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A method of estimating initial conditions of coupled map lattices based on time-varying symbolic dynamics*

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A novel computationally efficient algorithm in terms of the time-varying symbolic dynamic method is proposed to estimate the unknown initial conditions of coupled map lattices (CMLs). The presented method combines symbolic dynamics with time-varying control parameters to develop a time-varying scheme for estimating the initial condition of multi-dimensional spatiotemporal chaotic signals. The performances of the presented time-varying estimator in both noiseless and noisy environments are analysed and compared with the common time-invariant estimator. Simulations are carried out and the obtained results show that the proposed method provides an efficient estimation of the initial condition of each lattice in the coupled system. The algorithm cannot yield an asymptotically unbiased estimation due to the effect of the coupling term, but the estimation with the time-varying algorithm is closer to the Cramer–Rao lower bound (CRLB) than that with the time-invariant estimation method, especially at high signal-to-noise ratios (SNRs).

Keywords: coupled map lattices, symbolic dynamics, initial condition estimation
PACC: 0545

1. Introduction

Chaotic signals, generated from chaotic systems, have received much attention recently, and they are widely used in many engineering contexts such as physiology, weather forecasting, neural networks, economics and information processing.[1−6] Recent studies have mainly focused on the limit of the forward direction of chaotic maps while the initial process of iteration is always ignored.

Chaos provides a good model for many applications, but all the available information contained in the practical signals is not fully exploited. The reason is that chaos is basically a temporal phenomenon where the spatial structure is assumed to be unchanged. However, the spatial effects from the nearby area cannot be ignored since many signals are generated from the spatiotemporal dynamic system and correlated with each other. When a system is spatially extended with many effective degrees of freedom, spatiotemporal chaos should be considered.[3] To investigate spatiotemporal nonlinear dynamical behaviour, coupled map lattices (CMLs) are often used to model real systems. This problem has been of great interest in recent years.[1,3,5,6] Consequently, signal processing is always reduced to a problem of estimating the dynamical information of CMLs. If the map function of CMLs is known, the problem of signal estimation is equivalent to estimating the initial condition of the CML system in principle. Once it is available, all the information embedded in the system can be estimated. Therefore, the estimation of the unknown initial condition of CMLs is very important for spatiotemporal signal processing. Obviously, the higher the estimation accuracy is, the better the description of the system will be. For example, the human brain can be regarded as a complex time-varying CML system from the viewpoint of neuron dynamics. In the human brain, even a single neuron and a small ensemble of neurons are known to exhibit spatiotemporal chaotic behaviour.[7] With appropriate modelling, the inverse problem of the electroencephalograph (EEG) is reduced to the estimation of the initial conditions of the CMLs derived from the EEG time series, which may be of great significance and clinical importance in retrieving information on active sources inside the brain based on scalp potentials like EEG.

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Owing to the extremely sensitive characteristics of the initial conditions and the parameters of chaos, initial conditions which are close to each other will separate exponentially, which causes error propagation. In order to avoid error propagation and improve the estimation accuracy, some backward direction iteration technologies, such as dynamical programming implementation of maximum likelihood (ML) estimation, recursive ML algorithms and the halving method have been extensively studied.[8−13] On the other hand, the development of symbolic dynamics brings us some new light. In Ref.[14], a symbolic dynamics-based method was presented to provide an asymptotically unbiased estimation and attain the Cramer–Rao lower bound (CRLB) with a high signal-to-noise ratio (SNR). These algorithms are all based on the 1-D chaotic map, in which spatial effects among the signals are ignored. When a large number of individual chaotic lattices are coupled with CMLs, however, the effect of spatial coupling among the signals needs to be considered and the spatiotemporal system becomes a time-varying system. Both the control parameter and the coupling strength change with time at each lattice. Therefore, it is rather difficult to recover the initial condition of each lattice by using the 1-D chaotic map based on the algorithm directly. Only a coarse estimation based on the statistical property of the whole lattice can be recovered.[15,16]

