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# Regarding the $2^1S_0$ and $2^3S_1$ multiplets

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This paper discusses the masses and possible members of $2^1S_0$ and $2^3S_1$ meson multiplets in the framework of meson–meson mixing and Regge phenomenology. For the $2^1S_0$ meson nonet, $K^*(1460)$ may be the observed state of the $2^1S_0$ isodoublet with a mass of 1400 MeV and the mass of $\omega(1420)$ is 1412 MeV. For the $2^3S_1$ meson nonet, $K^*(1410)$ seems too light to be the $2^3S_1$ $n\overline{s}$ member. The fact that the agreement between the present findings and those given by experiments and other different approaches is satisfactory implies that the assignments in this paper may be reasonable.

**Keywords:** meson multiplet, meson mass, Regge trajectory
**PACC:** 1440, 3520X

## 1. Introduction

The investigation of the meson spectrum is of great importance for better understanding the dynamics of strong interactions, since the meson is an ideal laboratory for the study of strong interactions in the strongly coupled non-perturbative regime.[1] According to the recent issue of *Review of Particle Physics*,[2] there are many mesons, especially the radial states, so far absent from the Meson Summary Table. The members of $2^1S_0$ and $2^3S_1$ meson nonets are listed as $\pi(1300)$, $\eta(1295)$, $\eta(1475)$, $K(1460)$ and $\rho(1450)$, $\omega(1420)$, $\phi(1680)$, $K^*(1410)$, respectively. However, the mass of $K(1460)$ has not been decided, the experimental value is 1400 MeV or 1460 MeV. The mass of $K^*(1410)$ is in disagreement with the expectation of the quark model for the $2^3S_1$ kaon state, the fact that $K^*(1410)$ is lighter than $\rho(1450)$ does not coincide with the established meson nonets. The mass of $\omega(1420)$ is in the range 1400–1450 MeV but not an exact value.

These problems will be discussed in the framework of the Regge trajectory and meson–meson mixing below. The paper is organized as follows: introductions to Regge trajectory and meson–meson mixing are given in Section 2 and Section 3, respectively, then the multiplets of $2^1S_0$ and $2^3S_1$ are discussed in Section 4 and Section 5, respectively; there are also discussions and conclusions in the end.

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## 2. Regge phenomenology

Regge theory is concerned with the particle spectrum, the forces between particles and the high energy behaviour of scattering amplitudes.[3] One of the most distinctive features of Regge theory is the Regge trajectory by which the mass and the spin of a hadron are related.[4] Knowledge of Regge trajectories is useful not only for spectral purpose, but also for many non-spectral purposes. The intercepts and slopes of the Regge trajectories are of fundamental importance in hadron physics.[5] A series of recent papers[6−8] indicate that the quasi-linear Regge trajectory can, at least at present, give a reasonable description for the meson spectroscopy, and its predictions may be useful for the discovery of the meson states which have not yet been observed. By assuming the existence of the quasi-linear Regge trajectories for a meson multiplet, one can have

$$J = \alpha_{i\overline{i'}(N)}(0) + \alpha'_{i\overline{i'}(N)} M^2_{i\overline{i'}(N)}, \tag{1}$$

where $i(\overline{i'})$ refers to the quark (anti-quark) flavour, $i\overline{i'}(N)$ denotes the meson $i\overline{i'}$ with radial quantum number $N$ ($N = 1, 2, 3, \ldots$), $J$ and $M_{i\overline{i'}(N)}$ are respectively the spin and mass of the $i\overline{i'}(N)$ meson, $\alpha_{i\overline{i'}(N)}(0)$ and $\alpha'_{i\overline{i'}(N)}$ are respectively the intercept and slope of the trajectory on which the $i\overline{i'}(N)$ meson lies. For a meson multiplet, the parameters for different flavours can be related by the following relations (see Ref.[8] and references therein):

(1) additivity of intercepts,

$$\alpha_{i\overline{i'}(N)}(0)+\alpha_{j\overline{j'}(N)}(0)=2\alpha_{j\overline{i'}(N)}(0); \quad (2)$$

(2) additivity of inverse slopes,

$$\frac{1}{\alpha'_{i\overline{i'}(N)}}+\frac{1}{\alpha'_{j\overline{j'}(N)}}=\frac{2}{\alpha'_{j\overline{i'}(N)}}. \quad (3)$$

In the following text, we shall adopt one assumption: $\alpha'_{i\overline{i'}(N)}=\alpha'_{i\overline{i'}(1)}$, as adopted by Ref.[6]. Under this assumption, the slopes of the radial meson Regge trajectories are the same as those of the ground meson Regge trajectories.

