\ 0 \end{pmatrix} \mathrm{rect}(x_0, y_0), \tag{2}$$

where $\mathrm{rect}(x_0,\ y_0)$ is the hard-edge rectangular aperture function given by

$$\mathrm{rect}(x_0, y_0) = \begin{cases} 1,\ |x_0| < a, |y_0| < b, \\ 0,\ |x_0| > a, |y_0| > b. \end{cases} \tag{3}$$

The hard-edge rectangular aperture function can be expanded into a finite sum of complex Gaussian functions as[6−8]

$$\mathrm{rect}(x_0, y_0) = \sum_{h=1}^{N} A_h \exp\left(-\frac{B_h}{a^2} x_0^2\right) \sum_{g=1}^{N} A_g \exp\left(-\frac{B_g}{b^2} y_0^2\right), \tag{4}$$

where the complex-expanded Gaussian coefficients A_h, A_g, B_h and B_g can be obtained by optimization computation.[6] By using the vectorial Rayleigh–Sommerfeld integral formula, the nonparaxial vectorial apertured off-axis Lorentz beam propagating toward half free space $z > 0$ is found to be[9−11]

$$E_x(\boldsymbol{r}) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_x(x_0, y_0, 0) \frac{\partial \boldsymbol{G}(\boldsymbol{r}, \boldsymbol{\rho}_0)}{\partial z} \mathrm{d}x_0 \mathrm{d}y_0, \tag{5}$$

$$E_y(\boldsymbol{r}) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_y(x_0, y_0, 0) \frac{\partial \boldsymbol{G}(\boldsymbol{r}, \boldsymbol{\rho}_0)}{\partial z} \mathrm{d}x_0 \mathrm{d}y_0, \tag{6}$$

$$E_y(\boldsymbol{r}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(E_x(x_0, y_0, 0) \frac{\partial \boldsymbol{G}(\boldsymbol{r}, \boldsymbol{\rho}_0)}{\partial x} + E_y(x_0, y_0, 0) \frac{\partial \boldsymbol{G}(\boldsymbol{r}, \boldsymbol{\rho}_0)}{\partial y} \right) \mathrm{d}x_0 \mathrm{d}y_0, \tag{7}$$

where

$$\boldsymbol{G}(\boldsymbol{r}, \boldsymbol{\rho}_0) = \frac{\exp(\mathrm{i}k\,|\boldsymbol{r} - \boldsymbol{\rho}_0|)}{|\boldsymbol{r} - \boldsymbol{\rho}_0|}, \tag{8}$$

$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$, and $\boldsymbol{\rho}_0 = x_0\boldsymbol{i} + y_0\boldsymbol{j}$ with $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ being unit vectors of the x-, y- and z-axes in the Cartesian coordinate system, respectively, and $k = 2\pi/\lambda$ is the wave number with λ being the incident wavelength. The following approximation is valid beyond the paraxial approximation:[12−15]

$$\boldsymbol{G}(\boldsymbol{r}, \boldsymbol{\rho}_0) = \frac{1}{r} \exp\left[\mathrm{i}k\left(r + \frac{\boldsymbol{\rho}_0^2 - 2xx_0 - 2yy_0}{2r}\right)\right], \tag{9}$$

where $r = (x^2 + y^2 + z^2)^{1/2} = (\rho^2 + z^2)^{1/2}$, and $\rho_0^2 = (x_0^2 + y_0^2)^{1/2}$. Substituting expressions (2), (4) and (3) into expressions (5)–(7), one can obtain the propagation equation of a nonparaxial vectorial apertured off-axis Lorentz beam as

$$E_x(\boldsymbol{r}) = -\frac{\mathrm{i}E_0 z\exp(\mathrm{i}kr)}{\lambda r^2 w_{0x}w_{0y}}\sum_{h=1}^{N}A_h\int_{-\infty}^{\infty}\frac{1}{1+(x_0-x_\mathrm{d})^2/w_{0x}^2}\exp\left(\frac{\mathrm{i}k}{2r}(C_h x_0^2-2xx_0)\right)\mathrm{d}x_0$$
$$\times\sum_{g=1}^{N}A_g\int_{-\infty}^{\infty}\frac{1}{1+(y_0-y_\mathrm{d})^2/w_{0y}^2}\exp\left(\frac{\mathrm{i}k}{2r}(C_g y_0^2-2yy_0)\right)\mathrm{d}y_0, \tag{10}$$