To provide an optimal estimation of the unknown initial conditions for each lattice of the CML, an algorithm is presented in the present paper, based on the time-varying control parameter of the logistic map, where the information provided by the coupling term is fully employed. A time-varying symbolic dynamics-based efficient method is proposed for estimating the initial conditions of the CMLs for spatiotemporal chaotic signal processing, where the backward iteration algorithm is employed. Several computed results are obtained, which show that the proposed method is more efficient for estimation of the initial condition for CMLs. The performance in the presence of white Gaussian noise is also evaluated. The presented method cannot achieve asymptotically unbiased estimation, but it can avoid the influence of noise to some extent and approach the CRLB at high SNRs. Therefore, our approach is more applicable to the description of the dynamical properties of practical physical systems.

The remainder of the present paper is organized as follows. In Section 2 a computationally efficient algorithm with symbolic dynamics is developed and a time-varying control parameter for estimating the initial conditions of the spatiotemporal chaotic signal is defined. In Section 3, the problem of selecting the control parameter and the coupled strength is discussed and compared with different cases to investigate the behaviour of the algorithm. In Section 4 the performances of the time-varying and the time-invariant approaches are analysed by using several simulation results. The estimation errors are evaluated in noiseless and noisy environments, and the efficiency is addressed by using the CRLB for the proposed time-varying algorithm. Finally, conclusions drawn from the present study are given in Section 5.

2. Model and method

Symbolic dynamics is considered to be a coarse-grained description of trajectories of a general class of systems, which keeps both robust and statistical properties of the system invariant.[17] It provides a model for the orbits of the dynamical system to pass through a space of sequences. Obviously, the symbolic sequences depend on the initial condition of the trajectory. Based on the connection between the symbolic sequence and the initial condition of the CML system, an algorithm for estimating the unknown initial conditions of the CML system is derived.

Given a data set $\boldsymbol{X}$, the symbol sequence is achieved by quantifying $\boldsymbol{X}$ into a symbol. Let $\boldsymbol{E} = \{E_0, E_1, \ldots, E_{q-1}\}$ be a finite disjoint partition of a phase space $\boldsymbol{X}$, i.e. $\cap_{i=0}^{q-1} E_i = \boldsymbol{\Phi}$, and $\cup_{i=0}^{q-1} E_i = \boldsymbol{X}$. The time series $S = \{0, 1, 2, \ldots, q-1\}$ is defined as a symbol set of the given system. Consequently, if data point $x(n)$ at time n of the system trajectory is the ith element of the partition, we can assign a symbol $s(n) = i$, $i \in S$ by following some partition rules. Since the 1-D trajectory is mapped onto a logistic map, its phase space can be divided into two disjoint partitions $E_0 = [0, 0.5]$ and $E_1 = (0.5, 1]$. So the symbolic sequence is obtained according to the following relationship:

$$s(n) = \begin{cases} 0, & x(n) \in E_0, \\ 1, & x(n) \in E_1. \end{cases} \tag{1}$$

Thus, for the given 1-D dynamical function, any orbit starting from $x(0)$ can be encoded as a symbolic sequence $S = \{s(0), s(1), \ldots, s(q-1)\}$. Let f_i be the dynamical function of the 1-D chaotic map, assumed

to be invertible: the invertible function of the 1-D chaotic map is denoted by f_i^{-1}. Since the map function is one-to-one for generating a partition, the initial condition $x(0)$ of the CML system can be determined uniquely as $n \to \infty$[14]

$$x(0) = \lim_{n\to\infty} f_{s_{(0)}}^{-1} \circ f_{s_{(1)}}^{-1} \circ f_{s_{(2)}}^{-1} \circ \dots f_{s_{(n-1)}}^{-1}(x(n)), \quad (2)$$

where the symbol "$\circ$" denotes the intersection of the map. If the map is ergodic, then there always exists a generating partition. With the increase of n, equivalently, successive iterations of the map will provide increasingly more information about $x(0)$. Consequently, an average rate of the information production, given by the metric entropy $h(f)$ for generating partitions, is obtained and expressed as[14,15]