## 3. Meson–meson mixing scheme

It is well known that in a meson nonet, the pure isoscalar $n\overline{n}$ (hereafter, $n$ denotes $u-$ or $d-$ quark) and $s\overline{s}$ states can mix to produce the physical isoscalar states $f_0(M_1)$ and $f_0(M_2)$. In order to understand the physical scalar states, we shall discuss the mixing of the $n\overline{n}$ and $s\overline{s}$ states.

In the $N=(u\overline{u}+d\overline{d})/\sqrt{2}$, $S=s\overline{s}$ basis, the mass-squared matrix describing the mixing of the $f_0(M_1)$ and $f_0(M_2)$ can be written as[9,10]

$$\boldsymbol{M}^2=\begin{pmatrix} M_N^2+2\beta & \sqrt{2}\beta X \\ \sqrt{2}\beta X & 2M_{n\overline{s}}^2-M_N^2+\beta X^2 \end{pmatrix}, \quad (4)$$

where $M_N$ and $M_{n\overline{s}}$ are the masses of the states $N$ and $n\overline{s}$, respectively; $\beta$ denotes the total annihilation strength of the $q\overline{q}$ pair for the light flavours $u$ and $d$; $X$ describes the $SU(3)$-breaking ratio of the non-strange and strange quark propagators via the constituent quark mass ratio $m_u/m_s$. The masses of the two physical scalar states $f_0(M_1)$ and $f_0(M_2)$, $M_1$ and $M_2$, can be related to the matrix $\boldsymbol{M}^2$ by the unitary matrix $\boldsymbol{U}$

$$\boldsymbol{U}\boldsymbol{M}^2\boldsymbol{U}^{\dagger}=\begin{pmatrix} M_1^2 & 0 \\ 0 & M_2^2 \end{pmatrix}, \quad (5)$$

and the physical states $f_0(M_1)$ and $f_0(M_2)$ can be expressed as

$$\begin{pmatrix} f(M_1) \\ f(M_2) \end{pmatrix}=\boldsymbol{U}\begin{pmatrix} N \\ S \end{pmatrix}. \quad (6)$$

The constituent quark mass ratio can be determined within the non-relativistic constituent quark model(NRCQM). In NRCQM,[11,12] the mass of a $q\overline{q}$ state with $L=0$, $M_{q\overline{q}}$ is given by

$$M_{q\overline{q}}=m_q+m_{\overline{q}}+\Lambda\frac{s_q s_{\overline{q}}}{m_q m_{\overline{q}}}, \quad (7)$$

where $m$ and $s$ are the constituent quark mass and spin, $\Lambda$ is a constant. Since $s_q\cdot s_{\overline{q}}=-3/4$ for spin-0 mesons and $1/4$ for spin-1 mesons. In the $SU(2)$ flavour symmetry limit, one can have

$$X\equiv\frac{m_u}{m_s}=\frac{M_\pi+3M_\rho}{2M_K+6M_{K^*}-M_\pi-3M_\rho}=0.6298. \quad (8)$$

From Eqs.(4) and (5), one can have

$$2M_{n\overline{s}}^2+(2+X^2)\beta=M_1^2+M_2^2, \quad (9)$$

$$(M_N^2+2\beta)(2M_{n\overline{s}}^2-M_N^2+\beta X^2)-2\beta^2X^2=M_1^2M_2^2. \quad (10)$$

## 4. The $2^1S_0$ meson multiplet

Inserting the masses of $\pi(1300)$, $\eta(1295)$ and $\eta(1475)$ into Eqs.(6)–(10), one can have[1)]

$$M_{n\bar{s}}=1.3913\text{GeV},\quad \beta=-0.0077\text{GeV}^2, \quad (11)$$

and

$$\begin{pmatrix} \eta(1295) \\ \eta(1475) \end{pmatrix}=\begin{pmatrix} 0.9999 & 0.0137 \\ -0.0137 & 0.9999 \end{pmatrix}bigg(\begin{matrix} N \\ S \end{matrix}\Big). \quad (12)$$