$$E_y(\boldsymbol{r}) = 0, \tag{11}$$

$$E_z(\boldsymbol{r}) = \frac{\mathrm{i}E_0\exp(\mathrm{i}kr)}{\lambda r^2 w_{0x}w_{0y}}\sum_{h=1}^{N}A_h\int_{-\infty}^{\infty}\frac{x-x_0}{1+(x_0-x_\mathrm{d})^2/w_{0x}^2}\exp\left(\frac{\mathrm{i}k}{2r}(C_h x_0^2-2xx_0)\right)\mathrm{d}x_0,$$
$$\times\sum_{g=1}^{N}A_g\int_{-\infty}^{\infty}\frac{1}{1+(y_0-y_\mathrm{d})^2/w_{0y}^2}\exp\left(\frac{\mathrm{i}k}{2r}(C_g y_0^2-2yy_0)\right)\mathrm{d}y_0, \tag{12}$$

where

$$C_h = 1-\frac{2rB_h}{\mathrm{i}ka^2}\ \text{ and }\ C_g = 1-\frac{2rB_g}{\mathrm{i}kb^2}. \tag{13}$$

To obtain the analytical form of the x component for a vectorial apertured off-axis Lorentz beam, one should first perform the following integral:

$$T_1 = \int_{-\infty}^{\infty}\frac{1}{1+(x_0-x_\mathrm{d})^2/w_{0x}^2}\exp\left(\frac{\mathrm{i}k}{2r}(C_h x_0^2-2xx_0)\right)\mathrm{d}x_0$$
$$= \exp\left(\frac{\mathrm{i}k}{2r}(C_h x_\mathrm{d}^2-2xx_\mathrm{d})\right)\int_{-\infty}^{\infty}\frac{1}{1+x_1^2/w_{0x}^2}\exp\left(\frac{\mathrm{i}k}{2r}[C_h x_1^2-2(x-C_h x_\mathrm{d})x_1]\right)\mathrm{d}x_1. \tag{14}$$

Expression (14) can also be rewritten as

$$T_1 = w_{0x}^2\exp\left(-\frac{\mathrm{i}kx^2}{2C_h r}\right)\left[f_1\left(\frac{x-C_h x_\mathrm{d}}{C_h}\right)*f_2\left(\frac{x-C_h x_\mathrm{d}}{C_h}\right)\right], \tag{15}$$

with $f_1(\tau)$ and $f_2(\tau)$ defined respectively as

$$f_1(\tau) = \frac{1}{w_{0x}^2+\tau^2}, \tag{16}$$

and

$$f_2(\tau) = \exp\left(\frac{\mathrm{i}kC_h\tau^2}{2r}\right). \tag{17}$$

By using the convolution theorem of the Fourier transforms[16] expression (15) is found to be

$$T_1 = \frac{\pi w_{0x}}{2}[V_x^+(x-C_h x_\mathrm{d},z)+V_x^-(x-C_h x_\mathrm{d},z)]$$
$$\times\exp\left(-\frac{\mathrm{i}kx^2}{2C_h r}\right) \tag{18}$$

with $V_x^{\pm}(x,z)$ given by

$$V_x^{\pm}(x,z) = \exp\left[\frac{kC_h}{2\mathrm{i}r}\left(w_{0x}\pm\mathrm{i}\frac{x}{C_h}\right)^2\right]$$
$$\times\left\{1-\mathrm{erf}\left[\sqrt{\frac{kC_h}{2\mathrm{i}r}}\left(w_{0x}\pm\mathrm{i}\frac{x}{C_h}\right)\right]\right\}, \tag{19}$$

where

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x\exp(-s^2)\mathrm{d}s \tag{20}$$

is the usual error function.[17] Similarly, the following integral is obtained:

$$T_2 = \int_{-\infty}^{\infty}\frac{1}{1+(y_0-y_\mathrm{d})^2/w_{0y}^2}$$
$$\times\exp\left(\frac{\mathrm{i}k}{2r}(C_g y_0^2-2yy_0)\right)\mathrm{d}y_0$$
$$= \frac{\pi w_{0y}}{2}[V_y^+(y-C_g y_\mathrm{d},z)+V_y^-(y-C_g y_\mathrm{d},z)]$$
$$\times\exp\left(-\frac{\mathrm{i}ky^2}{2C_g r}\right) \tag{21}$$

with $V_y^{\pm}(y,z)$ given by

$$V_y^{\pm}(y,z) = \exp\left[\frac{kC_g}{2\mathrm{i}r}\left(w_{0y}\pm\mathrm{i}\frac{y}{C_g}\right)^2\right]$$
$$\times\left\{1-\mathrm{erf}\left[\sqrt{\frac{kC_g}{2\mathrm{i}r}}\left(w_{0y}\pm\mathrm{i}\frac{y}{C_g}\right)\right]\right\}. \tag{22}$$