$$-\frac{1}{N}\sum_{n=0}^{N-1} \ln \frac{\left|\cap_{i=0}^{n+1} f^{-i} E_{\{s_{(i)}\}}\right|}{\left|\cap_{i=0}^{n} f^{-i} E_{\{s_{(i)}\}}\right|} \approx h(f), \quad (3)$$

where $f^{-i}E_{\{s_{(i)}\}}$ represents the set points that are mapped onto the set $E_{\{s_{(i)}\}}$ after i iterations, $\cap_{i=0}^{n} f^{-i}E_{\{s_{(i)}\}}$ denotes the intersection of $f^{-i}E_{\{s_{(i)}\}}$ during n step iterations, and N is the total length of the iteration. The estimate error can therefore be bounded by

$$|x(0) - \hat{x}(0)| \le \left|\cap_{i=0}^{N-1} f^{-i} E_{\{s_{(i)}\}}\right| \approx D \cdot \mathrm{e}^{-h(f)N}, \quad (4)$$

where the constant D denotes the attractor size. Expression (4) indicates that the estimation error converges exponentially by using the proposed backward iteration method.

If there is no system coupling, the initial condition estimation for the CMLs is the same as that for a 1-D individual map based on expression (2). However, in practical applications, large numbers of individual lattices evolved by the 1-D map are usually coupled with each other to construct a CML system. When multiple 1-D maps are coupled with the CMLs, both the control parameters and coupling strength can change at each lattice of the system. For the CML system with multi-channel signals, it is a time-varying system and the effect of the spatial coupling among the lattices needs considering appropriately. To overcome this limitation, the time-varying characteristics of the CMLs need describing appropriately so that an optimal algorithm can be developed to estimate the unknown initial conditions of the spatiotemporal chaotic signal.

In the present study, the well-known global coupled map (GCM) is used as a model to analyse the problem of estimating the initial condition of the CML system, which is defined as[5]

$$\begin{aligned} & x_i(n+1) \\ &= (1-\varepsilon) f[x_i(n)] + \frac{\varepsilon}{L-1}\sum_{j=1, j\neq i}^{L} f[x_j(n)], \ \varepsilon \neq 0, \end{aligned} \quad (5)$$

where n is a discrete time step while i denotes the site index. The logistic map $f(x) = \lambda x(1-x)$ is used here in this paper in which λ represents the control parameter and ε denotes the coupling strength of the CMLs. L is the lattice size of the model. Without loss of generality, it is assumed that $\varepsilon \neq 0$ for the CML system. Moreover, the periodical boundary condition is assumed to be $x(i+L) = x(i)$.

Let $g(n)$ denote the coupling term among the lattices of the GCM system, which is described as

$$g(n) = \frac{\varepsilon}{L-1}\sum_{j=1, j\neq i}^{L} f[x_j(n)], \quad \varepsilon \neq 0\,. \quad (6)$$

Based on symbolic dynamic analysis, the true value of the initial condition $x_i(n)_{\mathrm{GCM}}$ of the GCM system at the ith lattice can be obtained as

$$\begin{aligned} & x_i(n)_{\mathrm{GCM}} \\ &= \frac{1}{2}\left[1 + (2s_i(n)-1)\sqrt{1 - \frac{4x_i(n+1)}{(1-\varepsilon)\lambda} + \frac{4g(n)}{(1-\varepsilon)\lambda}}\right], \\ & (\varepsilon \neq 1), \end{aligned} \quad (7)$$

where $s_i(n)$ denotes the symbolic time series of the system.

For the 1-D based signal estimation scheme, called the time-invariant symbolic dynamic method, i.e. the algorithm developed for individual 1-D chaotic maps without spatial coupling,[14] we have the estimation as

$$\begin{aligned} & \hat{x}_i(n)_{1-\mathrm{D}} \\ &= f_{s_i(n)}^{-1}[x_i(n+1)] \\ &= \frac{1}{2}\left[1 + (2s_i(n)-1)\sqrt{1 - 4x_i(n+1)/\lambda}\right]. \end{aligned} \quad (8)$$

