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The simplified material parameter equation for elliptical cylinder cloaks*

Ma Hua(马 华)[a)†], Qu Shao-Bo(屈绍波)[a)], Xu Zhuo(徐 卓)[b)],
Zhang Jie-Qiu(张介秋)[a)], and Wang Jia-Fu(王甲富)[a)]

[a)] *The College of Science, Air Force University of Engineering, Xi'an 710051, China*
[b)] *Electronic Materials Research Laboratory, Key Laboratory of Ministry of Education, Xi'an Jiaotong University, Xi'an 710049, China*

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We simplify the material parameter equation for elliptical cylinder cloaks under transverse-electric and transverse-magnetic models, respectively, and confirm these simplified equations by numerical simulations. As a result, the number of the component parameters is reduced from three to two, which simplifies the design of meta-materials and thus opens up the possibility of achieving elliptical cylinder cloaks in an easy way.

Keywords: invisibility, cloaks, metamaterials
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1. Introduction

Based on the high effectiveness of meta-materials in controlling electromagnetic (EM) waves, Pendry *et al* [1] and Leonhardt[2] separately proposed the concept of perfect invisibility achieved by meta-material cloaks. Also reported was the experimental circular cylinder cloak fabricated using a split resonator ring (SRR), achieving perfect invisibility with transverse electric (TE) polarized EM waves, and thus verifying the concept of meta-material cloaks.[3] Generally, the relative permittivity and permeability of the meta-material possessed by the cloak must be governed by material parameter equations (MPEs) that are derived according to the coordinate transformation (CT) associated with the cloak shape. The already obtained MPEs for, e.g., circular, triangular and elliptical cylinder cloaks,[4−8] all require that the meta-material must be anisotropic and position-dependent, and that more strictly the relative permittivity and permeability of such meta-material are respectively less than one in some region. Such rigorous requirements bring about great difficulties in designing and realizing cloaks. So, it is necessary to simplify the MPEs and thus to decrease the number of free designing variables in some special situations. For instance, under TE polarization, the MPE for circular cylinder cloaks has already been simplified successfully.[3]

In this paper, we expand Maxwell equations into their component forms in the elliptical cylinder coordinates, and simplify the MPE for elliptical cylinder cloaks by observing the key variables in connection with the TE and TM models, respectively. We also perform simulations to verify the simplified MPEs.

2. Theoretical analysis

Maxwell equations consist of six component equations. However, under TE and TM polarization, respectively, the number of equations reduces to three, which simplifies the analysis of EM fields. Under the TE model and in the elliptical cylinder coordinate system, Maxwell equations can be expressed as follows:

$$\frac{\partial}{\partial\eta}E_z = -\frac{\partial}{\partial t}\mu_0\mu_\xi c\sqrt{\frac{\xi^2-\eta^2}{1-\eta^2}}H_\xi,$$

$$\frac{\partial}{\partial\xi}E_z = \frac{\partial}{\partial t}\mu_0\mu_\eta c\sqrt{\frac{\xi^2-\eta^2}{\xi^2-1}}H_\eta,$$

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†E-mail: mahuar@163.com

$$\frac{\partial}{\partial\xi}\left(\sqrt{\frac{\xi^2-\eta^2}{1-\eta^2}}H_\eta\right)-\frac{\partial}{\partial\eta}\left(\sqrt{\frac{\xi^2-\eta^2}{\xi^2-1}}H_\xi\right)$$
$$=\frac{\partial}{\partial t}\varepsilon_0\varepsilon_z c\frac{\xi^2-\eta^2}{\sqrt{(\xi^2-1)(1-\eta^2)}}E_z, \quad (1)$$

where c is the focal length of the elliptical cylinder, and $(\xi,\ \eta,\ z)$ the position described by the elliptical cylinder coordinate system.

Suppose that $D_z=\varepsilon_0\varepsilon_z E_z$, and that $\partial/\partial t=-\mathrm{j}\omega$, then

$$\frac{\partial}{\partial\eta}\left(\frac{D_z}{\varepsilon_0\varepsilon_z}\right)=\mathrm{j}\omega\mu_0\mu_\xi c\sqrt{\frac{\xi^2-\eta^2}{1-\eta^2}}H_\xi,$$
$$\frac{\partial}{\partial\xi}\left(\frac{D_z}{\varepsilon_0\varepsilon_z}\right)=-\mathrm{j}\omega\mu_0\mu_\eta c\sqrt{\frac{\xi^2-\eta^2}{\xi^2-1}}H_\eta,$$
$$\frac{\partial}{\partial\xi}\left(\sqrt{\frac{\xi^2-\eta^2}{1-\eta^2}}H_\eta\right)-\frac{\partial}{\partial\eta}\left(\sqrt{\frac{\xi^2-\eta^2}{\xi^2-1}}H_\xi\right)$$
$$=-\mathrm{j}\omega c\frac{\xi^2-\eta^2}{\sqrt{(\xi^2-1)(1-\eta^2)}}D_z. \quad (2)$$