Therefore, the analytical expression of the x component for a vectorial apertured off-axis Lorentz beam beyond the paraxial approximation is obtained as

$$E_x(\boldsymbol{r}) = -\frac{\mathrm{i}\pi^2 E_0 z}{4\lambda r^2}\exp(\mathrm{i}kr)\sum_{h=1}^{N}\left\{A_h[V_x^+(x-C_hx_\mathrm{d},z)+V_x^-(x-C_hx_\mathrm{d},z)]\exp\left(-\frac{\mathrm{i}kx^2}{2C_hr}\right)\right\}$$
$$\times\sum_{g=1}^{N}\left\{A_g[V_y^+(y-C_gy_\mathrm{d},z)+V_y^-(y-C_gy_\mathrm{d},z)]\exp\left(-\frac{\mathrm{i}ky^2}{2C_gr}\right)\right\}. \tag{23}$$

To obtain the analytical longitudinal component of a vectorial apertured off-axis Lorentz beam, one should still perform the integral

$$T_3 = \int_{-\infty}^{\infty}\frac{x_0}{1+(x_0-x_\mathrm{d})^2/w_{0x}^2}\exp\left(\frac{\mathrm{i}k}{2r}(C_hx_0^2-2xx_0)\right)\mathrm{d}x_0$$
$$= x_\mathrm{d}T_1 + w_{0x}^2\exp\left(-\frac{\mathrm{i}kx^2}{2C_hr}\right)\left[f_3\left(\frac{x-C_hx_\mathrm{d}}{C_h}\right)*f_2\left(\frac{x-C_hx_\mathrm{d}}{C_h}\right)\right], \tag{24}$$

where $f_3(\tau)$ is defined by

$$f_3(\tau) = \frac{\tau}{w_{0x}^2+\tau^2}. \tag{25}$$

By using the convolution theorem of the Fourier transforms, expression (24) turns into

$$T_3 = x_\mathrm{d}T_1 + \frac{\mathrm{i}\pi w_{0x}^2}{2}[V_x^+(x-C_hx_\mathrm{d},z)-V_x^-(x-C_hx_\mathrm{d},z)]\exp\left(-\frac{\mathrm{i}kx^2}{2C_hr}\right). \tag{26}$$

As a result, the analytical longitudinal component of a vectorial apertured off-axis Lorentz beam is obtained as

$$E_z(\boldsymbol{r}) = \frac{\mathrm{i}\pi^2 E_0(\mathrm{i}kr)}{4\lambda r^2}\exp(\mathrm{i}kr)\sum_{h=1}^{N}\left\{A_h[(x-x_\mathrm{d}-\mathrm{i}w_{0x})V_x^+(x-C_hx_\mathrm{d},z)\right.$$
$$\left.+(x-x_\mathrm{d}+\mathrm{i}w_{0x})V_x^-(x-C_hx_\mathrm{d},z)]\exp\left(-\frac{\mathrm{i}kx^2}{2C_hr}\right)\right\}$$
$$\times\sum_{g=1}^{N}\left\{A_g[V_y^+(y-C_gy_\mathrm{d},z)+V_y^-(y-C_gy_\mathrm{d},z)]\exp\left(-\frac{\mathrm{i}ky^2}{2C_gr}\right)\right\}. \tag{27}$$

Expressions (23) and (27) are the main results obtained in this paper, which provide a general propagation expression of a vectorial apertured off-axis Lorentz beam beyond the paraxial approximation.

When $a\to\infty$ and $b\to\infty$, which means the case where there exists no rectangular aperture, expressions (23) and (27) are reduced into the following forms respectively:

$$E_x(\boldsymbol{r}) = -\frac{\mathrm{i}\pi^2 E_0 z}{4\lambda r^2}\exp\left(\mathrm{i}kr-\frac{\mathrm{i}k\rho^2}{2r}\right)\left\{\exp\left(\frac{k}{2\mathrm{i}r}[w_{0x}+\mathrm{i}(x-x_\mathrm{d})]^2\right)\left[1-\mathrm{erf}\left(\sqrt{\frac{k}{2\mathrm{i}r}}[w_{0x}+\mathrm{i}(x-x_\mathrm{d})]\right)\right]\right.$$
$$+\exp\left(\frac{k}{2\mathrm{i}r}[w_{0x}-\mathrm{i}(x-x_\mathrm{d})]^2\right)\left[1-\mathrm{erf}\left(\sqrt{\frac{k}{2\mathrm{i}r}}[w_{0x}-\mathrm{i}(x-x_\mathrm{d})]\right)\right]\left.\right\}\left\{\exp\left(\frac{k}{2\mathrm{i}r}[w_{0y}+\mathrm{i}(y-y_\mathrm{d})]^2\right)\right.$$
$$\times\left[1-\mathrm{erf}\left(\sqrt{\frac{k}{2\mathrm{i}r}}[w_{0y}+\mathrm{i}(y-y_\mathrm{d})]\right)\right]+\exp\left(\frac{k}{2\mathrm{i}r}[w_{0y}-\mathrm{i}(y-y_\mathrm{d})]^2\right)$$
$$\times\left.\left[1-\mathrm{erf}\left(\sqrt{\frac{k}{2\mathrm{i}r}}[w_{0y}-\mathrm{i}(y-y_\mathrm{d})]\right)\right]\right\}, \tag{28}$$