Defining $y_i(n)_{\mathrm{GCM}} = \sqrt{1 - \frac{4x_i(n+1)}{(1-\varepsilon)\lambda} + \frac{4g(n)}{(1-\varepsilon)\lambda}}$ and $\hat{y}_i(n)_{1-\mathrm{D}} = \sqrt{1 - \frac{4x_i(n+1)}{\lambda}}$, and comparing expression (7) with expression (8), it can be found that the difference between the true value of the system and the estimation by using the 1-D based time-invariant method is reduced to that between $y_i(n)_{\mathrm{GCM}}$ and $\hat{y}_i(n)_{1-\mathrm{D}}$. In fact, both $x_i(n+1)$ and the coupling

term $g(n)$ are relevant to $x_i(n)$. The $\hat{y}_i(n)_{1-\mathrm{D}}$, derived from the extension of the 1-D chaotic map directly, does not contain the effect of coupling, thereby causing a coarse estimation. To improve the performance of the estimation, $\hat{y}_i(n)_{1-\mathrm{D}}$ needs modifying so that the dynamical characteristics of the system can be appropriately captured. Comparing $y_i(n)_{\mathrm{GCM}}$ with $\hat{y}_i(n)_{1-\mathrm{D}}$, the following relationship can be concluded

$$\begin{aligned} &x_i(n+1) < \frac{g(n)}{\varepsilon} = \eta_n, \quad y_i(n)_{\mathrm{GCM}} > \hat{y}_i(n)_{1-\mathrm{D}}, \\ &x_i(n+1) = \frac{g(n)}{\varepsilon} = \eta_n, \quad y_i(n)_{\mathrm{GCM}} = \hat{y}_i(n)_{1-\mathrm{D}}, \\ &x_i(n+1) > \frac{g(n)}{\varepsilon} = \eta_n, \quad y_i(n)_{\mathrm{GCM}} < \hat{y}_i(n)_{1-\mathrm{D}}. \end{aligned} \tag{9}$$

The middle equation above indicates that the effect of the coupling term and the change of the corresponding control parameter counteract each other. For this case, the results based on expressions (7) and (8) are equivalent to each other. However, this is not a general case because the signal coupling of the spatiotemporal system ubiquitously exists in practical applications and these two terms. Therefore, we focus on the common case $y_i(n)_{\mathrm{GCM}} \neq \hat{y}_i(n)_{1-\mathrm{D}}$.

From expressions (7) and (8), it can be clearly seen that when a large number of 1-D chaotic maps are coupled with the GCM system, the coupling term plays an important role in estimating the initial conditions. To obtain a better estimation for each lattice, a time-varying algorithm is therefore developed so that we can fully use the information provided by the coupling term.

In general, the exact value of the coupling term for a practical system is unknown. However, when the coupling strength ε is not very strong, the influence of the term ε^2 can be ignored so that $f[x_j(n)]$ is substituted by $x_j(n+1)$ coarsely.[3] Thus, expression (5) can be approximately expressed as

$$\begin{aligned} &x_i(n+1) \\ &\approx (1-\varepsilon) f[x_i(n)] + \frac{\varepsilon}{L-1} \sum_{j=1, j\neq i}^{L} x_j(n+1). \end{aligned} \tag{10}$$

The magnitudes of both $y_i(n)_{\mathrm{GCM}}$ and $\hat{y}_i(n)_{1-\mathrm{D}}$ represent the relationship between the real data and the estimations based on the time-invariant algorithm. For the GCM system, if we consider mainly the effect of the coupling term and define a threshold as

$$\eta_n = \frac{g(n)}{\varepsilon} \approx \frac{1}{L-1} \sum_{j=1, j\neq i}^{L} x_j(n+1), \tag{11}$$

then a symbolic time series can be defined as

$$\begin{aligned} &\gamma_i(n+1) \\ &= \begin{cases} 1, x_i(n+1) > \eta_n \approx \dfrac{1}{L-1} \displaystyle\sum_{j=1, j\neq i}^{L} x_j(n+1), \\ 0, x_i(n+1) < \eta_n \approx \dfrac{1}{L-1} \displaystyle\sum_{j=1, j\neq i}^{L} x_j(n+1). \end{cases} \end{aligned} \tag{12}$$

