Vol 18 No 5, May 2009
1674-1056/2009/18(05)/1774-06
Chinese Physics B
© 2009 Chin. Phys. Soc.
and IOP Publishing Ltd

Fuzzy modelling and impulsive control of the hyperchaotic Lü system*

Zhang Xiao-Hong(张小洪)[a)†] and Li Dong(李 东)[b)]

[a)] *School of Software Engineering, Chongqing University, Chongqing* 400030, *China*

[b)] *College of Mathematics & Physics, Chongqing University, Chongqing* 400030, *China*

(Received 7 October 2008; revised manuscript received 1 November 2008)

This paper presents a novel approach to hyperchaos control of hyperchaotic systems based on impulsive control and the Takagi–Sugeno (T–S) fuzzy model. In this study, the hyperchaotic Lü system is exactly represented by the T–S fuzzy model and an impulsive control framework is proposed for stabilizing the hyperchaotic Lü system, which is also suitable for classes of T–S fuzzy hyperchaotic systems, such as the hyperchaotic Rössler, Chen, Chua systems and so on. Sufficient conditions for achieving stability in impulsive T–S fuzzy hyperchaotic systems are derived by using Lyapunov stability theory in the form of the linear matrix inequality, and are less conservative in comparison with existing results. Numerical simulations are given to demonstrate the effectiveness of the proposed method.

Keywords: hyperchaos control, impulsive control, T–S fuzzy model, linear matrix inequalities
PACC: 0545

1. Introduction

Since the interaction between fuzzy control systems and chaos was explored,[1] many fuzzy logic techniques have been developed in order to control chaos and synchronize chaotic systems represented by the T–S fuzzy model, including parallel distributed compensation,[2] adaptive fuzzy approach,[3,4] fuzzy observer,[5] impulsive method[6,7] and output feedback control.[8] The aforementioned methods, however, focused only on low-dimensional chaotic systems with only one positive Lyapunov exponent. Recently, since hyperchaotic systems with more than one positive Lyapunov exponent possess the characteristics of high capacity, high security and high efficiency, they have attracted great attention in the fields of nonlinear circuits,[9] secure communications,[10] lasers,[11] control,[12] and so on. Nowadays, there exist some well-known hyperchaotic systems such as the hyperchaotic Rössler system,[12,13] the hyperchaotic Chua's circuits,[9] the hyperchaotic Lorenz system[14] and the hyperchaotic Chen system.[15] Over the years, many hyperchaos control methods have been developed,[16] including conventional fuzzy control,[17,18] adaptive control[19] and passive control.[20] More recently, Lü proposed a new hyperchaotic system,[21] called the hyperchaotic Lü system and described by

$$\begin{aligned}\dot{x}(t) &= a[y(t) - x(t)] + w(t),\\ \dot{y}(t) &= -x(t)z(t) + cy(t),\\ \dot{z}(t) &= x(t)y(t) - bz(t),\\ \dot{w}(t) &= x(t)z(t) + kw(t),\end{aligned} \tag{1}$$

where a, b, c, and k are constant parameters, $x(t)$, $y(t)$, $z(t)$ and $w(t)$ are state variances.

In Ref.[22] linear feedback control, adaptive feedback control and the Pecora–Carroll method are used for hyperchaos synchronization of the Lü system. But hyperchaos control of the hyperchaotic Lü system based on fuzzy logic representation has not been reported. Moreover, most of the current research efforts on hyperchaos control are mainly involved with the design of continuous-time controllers for continuous-time fuzzy systems. Impulsive control as a discrete control method of continuous-time systems is one of the most attractive methods due to the following reasons: first the chaos control is achieved using only small control impulses; second it offers a direct method to modulate digital information onto a chaotic carrier for spread spectrum applications.[23] Hence, it is necessary to consider impulsive control of the hyperchaotic Lü system represented by the T–S fuzzy model.

In this paper, our main objectives are represent-

*Project supported by the National Natural Science Foundation of China (Grant No 60604007).
†Corresponding author. E-mail: xhongz@yahoo.com.cn

ing the hyperchaotic Lü system (1) based on the T–S fuzzy model and presenting a common impulsive fuzzy control framework for T–S fuzzy hyperchaotic systems, which is also suitable for many fuzzy hyperchaotic systems, such as the hyperchaotic Rössler, Chen, Lorenz, Chua systems and so on. At the same time, we will establish the stable sufficient conditions of the T–S fuzzy hyperchaotic systems based on Lyapunov stability theory and give some numerical simulations to verify the effectiveness of the proposed method.