$$E_z(\boldsymbol{r}) = \frac{\mathrm{i}\pi^2 E_0}{4\lambda r^2}\exp\left(\mathrm{i}kr-\frac{\mathrm{i}k\rho^2}{2r}\right)\left\{(x-x_\mathrm{d}-\mathrm{i}w_{0x})\exp\left(\frac{k}{2\mathrm{i}r}[w_{0x}+\mathrm{i}(x-x_\mathrm{d})]^2\right)\left[1-\mathrm{erf}\left(\sqrt{\frac{k}{2\mathrm{i}r}}[w_{0x}+\mathrm{i}(x-x_\mathrm{d})]\right)\right]\right.$$
$$\left.+(x-x_\mathrm{d}+\mathrm{i}w_{0x})\exp\left(\frac{k}{2\mathrm{i}r}[w_{0x}-\mathrm{i}(x-x_\mathrm{d})]^2\right)\left[1-\mathrm{erf}\left(\sqrt{\frac{k}{2\mathrm{i}r}}[w_{0x}-\mathrm{i}(x-x_\mathrm{d})]\right)\right]\right\}$$
$$\times\left\{\exp\left(\frac{k}{2\mathrm{i}r}[w_{0y}+\mathrm{i}(y-y_\mathrm{d})]^2\right)\left[1-\mathrm{erf}\left(\sqrt{\frac{k}{2\mathrm{i}r}}[w_{0y}+\mathrm{i}(y-y_\mathrm{d})]\right)\right]\right.$$
$$\left.+\exp\left(\frac{k}{2\mathrm{i}r}[w_{0y}-\mathrm{i}(y-y_\mathrm{d})]^2\right)\left[1-\mathrm{erf}\left(\sqrt{\frac{k}{2\mathrm{i}r}}[w_{0y}-\mathrm{i}(y-y_\mathrm{d})]\right)\right]\right\}. \tag{29}$$

The above expressions are just the propagation expression for a vectorial nonparaxial off-axis Lorentz beam in free space.

The far field and paraxial expressions of a vectorial apertured off-axis Lorentz beam can also be obtained under the far field and paraxial approximation conditions. In the far field regime, expression (9) can be further simplified into the following form:

$$\boldsymbol{G}(\boldsymbol{r},\boldsymbol{\rho}_0)=\frac{1}{r}\exp\left[\mathrm{i}k\left(r-\frac{xx_0+yy_0}{r}\right)\right]. \tag{30}$$

Accordingly, the far field components of a nonparaxial vectorial apertured off-axis Lorentz beam are reduced into the following forms:

$$\begin{aligned}
E_{xf}(\boldsymbol{r})=&-\frac{\mathrm{i}\pi^2E_0z}{4\lambda r^2}\exp(\mathrm{i}kr)\sum_{h=1}^{N}A_h\exp\left(-\frac{\delta_x^2x^2}{4B_hf_x^2r^2}\right)\left\{\exp\left[\frac{B_h}{\delta_x^2}\left(1+\frac{\delta_x^2x}{2B_hf_xr}-\mathrm{i}\frac{x_\mathrm{d}}{w_{0x}}\right)^2\right]\right.\\
&\times\left\{1-\mathrm{erf}\left[\sqrt{\frac{B_h}{\delta_x^2}}\left(1+\frac{\delta_x^2x}{2B_hfr_x}-\mathrm{i}\frac{x_\mathrm{d}}{w_{0x}}\right)\right]\right\}\\
&\left.+\exp\left[\frac{B_h}{\delta_x^2}\left(1-\frac{\delta_x^2x}{2B_hf_xr}+\mathrm{i}\frac{x_\mathrm{d}}{w_{0x}}\right)^2\right]\left\{1-\mathrm{erf}\left[\sqrt{\frac{B_h}{\delta_x^2}}\left(1-\frac{\delta_x^2x}{2B_hf_xr}+\mathrm{i}\frac{x_\mathrm{d}}{w_{0x}}\right)\right]\right\}\right\}\\
&\times\sum_{g=1}^{N}A_g\exp\left(-\frac{\delta_y^2y^2}{4B_gf_y^2r^2}\right)\left\{\exp\left[\frac{B_g}{\delta_y^2}\left(1+\frac{\delta_y^2y}{2B_gf_y^2r}-\mathrm{i}\frac{y_\mathrm{d}}{w_{0y}}\right)^2\right]\right.\\
&\times\left\{1-\mathrm{erf}\left[\sqrt{\frac{B_g}{\delta_y^2}}\left(1+\frac{\delta_y^2y}{2B_gf_y^2r}-\mathrm{i}\frac{y_\mathrm{d}}{w_{0y}}\right)\right]\right\}+\exp\left[\frac{B_g}{\delta_y^2}\left(1-\frac{\delta_y^2y}{2B_gf_y^2r}+\mathrm{i}\frac{y_\mathrm{d}}{w_{0y}}\right)^2\right]\\
&\left.\times\left\{1-\mathrm{erf}\left[\sqrt{\frac{B_g}{\delta_y^2}}\left(1-\frac{\delta_y^2y}{2B_gf_y^2r}+\mathrm{i}\frac{y_\mathrm{d}}{w_{0y}}\right)\right]\right\}\right\},
\end{aligned} \tag{31}$$