Expression (12) describes the relationship between the current lattice value and its adjacent ones, which coarsely reflects the value of the coupling term and provides a reference for the algorithm modification. Correspondingly, the time-varying control parameter of the CMLs is used as[18]

$$\lambda_i(n) = \lambda - [2\gamma_i(n+1) - 1]\mathrm{e}^{-c_1} \sin(\mathrm{e}^{-c_2 x_i(n)}), \tag{13}$$

where λ is the control parameter in the individual logistic map, and $\lambda = 4$ is selected for the logistic map. Both c_1 and c_2 are the parameters to be determined, and they vary from 1.0 to 10.0; $x_i(n)$ denotes the lattice value of the model. The time-varying control parameter $\lambda_i(n)$ changes exponentially, which is similar to the trend of change in estimation error. Therefore, the time-varying $\lambda_i(n)$ helps avoid an exponential increase in computational error to some extent. Correspondingly, expression (5) can be further expressed as

$$\begin{aligned} &\hat{x}_i(n)_{\mathrm{GCM}} \\ &= f^{-1}_{s_i(n)}[x_i(n+1)] \\ &= \frac{1}{2}\left[1 + [2s_i(n) - 1]\sqrt{1 - 4x_i(n+1)/\lambda_i(n)}\right]. \end{aligned} \tag{14}$$

Note that the estimate algorithm based on expression (8) can be regarded as the time-invariant symbolic dynamics-based algorithm, while expression (14) can be used as the time-varying symbolic dynamics-based method. Based on expressions (8) and (14), the relationship between the recovered data and the real data is derived as shown in Table 1, in which the $x_i(n)_{\mathrm{GCM}}$ represents the real data of the GCM system and $\hat{x}_i(n)_{1-\mathrm{D}}$ denotes the recovered data by expression (8), while $\hat{x}_i(n)_{\mathrm{GCM}}$ represents the recovered data by expression (14). From Table 1, it can be seen that this change is feasible. The relationship between estimator $\hat{x}_i(n)_{\mathrm{GCM}}$ and $\hat{x}_i(n)_{1-\mathrm{D}}$ stays the same as that between the system's true value $x_i(n)_{\mathrm{GCM}}$ and the estimation from $\hat{x}_i(n)_{1-\mathrm{D}}$.

Table 1. Relationship between estimated data and real data.

$\gamma_i(n+1)$	$s_i(n)$	$\lambda_i(n)$	comparison $x_i(n)_{\mathrm{GCM}}$ with $\hat{x}_i(n)_{1-\mathrm{D}}$, $\hat{x}_i(n)_{\mathrm{GCM}}$ with $\hat{x}_i(n)_{1-\mathrm{D}}$
0	0	$\lambda_i(n) > \lambda$	$x_i(n)_{\mathrm{GCM}} < \hat{x}_i(n)_{1-\mathrm{D}}$, $\hat{x}_i(n)_{\mathrm{GCM}} < \hat{x}_i(n)_{1-\mathrm{D}}$
0	1	$\lambda_i(n) > \lambda$	$x_i(n)_{\mathrm{GCM}} > \hat{x}_i(n)_{1-\mathrm{D}}$, $\hat{x}_i(n)_{\mathrm{GCM}} > \hat{x}_i(n)_{1-\mathrm{D}}$
1	0	$\lambda_i(n) < \lambda$	$x_i(n)_{\mathrm{GCM}} > \hat{x}_i(n)_{1-\mathrm{D}}$, $\hat{x}_i(n)_{\mathrm{GCM}} > \hat{x}_i(n)_{1-\mathrm{D}}$
1	1	$\lambda_i(n) < \lambda$	$x_i(n)_{\mathrm{GCM}} < \hat{x}_i(n)_{1-\mathrm{D}}$, $\hat{x}_i(n)_{\mathrm{GCM}} < \hat{x}_i(n)_{1-\mathrm{D}}$

The proposed time-varying symbolic dynamics-based method significantly reduces the biases induced by the constant parameter λ and properly makes a balance between the effect of the local map and the coupling term, which enables us to capture the spatiotemporal dynamical information provided by the system successfully. So the proposed time-varying parameter-based method is more suitable for describing the real system and gives better estimation behaviour.