2. Fuzzy modelling of the hyperchaotic Lü system and impulsive fuzzy control model

The T–S fuzzy model originates from Takagi and Sugeno, which is described by fuzzy IF-THEN rules where the consequent parts represent local linear models. A general T–S fuzzy system is described as follows:

Plant Rule i**:** IF $z_1(t)$ is M_{i1} and $\ldots$ and $z_p(t)$ is M_{ip}

$$\text{THEN} \quad \dot{\boldsymbol{X}}(t) = \boldsymbol{A}_i X(t) \quad i = 1, 2, \ldots, r, \qquad (2)$$

where $M_{ik} (i = 1, 2, \ldots, r,\ k = 1, 2, \ldots, p)$ are the fuzzy sets and r is the number of IF-THEN rules, $\boldsymbol{X}(t) \in R^n$ is the state vector, $\boldsymbol{A}_i \in R^{n\times n}$ is a constant matrix with appropriate dimensions, $z_1(t) \sim z_p(t)$ are the premised variables. Using the defuzzifier, we infer the final output of the fuzzy system as

$$\dot{\boldsymbol{X}}(t) = \sum_{i=1}^{r} h_i(z(t)) \boldsymbol{A}_i \boldsymbol{X}(t), \qquad (3)$$

where $h_i(z(t)) = w_i(z(t)) / \sum\limits_{i=1}^{r} w_i(z(t))$ for all t, $w_i(z(t)) = \prod\limits_{k=1}^{p} M_{ik}(z_k(t))$, $\sum\limits_{i=1}^{r} w_i(z(t)) > 0$, $w_i(z(t)) \geq 0$, $i =, 2, \ldots, r$. From these formulas we have $\sum\limits_{i=1}^{r} h_i(z(t)) = 1$, $h_i(z(t)) \geq 0$, $i = 1, 2, \ldots, r$.

Since the state variances of the chaotic systems are bounded, assume that $x(t) \in [-m, m]$ in Eq.(1) and $m > 0$. Following the above T–S fuzzy rules, we can exactly express the hyperchaotic Lü system (1) as:

Plant Rule i**:** IF $x(t)$ is M_i

$$\text{THEN} \quad \dot{\boldsymbol{X}}(t) = \boldsymbol{A}_i \boldsymbol{X}(t), \quad j = 1, 2, \ldots. \qquad (4)$$

Where i=1, 2, $\boldsymbol{X}(t) = [x(t),\ y(t),\ z(t),\ w(t)]^{\mathrm{T}}$,

$$\boldsymbol{A}_1 = \begin{bmatrix} -a & a & 0 & 1 \\ 0 & c & -m & 0 \\ 0 & m & -b & 0 \\ 0 & 0 & m & k \end{bmatrix},$$

$$\boldsymbol{A}_2 = \begin{bmatrix} -a & a & 0 & 1 \\ 0 & c & m & 0 \\ 0 & -m & -b & 0 \\ 0 & 0 & -m & k \end{bmatrix},$$

$$M_1(x(t)) = \frac{1}{2}(1 + x(t)/m),$$

$$M_2(x(t)) = \frac{1}{2}(1 - x(t)/m).$$

According the general defuzzifier method, the following dynamic global model of Eq.(4) can be obtained:

$$\dot{\boldsymbol{X}}(t) = \sum_{i=1}^{2} M_i(x(t)) \boldsymbol{X}(t). \qquad (5)$$

Remark 1 It is easily shown that fuzzy system (2) exactly represents the hyperchaotic Lü system (1). When $a = 36$, $c = 3$, $b = 20$, $k = 1$, $m = 40$, the hyperchaotic behaviour of Eq.(4) is shown in Fig.1. Figures 1(b), 1(c) and 1(d), respectively, show the projection for the phase diagrams of (v_0) in xyz, xyw and yzw spaces.