$$\begin{aligned}
E_{zf}(\boldsymbol{r})=&\frac{\mathrm{i}\pi^2E_0\exp(\mathrm{i}kr)}{4\lambda r^2}\sum_{h=1}^{N}A_h\left\{(x-x_\mathrm{d}-\mathrm{i}w_{0x})\exp\left[\frac{B_h}{\delta_x^2}\left(1+\frac{\delta_x^2x}{2B_hf_xr}-\mathrm{i}\frac{x_\mathrm{d}}{w_{0x}}\right)^2\right]\right.\\
&\times\left\{1-\mathrm{erf}\left[\sqrt{\frac{B_h}{\delta_x^2}}\left(1+\frac{\delta_x^2x}{2B_hf_xr}-\mathrm{i}\frac{x_\mathrm{d}}{w_{0x}}\right)\right]\right\}+(x-x_\mathrm{d}+\mathrm{i}w_{0x})\exp\left[\frac{B_h}{\delta_x^2}\left(1-\frac{\delta_x^2x}{2B_hf_xr}+\mathrm{i}\frac{x_\mathrm{d}}{w_{0x}}\right)^2\right]\\
&\left.\times\left\{1-\mathrm{erf}\left[\sqrt{\frac{B_h}{\delta_x^2}}\left(1-\frac{\delta_x^2x}{2B_hf_xr}+\mathrm{i}\frac{x_\mathrm{d}}{w_{0x}}\right)\right]\right\}\right\}\exp\left(-\frac{\delta_x^2x^2}{4B_hf_x^2r^2}\right)\\
&\times\sum_{g=1}^{N}A_g\exp\left(-\frac{\delta_y^2y^2}{4B_gf_y^2r^2}\right)\left\{\exp\left[\frac{B_g}{\delta_y^2}\left(1+\frac{\delta_y^2y}{2B_gf_y^2r}-\mathrm{i}\frac{y_\mathrm{d}}{w_{0y}}\right)^2\right]\left\{1-\mathrm{erf}\left[\sqrt{\frac{B_g}{\delta_y^2}}\left(1+\frac{\delta_y^2y}{2B_gf_y^2r}-\mathrm{i}\frac{y_\mathrm{d}}{w_{0y}}\right)\right]\right\}\right.\\
&\left.+\exp\left[\frac{B_g}{\delta_y^2}\left(1-\frac{\delta_y^2y}{2B_gf_y^2r}+\mathrm{i}\frac{y_\mathrm{d}}{w_{0y}}\right)^2\right]\left\{1-\mathrm{erf}\left[\sqrt{\frac{B_g}{\delta_y^2}}\left(1-\frac{\delta_y^2y}{2B_gf_y^2r}+\mathrm{i}\frac{y_\mathrm{d}}{w_{0y}}\right)\right]\right\}\right\},
\end{aligned} \tag{32}$$

where

$$f_x=1/kw_{0x},\quad f_y=1/kw_{0y},\quad \delta_x=a/w_{0x},\quad \text{and}\quad \delta_y=b/w_{0y} \tag{33}$$

with f_x and f_y being the f parameters and δ_x and δ_y being the truncation parameters in the x- and y-directions, respectively.

The propagation equation of a vectorial apertured off-axis Lorentz beam in the paraxial regime can be treated as a special case by using the paraxial expansion

$$r=z+\frac{\rho^2}{2z}. \tag{34}$$

Replacing r in the first phase part of expression (23) with expression (34) and other terms with z, one can

obtain the paraxial apertured off-axis Lorentz beam as

$$E_{xp}(\boldsymbol{r}) = -\frac{\mathrm{i}\pi^2 E_0}{4\lambda z}\exp\left[\mathrm{i}k\left(z+\frac{\rho^2}{2z}\right)\sum_{h=1}^{N}\left\{A_h[P_x^+(x-C_{hp}x_\mathrm{d},z)+P_x^-(x-C_{hp}x_\mathrm{d},z)]\exp\left(-\frac{\mathrm{i}kx^2}{2C_{hp}z}\right)\right\}\right.$$
$$\left.\times\sum_{g=1}^{N}\left\{A_g[P_y^+(y-C_{gp}y_\mathrm{d},z)+P_y^-(y-C_{gp}y_\mathrm{d},z)]\exp\left(-\frac{\mathrm{i}ky^2}{2C_{gp}z}\right)\right\}\right. \tag{35}$$