3. Selection of parameters

It is necessary to carefully select the time-varying parameters $\lambda_i(n)$ according to the dynamical characteristics of the considered system so that a better estimation can be obtained. Without loss of generality, the initial conditions of the GCM system are assumed to be uniformly distributed within [0, 1] in steps of 0.01. The total number of lattices L is 100. All lattices of the GCM are considered according to expression (5) and each value of the lattices is converted into a symbol with binary 0 or 1 to develop the algorithm for estimating the initial conditions of the GCM.

To evaluate the performance of the estimated result using different model parameters, the bias of the estimated values is defined as

$$e(n) = x(n) - \hat{x}(n), \quad n = 1, \ldots, L. \tag{15}$$

First, let the length of iteration $N = 30$ and the coupling strength $\varepsilon = 0.1$. The biases with two specified parameters in expression (13) are shown in Fig.1, by evaluating the performance in two cases, i.e. case I with $c_1 = 1$ and $c_2 = 1$ and case II with $c_1 = 2$ and $c_2 = 2$. It can be seen from this example that the selection of the parameters of c_1 and c_2 significantly affects the result of the estimated bias. Different systems may have different dynamical characteristics. Therefore, the selection of c_1 and c_2 depends on the characteristics of the considered system. To achieve a better estimation result, parameters c_1 and c_2 both need carefully selecting based on the system under consideration. If the dynamical property of the considered system changes slowly, then larger c_1 and/or c_2 should be selected. Otherwise, smaller c_1 and/or c_2 should be adopted.

It is observed from Fig.1 that the estimation errors are distributed symmetrically at a value of 0.5. If $x(n) \in E_0$, the estimated data are slightly lower than the given values, while for $x(n) \in E_1$ the estimated data are slightly higher than the given values. The maximum estimation error happens at a value of 0.5. The occurrence of the symbol errors is the main reason for inducing the estimation error as discussed in Ref. [14]. More specifically, if the initial conditions lie at about 0.5 which is close to the threshold of the symbolization, it is much easier to use totally different inverse mapping due to the symbol errors caused by the coupling term. If this happens, the estimation error would be much larger. For case I, i.e. a faster time-varying control parameter is taken, we observe that the estimation is very small even if $x(n) \to 0.5$ (only about 0.03) compared with that for case II (about 0.11), in which a slower time-varying control parameter is used. This implies that a faster time-varying $\lambda_i(n)$ can characterize the properties of this kind of system much better.

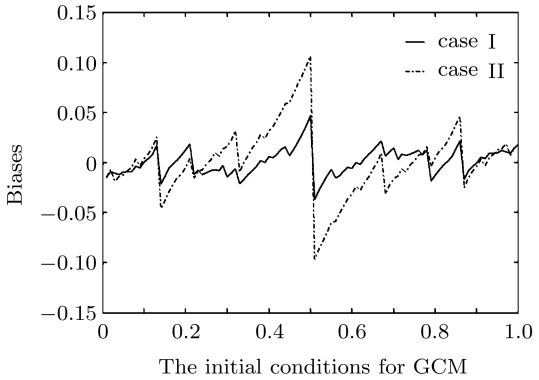


Fig.1. The biases between the estimated and the given data under different parameters. Case I: $c_1 = 1$, $c_2 = 1$ and case II: $c_1 = 2$, $c_2 = 2$.

Next, we investigate the influence of coupling strength on the performance for estimating the initial

conditions of the GCM when using both time-varying and time-invariant methods. Without loss of generality, the initial conditions of GCM are set to be the Gaussian distribution with $x(n) \sim N(0.5, 0.05^2)$ and the parameters of the coupled map for the initial signal are set as $N = 30$ and $L = 20000$. The mean relative error (MRE) is used for evaluating the performance of the presented method, which is defined as

$$\mathrm{MRE} = \frac{\sum_{k=1}^{L} [x(k) - \hat{x}(k)]^2}{\sum_{k=1}^{L} x^2(k)}, \tag{16}$$