In the following, we will design an impulsive control model based on Eq.(4) for hyperchaos control of the hyperchaotic Lü system. Here, we only need to introduce impulsive jump items into each local subsystem of Eq.(4), and the impulsive T–S fuzzy model can be expressed as follows:

Plant Rule i**:** IF $z_i(t)$ is M_i

$$\text{THEN} \quad \begin{cases} \dot{\boldsymbol{X}}(t) = \boldsymbol{A}_i \boldsymbol{X}(t), & t \neq \tau_j, \\ \Delta \boldsymbol{X}|_{t=\tau_j} = \boldsymbol{X}(t_j^+) - \boldsymbol{X}(t_j^-) \equiv U(j, \boldsymbol{X}(t)), \ t = \tau_j, & i = 1, 2, \ldots, r, \quad j = 1, 2, \ldots \\ \boldsymbol{X}(t_0^+) = \boldsymbol{X}_0, \end{cases} \qquad (6)$$

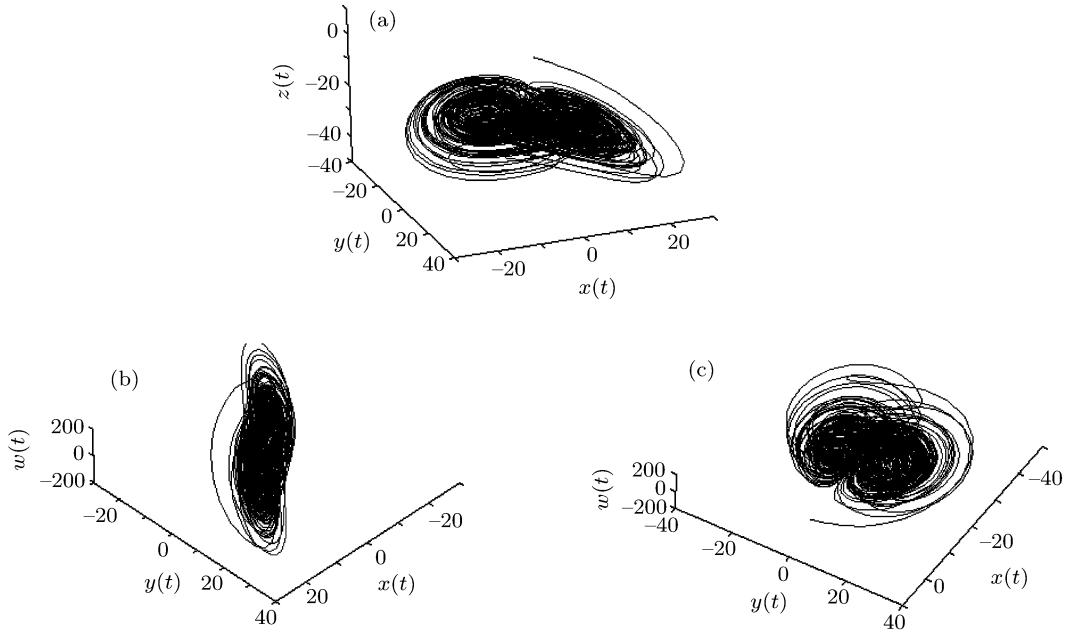


Fig.1. Projection for hyperchaotic attractor of Eq.(4): (a) xyz, (b) xyw and (c) yzw.

where $U(j, \boldsymbol{X})$ denotes a local impulsive plant in Eq.(4), $r = 2$. Generally we can let $U(j, \boldsymbol{X}(t)) = \boldsymbol{B}_i\boldsymbol{X}(t)$, and then the final output of Eq.(2) can be inferred as:

$$\begin{aligned}
&\dot{\boldsymbol{X}}(t) = \sum_{i=1}^{r} M_i(z(t))\boldsymbol{A}_i\boldsymbol{X}(t), \quad t \neq \tau_j,\\
&\Delta\boldsymbol{X}(t) = \sum_{i=1}^{r} M_i(z(t))\boldsymbol{B}_i\boldsymbol{X}(t), \quad t = \tau_j,\\
&\boldsymbol{X}(t_0^+) = \boldsymbol{X}_0.
\end{aligned} \tag{7}$$

Remark 2 When $z_i(t) = x(t)$, Eq.(7) represents the hyperchaotic Lü system as well as the hyperchaotic Rössler, Chua systems;[18] when $z_1(t) = x(t)$, $z_2(t) = x(t)$, $z_3(t) = y(t)$ and $z_4(t) = y(t)$, Eq.(7) represents the hyperchaotic Chen and Lorenz systems.[17] Therefore, Eq.(7) provides a unified impulsive control framework for a class of fuzzy hyperchaotic systems.