From Eq.(1), the formulas are given as

$$0=\alpha_{n\overline{n}(2)}(0)+\alpha'_{n\overline{n}(2)}M^2_{n\overline{n}(2)}, \quad (13)$$

$$0=\alpha_{n\overline{s}(2)}(0)+\alpha'_{n\overline{s}(2)}M^2_{n\overline{s}(2)}, \quad (14)$$

$$0=\alpha_{c\overline{c}(2)}(0)+\alpha'_{c\overline{c}(2)}M^2_{c\overline{c}(2)}, \quad (15)$$

$$0=\alpha_{c\bar{b}(2)}(0)+\alpha'_{c\bar{b}(2)}M^2_{c\bar{b}(2)}. \quad (16)$$

Though the $c\bar{b}$ state of $2^1S_0$ has not been observed experimentally, the mass of this state is given as follows: 6.85 GeV[13], 6.855 GeV,[14] 6.8528 GeV,[15] 6.856 GeV,[16] 6.835 GeV,[17] 6.867 GeV,[18] 6.879 GeV,[19] 6.856 GeV,[20] These results are in good agreement, we take the average of these results 6.8646 GeV as the mass of the $c\bar{b}$ state of $2^1S_0$. Inserting the masses of $\pi(1300)$, $\eta_c(2S)$ and $M_{n\bar{s}}=1.3913$ GeV, $M_{c\bar{b}}=6.8646$ GeV into Eqs.(13)–(16), with the help of Eqs.(2) and (3), and the slopes of the $1^1S_0$ meson Regge trajectories exacted by Ref.[8], as shown in Table 1, one can exact all the intercepts of the $2^1S_0$ trajectories as shown in Table 2.

[1)] Hereafter, all the masses used as input for our calculation are taken from Particle Data Group (PDG) 2006.[2]

**Table 1.** The slopes of the $1^1S_0$ Regge trajectories.

| | $n\overline{n}$ | $n\overline{s}$ | $c\overline{c}$ |
|---|---|---|---|
| $\alpha'_{i\overline{i}(1)}/\mathrm{GeV}^{-2}$ | 0.7218 | 0.6902 | 0.4075 |
| | $b\overline{b}$ | $c\overline{n}$ | $c\overline{s}$ |
| $\alpha'_{i\overline{i}(1)}/\mathrm{GeV}^{-2}$ | 0.2079 | 0.5209 | 0.5043 |
| | $n\overline{b}$ | $s\overline{b}$ | $c\overline{b}$ |
| $\alpha'_{i\overline{i}(1)}/\mathrm{GeV}^{-2}$ | 0.3228 | 0.3164 | 0.2753 |

**Table 2.** The intercepts of the $2^1S_0$ Regge trajectories.

| | $n\overline{n}$ | $n\overline{s}$ | $c\overline{c}$ |
|---|---|---|---|
| $\alpha_{i\overline{i}(1)}(0)$ | –1.2198 | –1.3361 | –5.3933 |
| | $b\overline{b}$ | $c\overline{n}$ | $c\overline{s}$ |
| $\alpha_{i\overline{i}(1)}(0)$ | –20.477 | –3.3066 | –3.4228 |
| | $n\overline{b}$ | $s\overline{b}$ | $c\overline{b}$ |
| $\alpha_{i\overline{i}(1)}(0)$ | –10.848 | –10.965 | –12.935 |

Based on these parameters, the estimated masses of states belonging to the $2^1S_0$ $q\overline{q}$ multiplet and those given by other references are shown in Table 3. The masses used as inputs for our calculation are shown in boldface.

**Table 3.** The masses of the states belonging to $2^1S_0$ $q\overline{q}$ multiplet. All in GeV.

| | $M_{n\overline{n}}$ | $M_{n\overline{s}}$ | $M_{c\overline{n}}$ | $M_{c\overline{s}}$ | $M_{n\overline{b}}$ | $M_{s\overline{b}}$ | $M_{c\overline{c}}$ | $M_{b\overline{b}}$ | $M_{c\overline{b}}$ |
|---|---|---|---|---|---|---|---|---|---|
| this work | **1.3** | 1.3913 | 2.5195 | 2.6052 | 5.7972 | 5.8868 | **3.638** | 9.9244 | **6.8546** |
| Exp.[2] | 1.3 | | | | | | 3.638 | | |
| Ref.[13] | | | 2.50 | 2.61 | 5.83 | 5.93 | | | 6.85 |
| Ref.[15] | | | | | | | 3.6179 | 9.9905 | 6.8528 |
| Ref.[16] | | | | | | | 3.608 | 9.963 | 6.856 |
| Ref.[19] | | | | | | | 3.6359 | 10.019 | 6.879 |
| Ref.[20] | | | | | | | | 9.963 | 6.856 |
| Ref.[21] | | | 2.579 | 2.67 | 5.883 | 5.971 | | | |
| Ref.[22] | | | 2.5477 | 2.6565 | 5.8921 | 6.0009 | | | |
| Ref.[23] | | | 2.589 | 2.7 | 5.886 | 5.985 | | | |