where $x(k)$ is the true value and $\hat{x}(k)$ is the estimation. The simulation result is shown in Fig.2 by comparing the results obtained using the proposed time-varying method with those using the common time-invariant method, with the coupling strength changing from 0.1 to 0.9. Figure 2 shows that for the considered circumstance, i.e. the coupling strength changing from 0.1 to 0.9, the MRE of the time-varying scheme is much smaller than that obtained by using the conventional time-invariant method over a wide range of coupling strengths. It also indicates that the MRE increases with increasing coupling strength since the proposed scheme is derived based on the weakly coupled system as mentioned in Section 2.

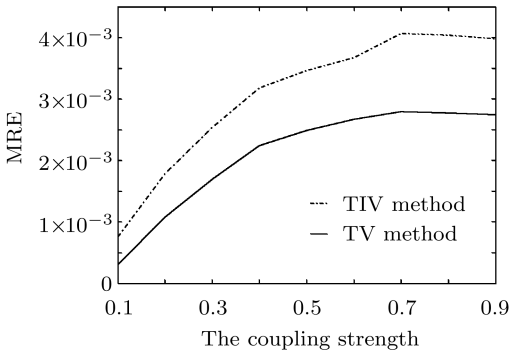


Fig.2. Comparison between the MREs obtained using the time-invariant (TIV) and those using the time-varying (TV) methods with different coupling strengths.

4. Performance characteristics

4.1. Performance without noise

Figure 3 represents the distributions of the given data and the estimated data obtained by using time-invariant and time-varying symbolic dynamics-based methods for the GCM system in a noiseless environment separately. It shows that the behaviour of the proposed time-varying scheme is much better than that of the time-invariant algorithm. Similarly to Fig.1, we observe that the estimation errors are also symmetrically distributed at a value of 0.5 and reach a maximum at the threshold of 0.5. For the time-varying method, even if $x(n) \to 0.5$, the absolute error is still controlled within 0.015, showing its fitness better than that of the given signal.

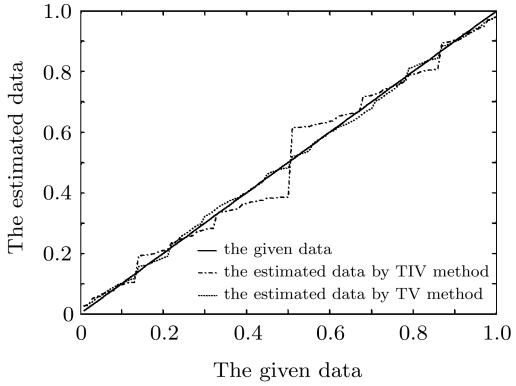


Fig.3. Estimation results obtained by using time-invariant (TIV) and time-varying (TV) methods.

4.2. Performance in white Gaussian noise

In many engineering applications, the existence of noise may cause poor performance in practice. The situation may be worse when the SNR is too low. It is necessary to investigate the robustness against noise when the unknown initial condition of the CML system is estimated. Here we evaluate the performance with one channel signal plus white Gaussian noise sequences. The data have the following form:

$$\begin{aligned} &y_i(n) = x_i(n) + w_i(n), \\ &n = 0, 1, \ldots, N-1;\ i = 0, 1, \ldots, L-1, \end{aligned} \tag{17}$$

where $x_i(n)$ is a logistic map sequence generated by iterating $x_i(n) \in [0, 1]$ and $w_i(n)$ denotes a stationary zero-mean white Gaussian sequence with variance σ^2.

To start with, we discuss the influence of the finite iteration times N on the performance of the method. The absolute bias between the estimated data and the real ones is expressed as a function of N at different SNR levels determined from Monte Carlo simulations, corresponding to 20, 30 and 40 dB. The biases obtained by using the time-invariant scheme and the time-varying scheme are shown in Figs.4(a) and 4(b), respectively. The simulations indicate that the two algorithms have similar trends of change in performance. When SNR = 20 dB and the iteration step

is less than 25, the biases of the two methods range within a small region. However, when the iteration step is above this threshold (25), the estimated error increases abruptly and then converges gradually with the iteration step increasing. In addition, when SNR is 30 dB or 40 dB, the biases of the time-invariant method stay within 0.03. In contrast to the biases obtained by using the time-invariant method, the biases obtained by using the time-varying method are still in a smaller value less than 0.005. Finally, when N is sufficiently large (larger than 10), the estimation error caused by finite N can be neglected. In this case, the estimation errors coming from the symbol errors and the coupling terms are the main factors of introduced estimation errors. So, $N = 10$ is recommended when using our method.