3. Impulsive fuzzy stability of hyperchaotic systems

In this section, we will focus on the stability of Eq.(7) and derive sufficient conditions for suppressing hyperchaos of Eq.(7) based on Lyapunov theory.

Theorem 1 Let a $n \times n$ matrix $\boldsymbol{P}$ be symmetric and positive definite. If there exist the positive constant scalars m, σ, γ and θ such that

(i)
$$\boldsymbol{P}\boldsymbol{A}_i + \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P} - \sigma\boldsymbol{P} \leq 0, \tag{8}$$

(ii)
$$(\boldsymbol{I} + \boldsymbol{B}_i)^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{I} + \boldsymbol{B}_i) - \theta\boldsymbol{P} \leq 0, \tag{9}$$

then the origin of impulsive fuzzy systems (7) is globally asymptotically stable if $\sigma(\tau_{j+1} - \tau_j) + \ln(\gamma\theta) \leq 0$ and $0 < \gamma < 1$ for any $j \in N$, $0 < \tau_{j+1} - \tau_j \leq d$.

Proof Let the candidate Lyapunov function be of the following form:

$$V(t, \boldsymbol{X}) = \frac{1}{2}\boldsymbol{X}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{X}.$$

For $t \neq \tau_j$, the derivative of $V(t, X)$ along the trajectory of Eq.(7) is

$$\begin{aligned}
D^+V(t, \boldsymbol{X}) =& \frac{1}{2}\sum_{i=1}^{r} M_i(x(t))\boldsymbol{X}^{\mathrm{T}}(\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_i)\boldsymbol{X}\\
\leq& \frac{1}{2}\sigma\boldsymbol{X}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{X} = \sigma V(t, \boldsymbol{X}).
\end{aligned} \tag{10}$$

This implies that

$$\begin{aligned}
V(t, \boldsymbol{X}) \leq& V(t_{j-1}^+, \boldsymbol{X})\exp(\sigma(t - t_{j-1})), \quad t \in (t_{j-1}, t_j],\\
& j = 1, 2, \ldots.
\end{aligned} \tag{11}$$

By Eq.(9), for $t = \tau_j$, we have

$$
\begin{aligned}
&V(\tau_j^+, \boldsymbol{X}(\tau_j) + \sum_{i=1}^{r} M_i(z(\tau_j))\boldsymbol{B}_i\boldsymbol{X}(\tau_j)) \\
=&\frac{1}{2}\boldsymbol{X}^{\mathrm{T}}(\tau_j)\left(\boldsymbol{I} + \sum_{i=1}^{r} M_i(z(\tau_j))\boldsymbol{B}_i\right)^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{I} + \sum_{i=1}^{r} M_i(z(\tau_j))\boldsymbol{B}_i)\boldsymbol{X}(\tau_j) \\
=&\frac{1}{2}\boldsymbol{X}^{\mathrm{T}}(\tau_j)\sum_{i=1}^{r} M_i(z(\tau_j))(\boldsymbol{I} + \boldsymbol{B}_i)^{\mathrm{T}}\boldsymbol{P}\sum_{i=1}^{r} M_i(z(\tau_j))(\boldsymbol{I} + \boldsymbol{B}_i)\boldsymbol{X}(\tau_j) \\
=&\frac{1}{2}\boldsymbol{X}^{\mathrm{T}}(\tau_j)\sum_{i=1}^{r} M_i(z(\tau_j))(\boldsymbol{I} + \boldsymbol{B}_i)^{\mathrm{T}}\boldsymbol{P}\sum_{i=1}^{r} M_i(z(\tau_j))(\boldsymbol{I} + \boldsymbol{B}_i)\boldsymbol{X}(\tau_j) \\
=&\frac{1}{2}\boldsymbol{X}^{\mathrm{T}}(\tau_j)\left\{\begin{array}{l}\displaystyle\sum_{i=1}^{r} M_i^2(z(\tau_j))(\boldsymbol{I} + \boldsymbol{B}_i)^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{I} + \boldsymbol{B}_i) + \\ \displaystyle\sum_{1\le i<k\le r} M_i(z(\tau_j))M_k(z(\tau_j))((\boldsymbol{I} + \boldsymbol{B}_i)^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{I} + \boldsymbol{B}_k) + (\boldsymbol{I} + \boldsymbol{B}_k)^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{I} + \boldsymbol{B}_i))\end{array}\right\}\boldsymbol{X}(\tau_j) \\
\le&\frac{1}{2}\boldsymbol{X}^{\mathrm{T}}(\tau_j)\left\{\begin{array}{l}\displaystyle\sum_{i=1}^{r} M_i^2(z(\tau_j))(\boldsymbol{I} + \boldsymbol{B}_i)^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{I} + \boldsymbol{B}_i) + \\ \displaystyle\sum_{1\le i<k\le r} M_i(z(\tau_j))M_k(z(\tau_j))((\boldsymbol{I} + \boldsymbol{B}_i)^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{I} + \boldsymbol{B}_i) + (\boldsymbol{I} + \boldsymbol{B}_k)^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{I} + \boldsymbol{B}_k))\end{array}\right\}\boldsymbol{X}(\tau_j) \\
=&\frac{1}{2}\boldsymbol{X}^{\mathrm{T}}(\tau_j)\sum_{i=1}^{r} M_i(z(\tau_j))(\boldsymbol{I} + \boldsymbol{B}_i)^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{I} + \boldsymbol{B}_i)\boldsymbol{X}(\tau_j) \le \theta V(\tau_j, \boldsymbol{X}).
\end{aligned} \tag{12}
$$