## 5. The $2^3S_1$ meson multiplet

Inserting the masses of $\rho(1450)$, $\phi(1680)$ and $M_{n\bar{s}}$=1.58GeV given by Godfrey and Isgur in the relativized quark model[24] into Eqs.(6)–(10), one can have

$$M_1 = 1.4115 \text{ GeV}, \quad \beta = -0.07435 \text{ GeV}^2, \tag{17}$$

and

$$\begin{pmatrix} \omega(1420) \\ \phi(1680) \end{pmatrix} = \begin{pmatrix} 0.9968 & 0.08 \\ 0.08 & 0.9968 \end{pmatrix} \begin{pmatrix} N \\ S \end{pmatrix}. \tag{18}$$

From Eq.(1), one can have

$$1 = \alpha_{n\overline{n}(2)}(0) + \alpha'_{n\overline{n}(2)} M^2_{n\overline{n}(2)}, \tag{19}$$

$$1 = \alpha_{n\overline{s}(2)}(0) + \alpha'_{n\overline{s}(2)} M^2_{n\overline{s}(2)}, \tag{20}$$

$$1 = \alpha_{c\overline{c}(2)}(0) + \alpha'_{c\overline{c}(2)} M^2_{c\overline{c}(2)}, \tag{21}$$

$$1 = \alpha_{b\overline{b}(2)}(0) + \alpha'_{b\overline{b}(2)} M^2_{b\overline{b}(2)}. \tag{22}$$

Inserting the mass of $\rho(1450)$, $\psi(2S)$, $\gamma(2S)$ and $M_{n\bar{s}}$ = 1.58 GeV into Eqs.(19)–(22), with the help of Eqs.(2) and (3), and the slopes of the $1^3S_1$ meson Regge trajectory exacted by Ref.[8], as shown in Table 4, one can exact all the intercepts of the $2^3S_1$ Regge trajectories as shown in Table 5. Based on these parameters, the estimated masses of states belonging to $2^3S_1$ $q\overline{q}$ multiplet and those given by other references are shown in Table 6. The masses used as inputs for our calculation are shown in boldface.

**Table 4.** The slopes of the $1^3S_1$ Regge trajectories.

| | $n\overline{n}$ | $n\overline{s}$ | $c\overline{c}$ |
|---|---|---|---|
| $\alpha'_{i\overline{i}(1)}/\mathrm{GeV}^{-2}$ | 0.8830 | 0.8493 | 0.4364 |
| | $b\overline{b}$ | $c\overline{n}$ | $c\overline{s}$ |
| $\alpha'_{i\overline{i}(1)}/\mathrm{GeV}^{-2}$ | 0.2049 | 0.5841 | 0.5692 |
| | $n\overline{b}$ | $s\overline{b}$ | $c\overline{b}$ |
| $\alpha'_{i\overline{i}(1)}/\mathrm{GeV}^{-2}$ | 0.3326 | 0.3277 | 0.2789 |

**Table 5.** The intercepts of the $2^3S_1$ Regge trajectories.

| | $n\overline{n}$ | $n\overline{s}$ | $c\overline{c}$ |
|---|---|---|---|
| $\alpha_{i\overline{i}(1)}(0)$ | –0.8951 | –1.1202 | –4.9295 |
| | $b\overline{b}$ | $c\overline{n}$ | $c\overline{s}$ |
| $\alpha_{i\overline{i}(1)}(0)$ | –19.585 | –2.9123 | –3.1374 |
| | $n\overline{b}$ | $s\overline{b}$ | $c\overline{b}$ |
| $\alpha_{i\overline{i}(1)}(0)$ | –10.24 | –10.465 | –12.262 |