with $P_x^{\pm}(x,z)$ and $P_y^{\pm}(y,z)$ given by

$$P_x^{\pm}(x,z)=\exp\left[\frac{kC_{hp}}{2\mathrm{i}z}\left(w_{0x}\pm\mathrm{i}\frac{x}{C_{hp}}\right)^2\right]\left\{1-\mathrm{erf}\left[\sqrt{\frac{kC_{hp}}{2\mathrm{i}z}}\left(w_{0x}\pm\mathrm{i}\frac{x}{C_{hp}}\right)\right]\right\}, \tag{36}$$

$$P_y^{\pm}(y,z)=\exp\left[\frac{kC_{gp}}{2\mathrm{i}z}\left(w_{0y}\pm\mathrm{i}\frac{y}{C_{gp}}\right)^2\right]\left\{1-\mathrm{erf}\left[\sqrt{\frac{kC_{gp}}{2\mathrm{i}z}}\left(w_{0y}\pm\mathrm{i}\frac{y}{C_{gp}}\right)\right]\right\}, \tag{37}$$

where

$$C_{hp}=1-\frac{2zB_h}{\mathrm{i}ka^2},\quad\text{and}\quad C_{gp}=1-\frac{2zB_g}{\mathrm{i}kb^2}. \tag{38}$$

The longitudinal component is omitted within the framework of the paraxial approximation as it is negligible compared with the x component.[18]

3. Numerical calculation results and analyses

For the sake of intuition, the normalized light intensity distribution of a vectorial apertured off-axis Lorentz beam in the reference planes is depicted in Figs.1–5, where the light intensity is given by $I = |E_x|^2 + |E_y|^2 + |E_z|^2$. The solid curve represents the nonparaxial result obtained by using expressions (23) and (27). The dotted curve denotes the paraxial result calculated by using expression (35), which is given here for comparison. The reference plane is $z = 100\lambda$ in Figs.1–4. $x_\mathrm{d} = y_\mathrm{d} = 0.1\lambda$, $\delta_x = \delta_y = 2$, and $f_x = f_y = 0.02$ in Fig.1. In Fig.2, $f_x = f_y = 0.5$, and the rest of parameters are the same as those in Fig.1. $x_\mathrm{d} = y_\mathrm{d} = 2\lambda$ in Fig.3, and the other parameters are the same as those in Fig.2. $x_\mathrm{d} = y_\mathrm{d} = 0.1\lambda$, $\delta_x = \delta_y = 3$, and $f_x = f_y = 0.5$ in Fig.4. $x_\mathrm{d} = y_\mathrm{d} = 0.1\lambda$, $\delta_x = \delta_y = 2$, and $f_x = f_y = 0.5$ in Fig.5. Moreover, the different reference planes $z = 50\lambda$ and $z = 1000\lambda$ are taken into account. If f_x and f_y are small, the nonparaxial result is in good agreement with the paraxial result, which is shown in Fig.1. Figure 2 indicates the difference between the nonparaxial and paraxial results becomes noticeable if f_x and f_y are large. Comparing Fig.2 with Fig.3, one can see the influence of off-axis displacements on the propagation behaviour. As the off-axis displacement increases, the difference between the nonparaxial and paraxial results decreases in the central area, while their difference is augmented

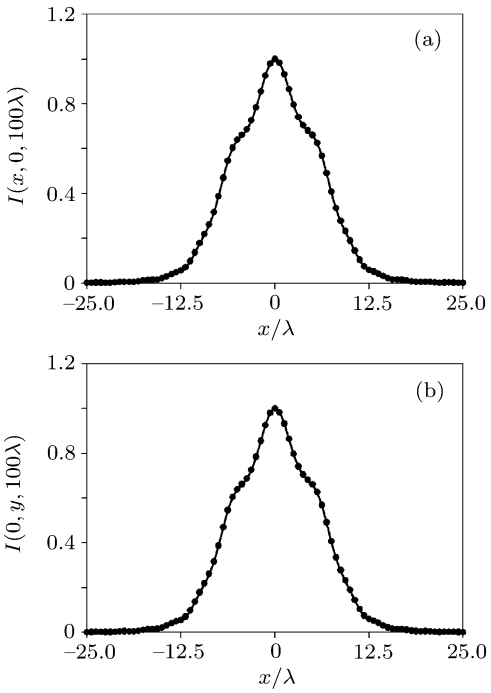


Fig.1. Normalized intensity distribution of an apertured off-axis Lorentz beam in the reference plane $z = 100\lambda$ with $x_\mathrm{d} = y_\mathrm{d} = 0.1\lambda$, $\delta_x = \delta_y = 2$, and $f_x = f_y = 0.02$ for (a) $I(x,\ 0,\ 100\ \lambda)$ and (b) $I(0,\ y,\ 100\ \lambda)$, where the solid and dotted curves denote the nonparaxial and paraxial results, respectively.