When $j = 1$ in inequality (11), then for any $t \in (t_0, t_1]$

$$
V(t_1, \boldsymbol{X}) \le V(t_0, \boldsymbol{X})\exp(\sigma(t_1 - t_0)). \tag{13}
$$

By Eqs.(12) and (13), we have

$$
V(t_1^+, \boldsymbol{X}) \le \theta V(t_1, \boldsymbol{X}) \le \theta V(t_0, \boldsymbol{X})\exp(\sigma(t_1 - t_0)).
$$

In the same way, for any $t \in (t_1, t_2]$ we have

$$
\begin{aligned}
&V(t, \boldsymbol{X}) \le V(t_1^+, \boldsymbol{X})\exp(\sigma(t - t_1)) \\
\le&\theta V(t_0, \boldsymbol{X})\exp(\sigma(t_1 - t_0))\exp(\sigma(t - t_1)) = \theta V(t_0, \boldsymbol{X})\exp(\sigma(t - t_0))
\end{aligned}
$$

and $V(t_2^+, \boldsymbol{X}) \le \theta V(t_2, \boldsymbol{X}) \le \theta^2 V(t_0, \boldsymbol{X})\exp(\sigma(t - t_0))$.
In general, for any $t \in (t_j, t_{j+1}]$ one finds that

$$
V(t, \boldsymbol{X})) \le \theta^j V(t_0, \boldsymbol{X})\exp(\sigma(t - t_0)). \tag{14}
$$

By $\sigma(\tau_{j+1} - \tau_j) + \ln(\gamma\theta) \le 0$, for any $t \in (t_j, t_{j+1}]$ we have

$$
\begin{aligned}
V(t, \boldsymbol{X}) \le&\theta^j V(t_0, \boldsymbol{X})\exp(\theta(t - t_0)) \\
\le&V(t_0, \boldsymbol{X})\left[\exp(-\theta(t_1 - t_0))/\gamma\right]\left[\exp(-\theta(t_2 - t_1))/\gamma\right]\ldots \\
&\left[\exp(-\theta(t_j - t_{j-1}))/\gamma\right]\left[\exp(\theta(t - t_0))/\gamma\right] \\
\le&(1/\gamma)^j\, V(t_0, \boldsymbol{X})\exp(-\theta(t_j - t_0))\exp(\theta(t - t_0)) \\
\le&(1/\gamma)^j\, V(t_0, \boldsymbol{X})\exp(\theta d).
\end{aligned} \tag{15}
$$

This leads to

$$
\|\boldsymbol{X}(t)\| \le \lambda_{\max}(\boldsymbol{P})V(t_0, \boldsymbol{X})\exp(\alpha d)\,(1/\gamma)^j\,, \tag{16}
$$

where $\lambda_{\max}(\boldsymbol{P})$ is largest eigenvalue of $\boldsymbol{P}$. From Eq.(16) it follows that the conclusion holds.