Letting ε_z be spatially uniform, we observe Eq.(2) and find that the behaviour of the EM waves is determined by $(\mu_\xi\varepsilon_z)$ and $(\mu_\eta\varepsilon_z)$. Reference [6] has already provided the MPEs for elliptical cylinder cloaks, so we can obtain the value of $(\mu_\xi\varepsilon_z)$ and $(\mu_\eta\varepsilon_z)$ by using the MPEs. The results are

$$\mu_\xi\varepsilon_z=\frac{\alpha^2-\eta^2}{\xi^2-\eta^2},$$
$$\mu_\eta\varepsilon_z=\frac{\xi^2-1}{\alpha^2-1}\frac{\alpha^2-\eta^2}{\xi^2-\eta^2}\left(\frac{\xi_2-1}{\xi_2-\xi_1}\right)^2, \quad (3)$$

where $\alpha=(\xi-\xi_1)[(\xi_2-1)/(\xi_2-\xi)]+1$, and,

$$\xi_1\le\xi\le\xi_2,\quad -1\le\eta\le 1. \quad (4)$$

Assigning an arbitrary constant ε_z, for example, ε_z=1, we obtain:

$$\varepsilon_z=1,$$
$$\mu_\xi=\frac{\alpha^2-\eta^2}{\xi^2-\eta^2},$$
$$\mu_\eta=\frac{\xi^2-1}{\alpha^2-1}\frac{\alpha^2-\eta^2}{\xi^2-\eta^2}\left(\frac{\xi_2-1}{\xi_2-\xi_1}\right)^2. \quad (5)$$

The above method is also valid in dealing with the TM model. In this case, the Maxwell equations are formally reduced to Eq.(1) with the fields and the constitutive parameters being replaced according to the exact variable exchanges as following:

$$[E_z,-H_\xi,-H_\eta,\varepsilon_0,\varepsilon_z,\mu_0,\mu_\xi,\mu_\eta]$$
$$\leftrightarrow[H_z,E_\xi,E_\eta,\mu_0,\mu_z,\varepsilon_0,\varepsilon_\xi,\varepsilon_\eta]. \quad (6)$$

Similarly, Maxwell equations in the TM model are determined by $(\varepsilon_\xi\mu_z)$ and $(\varepsilon_\eta\mu_z)$, and $(\mu_z,\varepsilon_\xi,\ \varepsilon_\eta)$ can therefore be obtained from Eq.(5) with the variable exchanges according to Eq.(6):

$$\mu_z=1,$$
$$\varepsilon_\xi=\frac{\alpha^2-\eta^2}{\xi^2-\eta^2},$$
$$\varepsilon_\eta=\frac{\xi^2-1}{\alpha^2-1}\frac{\alpha^2-\eta^2}{\xi^2-\eta^2}\left(\frac{\xi_2-1}{\xi_2-\xi_1}\right)^2. \quad (7)$$

By the above procedures, in the TE and TM models, we reduce the number of component parameters from three to two, which opens up a possibility of achieving cloaks more easily. As confirmation of these procedures, we perform the simulation for Eq.(5) and compare it with the results of Ref.[6].

3. Simulation and discussion

In the simulation, the commercial electromagnetic simulation software, COMSOL$^{\mathrm{TM}}$, is used. We configure a computational domain in which an elliptical cylinder region is cloaked by a series of elliptical shells holding the same focus, and launch a 2 GHz TE polarized time-harmonic uniform plane wave into the cloak. Clearly, such configurations are the same as that of Ref.[6]. By doing so, it is convenient to make a comparison in terms of the cloaking performance between the cloaks that are constructed based on the original equations in Ref.[6] and on the simplified Eq.(5) in this paper.

Considering the low symmetry of elliptical cylinders and the variation in scattering with incident beam direction, we perform the simulation in the following four cases in which the simulations are done for four incident angles –0°, 30°, 45°, 90°, respectively. Note that the incident angle is characterized by the angle between the elliptic major axis and the incident wave beam. The cloaking materials are also assumed to be lossless, by which we configure the same numerical condition as that in Ref.[6] and therefore make the comparison by focusing on the shapes of phase fronts and the paths of EM power-flow.