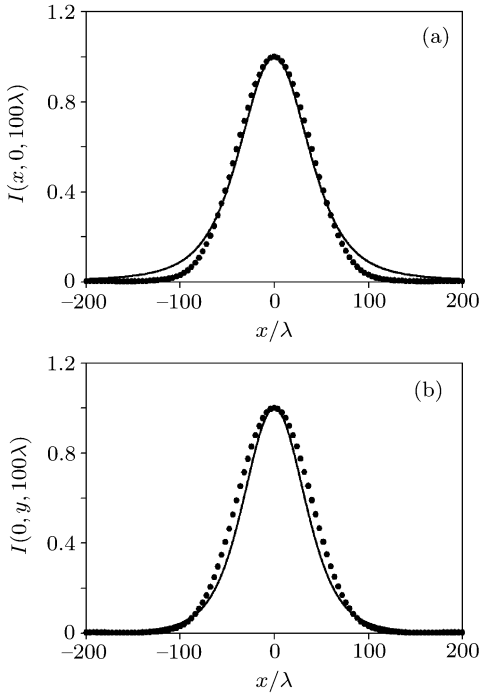


Fig.2. Normalized intensity distribution of an apertured off-axis Lorentz beam in the reference plane $z = 100\lambda$ with $x_{\rm d} = y_{\rm d} = 0.1\lambda$, $\delta_x = \delta_y = 2$, and $f_x = f_y = 0.5$ for (a) $I(x,\ 0,\ 100\ \lambda)$ and (b) $I(0,\ y,\ 100\lambda)$, where the solid and dotted curves denote the nonparaxial and paraxial results, respectively.

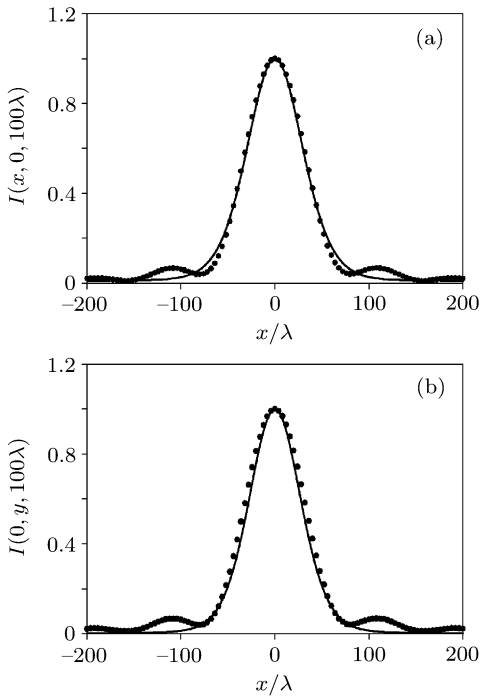


Fig.3. Normalized intensity distribution of an apertured off-axis Lorentz beam in the reference plane $z = 100\lambda$ with $x_{\rm d} = y_{\rm d} = 2\lambda$, $\delta_x = \delta_y = 2$, and $f_x = f_y = 0.5$ for (a) $I(x, 0, 100\lambda)$ and (b) $I(0, y, 100\lambda)$, where the solid and dotted curves denote the nonparaxial and paraxial results, respectively.

in the periphery. Comparing Fig.4 with Fig.2, it is seen that the difference between the nonparaxial and paraxial results becomes evident as the truncation parameter decreases. The stronger the truncation is, the larger the difference will be. However, the truncation parameter may not be set to be smaller than unity, as otherwise expression (3) is not satisfied. As shown in Figs.2 and 5, the difference between the nonparaxial and paraxial results is independent of axial propagation distance z. Therefore, the f parameter, the off-axis displacement, and the truncation parameter play an important role in determining the nonparaxiality of a vectorial apertured off-axis Lorentz beam.