Remark 3 Obviously calculating $\boldsymbol{P}$ only depends on several simple linear matrix inequalities with respect to $\boldsymbol{P}$; hence, our result is less conservative in comparison with the existing ones.[21,22] Moreover, $\boldsymbol{B}_i$ may be chosen as very simple diagonal matrixes, and then solving the linear matrix inequalities will become easier.

Remark 4 Since Eq.(7) represents a class of hyperchaotic systems, such as the hyperchaotic Rössler, Chen, Lorenz, Chua systems and so on, our result is general.

4. Numerical simulations

In this section numerical simulations are given to verify the effectiveness of impulsive fuzzy control for achieving the stability of hyperchaotic systems. The fourth-order Runge–Kutta integration method is used to solve the system with time step equal to 0.001 and the parameters of the hyperchaotic Lü system are chosen as $a = 36$, $c = 3$, $b = 20$, $k = 1$, $m = 40$ so that hyperchaotic behaviour is exhibited.

In our simulations, the initial values are $\boldsymbol{X}_0 = [-5\ 9,\ 8,\ 5]$ and we choose $\sigma = 50$, $\theta = 50$, $\gamma = 0.001$, $\boldsymbol{B}_1 = -\mathrm{diag}([0.5,\ 0.6,\ 0.7,\ 0.6])$, $\boldsymbol{B}_2 = -\mathrm{diag}([0.5,\ 0.7,\ 0.7,\ 0.5])$. Using the LMI toolbox provided by MATLAB, we obtain

$$\boldsymbol{P} = \begin{bmatrix} 0.0128 & 0.0051 & 0 & -0.0012 \\ 0.0051 & 0.0159 & 0 & -0.0045 \\ 0 & 0 & 0.0145 & 0 \\ -0.0012 & -0.0045 & 0 & 0.0211 \end{bmatrix}.$$

And then we have $0 < \tau_j - \tau_{j-1} \leq -\ln(\gamma\theta)/\sigma = 0.0599$. Therefore, in our simulation impulsive instance is selected as a fixed length 0.015. Figure 2(a) shows the behaviours of the state $(x,\ y,\ z,\ w)$ of the controlled hyperchaotic system (7). Figures 2(b), 2(c) and 2(d), respectively, show the projection for the phase diagrams of the controlled hyperchaotic system in xyz, xyw and yzw spaces.