**Table 6.** The masses of the states belonging to the $2^3S_1$ $q\overline{q}$ multiplet. All in GeV.

| | $M_{n\overline{n}}$ | $M_{n\overline{s}}$ | $M_{c\overline{c}}$ | $M_{b\overline{b}}$ | $M_{c\overline{n}}$ | $M_{c\overline{s}}$ | $M_{n\overline{b}}$ | $M_{s\overline{b}}$ | $M_{c\overline{b}}$ |
|---|---|---|---|---|---|---|---|---|---|
| this work | **1.465** | **1.58** | **3.686** | **10.023** | 2.5881 | 2.6961 | 5.8124 | 5.915 | 6.8958 |
| Expt.[2] | 1.465 | | 3.686 | 10.023 | | | | | |
| Ref.[8] | 1.465 | 1.573 | 3.686 | 10.023 | 2.588 | 2.69 | 5.813 | 5.91 | 6.894 |
| Ref.[13] | | | 3.73 | 10.02 | 2.62 | 2.73 | 5.87 | 5.97 | 6.9 |
| Ref.[14] | 1.45 | | 3.68 | 10.00 | 2.64 | 2.73 | 5.93 | 6.01 | 6.89 |
| Ref.[15] | | | 3.686 | 10.016 | | | | | 6.886 |
| Ref.[16] | | | 3.693 | 10.027 | | | | | 6.914 |
| Ref.[17] | | | 3.686 | 10.023 | | | | | 6.881 |
| Ref.[19] | | | 3.719 | 10.023 | | | | | 6.935 |
| Ref.[20] | | | 3.686 | 10.007 | | | | | 6.899 |
| Ref.[25] | | | 3.69 | 10.023 | | | | | 6.921 |
| Ref.[21] | | | | | 2.629 | 2.716 | 5.898 | 5.984 | |
| Ref.[22] | | | | | 2.647 | 2.758 | 5.924 | 6.036 | |
| Ref.[23] | | | | | 2.692 | 2.806 | 5.92 | 6.019 | |

## 6. Discussions

Considering the situation that the mass of $\pi(1300)$ is equal to that of $\eta(1295)$, one finds that $\eta(1295)$ and $\eta(1475)$ show ideal mixing.[26] From Eq.(12), one can find that $\eta(1295)$ which is composed mostly of $s\overline{s}$ quarks and $\eta(1475)$ are mainly non-strange quarkonia, this result is in good agreement with the experimental data. The Eq.(11) shows that if $K^*(1460)$ is the observed state of $2^1S_0$ isodoublet, it could have a mass of 1400 MeV. The fact that $\phi(1680)$ is observed in $K\overline{K}$ and $KK^*$ but not in $\omega\pi\pi$[2] implies that $\phi(1680)$ is mainly strange quarkonia. Also, the decay modes of $\omega(1420)$[2] indicate that $\omega(1420)$ is composed mostly of $n\overline{n}$. Based on the mass of the $2^3S_1$ kaon state, 1580 MeV, the result of Eq.(18) is derived. It shows the fact that $\omega(1420)$ is mainly $n\overline{n}$ and $\phi(1680)$ is dominantly $s\overline{s}$, which is in good agreement with the experiment. Also, the results tell us that the mass of $\omega(1420)$ may be 1412 MeV and $K^*(1410)$ seems too light to be the $2^3S_1$ $n\overline{s}$ member. However, the decay modes of $K^*(1410)$ are inconsistent with the prediction of the $^3P_0$ decay model for the first radial excitation of $K^*(892)$. So, it is suggested that if the $K^*(1410)$ is indeed a real $J^P = 1^-$ resonance, the low mass may be due to the presence of additional hybrid mixing states.[27]

## 7. Conclusions

In the framework of meson–meson mixing and Regge phenomenology, the masses of the $2^1S_0$ and $2^3S_1$ multiplets are calculated. The results show that, for the $2^1S_0$ meson nonet, $K^*(1460)$ may be the observed state of the $2^1S_0$ isodoublet with a mass of 1400 MeV and the mass of $\omega(1420)$ is 1412 MeV. For the $2^3S_1$ meson nonet, the ideal mass of the kaon state is 1580 MeV, $K^*(1410)$ seems too light to be the $2^3S_1$ $n\overline{s}$ member. From Tables 3 and 6, the fact that the agreement between the present findings and those given by other different approaches is satisfactory implies that the assignments in this paper may be reasonable.

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