Figure 1 shows the electric-field distribution inside and outside the cloak, respectively, where permittivity and permeability of the cloak are governed by Eq.(5). Comparing this figure with Fig.2 in Ref.[6], we find the same cloaking performance: outside the cloaking shell, the wave fronts remain planar and the scattering is quite small, whereas inside the cloak,

the phase fronts are warped, which causes power-flow to bend around the inner shell. In detail, Figs.1(a) and 1(b) show the electric-field distribution when the wave propagates along the coordinate aligned orientations, and Figs.1(c) and 1(d) show the results when the wave propagates along the non-coordinate aligned orientations. Generally, scattering outside the cloak for the non-coordinate aligned orientations is greater than that for the coordinate aligned orientations, but this difference can be reduced by improving computation methods and increasing the number of simulated field elements. Figure 1 is an optimized result, which shows less scattering in all these four directions and thus confirms the cloaking effect.

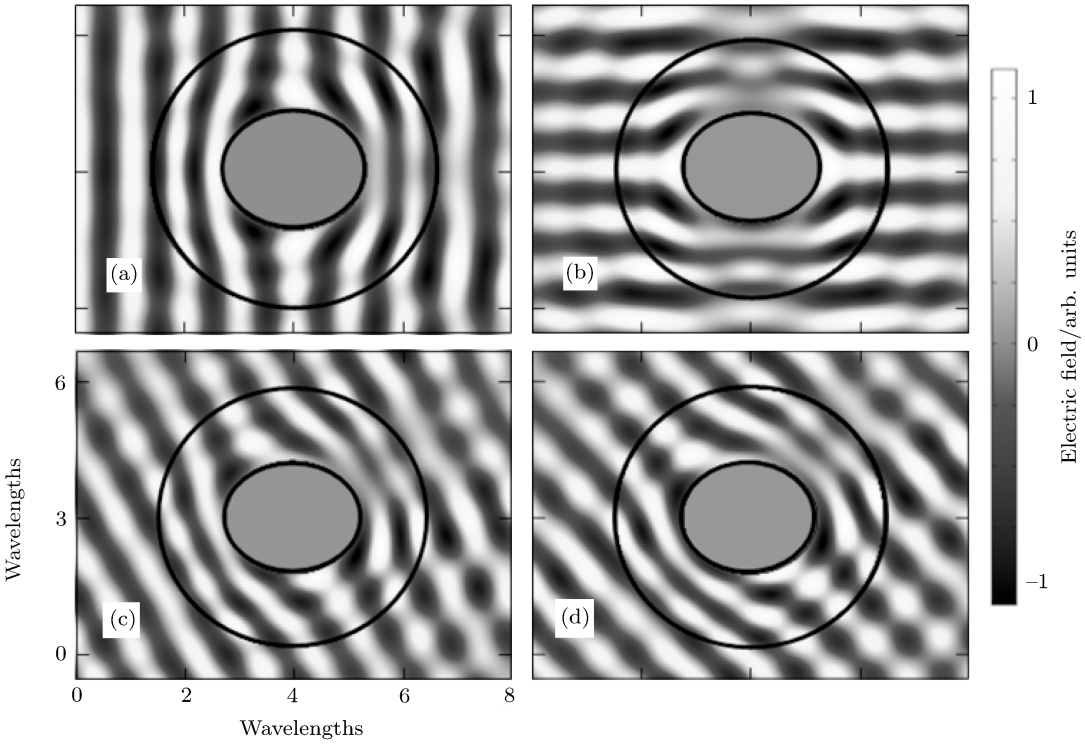


Fig.1. The electric-field distributions inside and outside the elliptical cylinder cloak; here four cases are considered in terms of the incident EM wave angle with respect to the major axis: 0° (a), 90° (b), 30° (c) and 45° (d), respectively.

4. Conclusion

We simplified the MPE for elliptical cylinder cloaks in the TE and TM models, respectively, and verified those simplified equations through the numerical simulation. In doing so, our goal was to decrease the number of component parameters and thus simplify the design of meta-materials. As a result, the number is reduced from three to two, thus offering the possibility of achieving elliptical cylinder cloaks in a practical way. In future research, we will attempt the realization of elliptical cylinder cloaks by adopting these simplified equations.

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