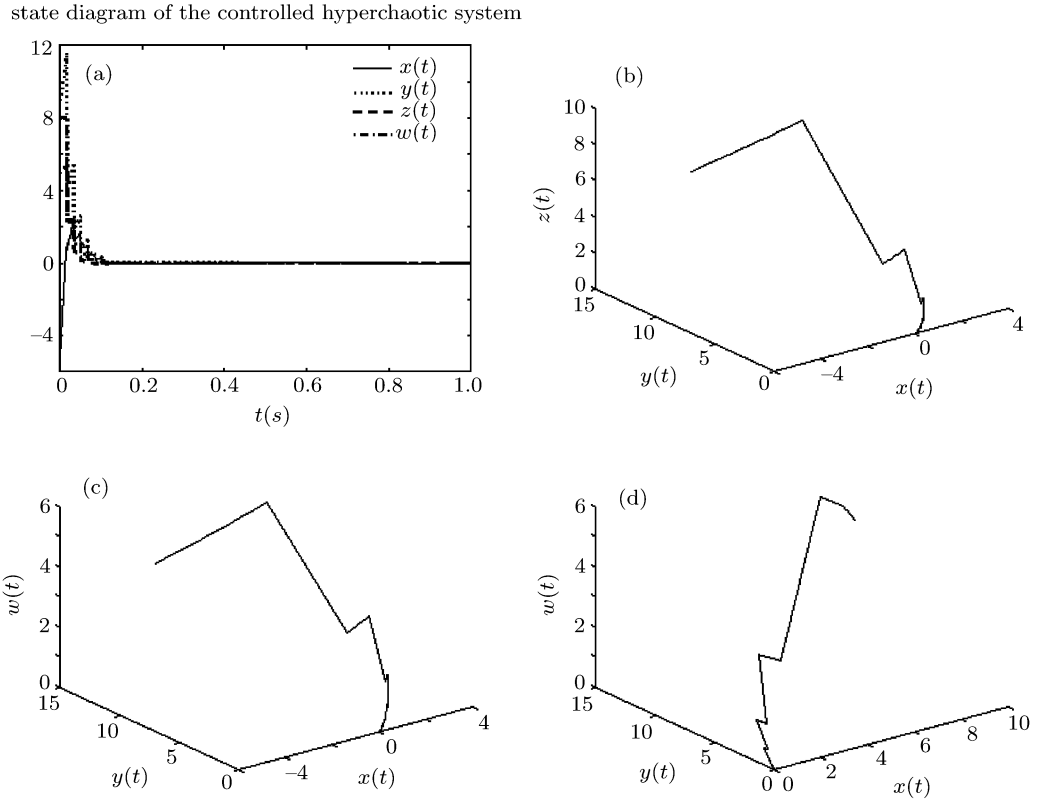


Fig.2. (a) The behaviours of the state $(x,\ y,\ z,\ w)$ of the controlled hyperchaotic system (7). Figure 2(b), 2(c) and 2(d) are respectively the phase diagrams of the controlled hyperchaotic system in xyz, xyw and yzw spaces.

5. Conclusions

In this paper, we design an impulsive T–S fuzzy framework, which generalizes the existing fuzzy hyperchaotic systems, for controlling Lü hyperchaos based on impulsive control and the T–S fuzzy model. Sufficient conditions for achieving hyperchaos control of the hyperchaotic Lü system are derived by using Lyapunov stability theory in the form of a linear matrix inequality. The obtained results are also applicable to a class of the hyperchaotic systems represented by the T–S fuzzy model, such as the hyperchaotic Rössler, Chen, Lorenz, Chua systems and so on. Thus our results are general. Moreover, our results are less conservative in comparison with existing ones. Finally, numerical simulations are provided to demonstrate the effectiveness of the proposed control techniques.

References

[1] Wang H O 1996 *IEEE Symposium on Circuits Syst.* Atlanta p209
[2] Tanaka T K, Ikeda T and Wang H O 1998 *IEEE Trans. Circuits Systems* I **45** 1021
[3] Calvo O 1998 *Int. J. Bifurcat. Chaos* **8** 1743
[4] Guan X 2003 *Fuzzy Sets Syst.* **139** 81
[5] Lian K Y, Chiu C S, Chiang T S and Liu P 2001 *IEEE Trans. Fuzzy Syst.* **9** 539
[6] Wang Y W, Guan Z H and Wang H O 2005 *Phys. Lett.* A **339** 325
[7] Liu X W and Zhong S M 2007 *Phys. Lett.* A **370** 260
[8] Chang W and Jin B P 2003 *Info. Sci.* **151** 227
[9] Cafagna D and Grassi G 2003 *Int. J. Bifur. Chaos* **13** 2889
[10] Udaltsov V S, Goedgebuer J P, Larger L, Cuenot J B, Levy P and Rhodes W T 2003 *Opt. Spectrosc.* **95** 114
[11] Shahverdiev E M, Nuriev R A, Hashimov R H and Shore K A 2004 arXiv:nlin.CD/0404053 29
[12] Hsieh J Y, Hwang C C, Wang A P and Li W J 1999 *Int. J. Control* **72** 882
[13] Rössler O E 1979 *Phys. Lett.* A **71** 155
[14] Li Y, Tang W K S and Chen G 2005 *Int. J. Circuit Theory Appl.* **33** 235
[15] Li Y, Tang W K S and Chen G 2005 *Int. J. Bifur. Chaos* **15** 3367
[16] Zhou X B, Wu Y, Li Y and Xue H Q 2008 *Appl. Math. Comp.* **203** 80
[17] Zhao Y, Zhang H G and Liu D R 2008 *Proc. 7th World Congress Intell. Cont. Auto.* Chongqing, China p19
[18] Zhang H B, Liao X F and Yu J B 2005 *Chaos, Solitons and Fractals* **26** 835
[19] Cai G L, Zheng S and Tian L X 2008 *Chin. Phys.* B **17** 2412
[20] Zhang Q J and Lu J 2008 *Chin. Phys.* B **17** 492
[21] Chen A, Lu J and Yu S 2006 *Physica* A **364** 103
[22] Yassen M T 2008 *Nonlinear Analysis* **68** 3592
[23] Haeri M and Dehghani M 2008 *Choas, Solitons and Fractals* **38** 120