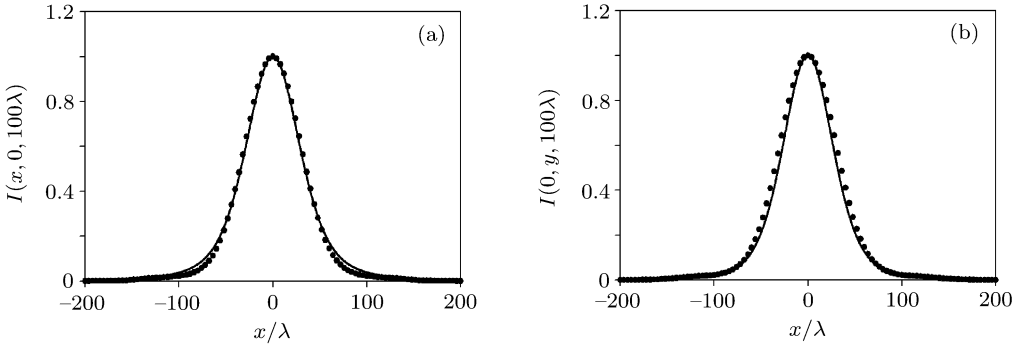


Fig.4. Normalized intensity distribution of an apertured off-axis Lorentz beam in the reference plane $z = 100\lambda$ with $x_{\rm d} = y_{\rm d} = 0.1\lambda$, $\delta_x = \delta_y = 3$, and $f_x = f_y = 0.5$ for (a) $I(x,\ 0,\ 100\lambda)$ and (b) $I(0,\ y,\ 100\lambda)$, where the solid and dotted curves denote the nonparaxial and paraxial results, respectively.

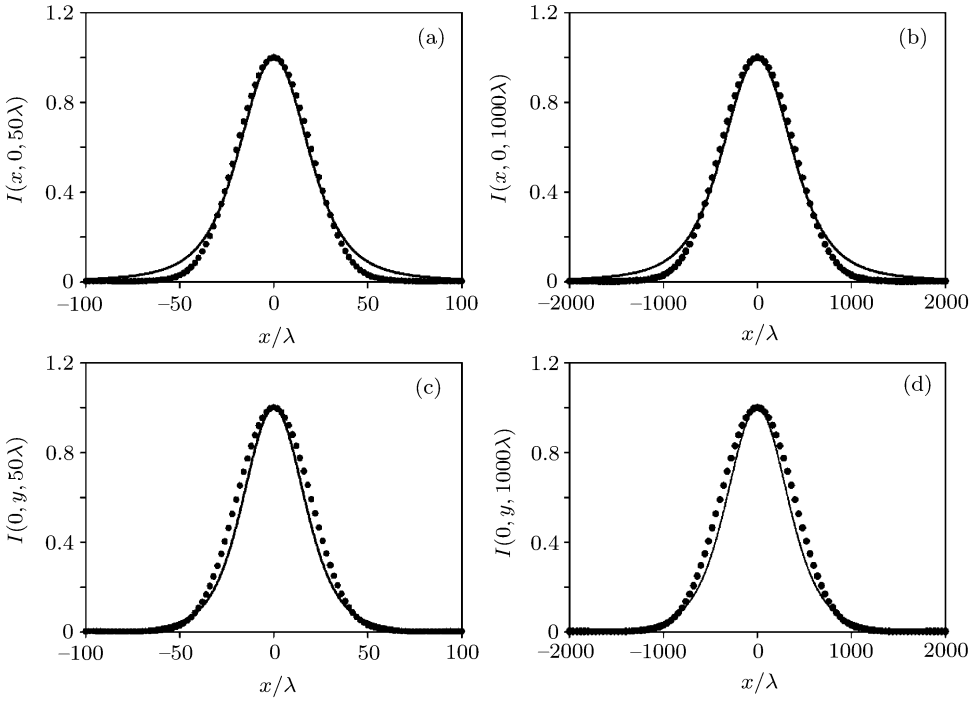


Fig.5. Normalized intensity distribution of an apertured off-axis Lorentz beam in the different reference planes, with $x_{\rm d} = y_{\rm d} = 0.1\lambda$, $\delta_x = \delta_y = 2$, and $f_x = f_y = 0.5$ for (a) $I(x, 0, 50\lambda)$, (b) $I(x, 0, 1000\lambda)$, (c) $I(0, y, 50\lambda)$, and (d) $I(0, y, 1000\lambda)$, where the solid and dotted curves denote the nonparaxial and paraxial results, respectively.

4. Conclusions

Based on the vectorial Rayleigh–Sommerfeld integral formula and the complex Gaussian expansion of the hard-edge aperture function, an analytical propagation equation for a nonparaxial vectorial off-axis Lorentz beam passing through a rectangular aperture is derived in this paper. As special cases of the general formulae, the unapertured case, the far field expression and the scalar paraxial result are also presented. According to the derived formulae, numerical examples are given to show the propagation characteristics of a nonparaxial vectorial off-axis Lorentz beam through a rectangular aperture. The numerical results indicate that the f parameter, the off-axis displacement and the truncation parameter play an important role in determining the nonparaxiality and the nonparaxial propagation behaviour of a vectorial apertured off-axis Lorentz beam.

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