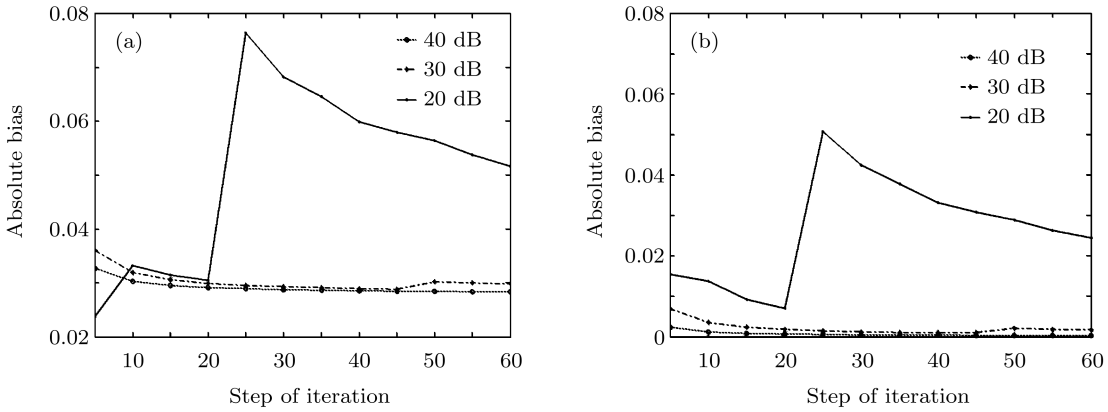


Fig.4. Performance comparison between the time-invariant algorithm (a) and the time-varying method (b).

It has been shown that the symbolic dynamics-based method of estimating the initial condition of 1-D chaotic maps can produce an asymptotically unbiased estimation and achieves the CRLB at high SNRs.[14] However, for CMLs, due to the effect of the coupling term, it is rather difficult to obtain an unbiased and consistent estimation. However, if the gap between the CRLB and the actual performance is small at high SNRs, the method is still applicable. In the present study, we use one channel signal to investigate the relationship between the estimator errors and the SNRs for CMLs. In particular, the estimator variance is determined with Monte Carlo simulations using iteration step $N = 10$ for estimating one channel signal with initial value $x(0) = 0.61$.

The CRLB for $\hat{x}(0)$ onto the logistic map is given as[8]

$$I^{-1}[x(0)] = \frac{\sigma^2 x(1)}{\sum_{n=0}^{N-1} 4^n x(n+1)}. \tag{18}$$

The simulation is carried out and the result is shown in Fig.5 in which the vertical axis denotes the inverse estimation variance, while the horizontal axis represents the SNR. For comparison, the estimation variances

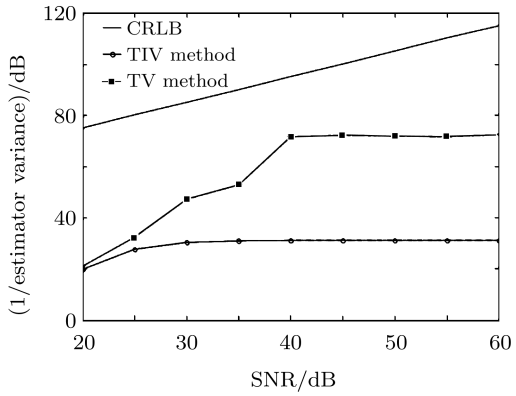


Fig.5. Comparison between the performance of the time-varying and time-invariant methods with different SNRs.

obtained by using the time-invariant method and the time-varying method are both depicted. The generalization performance illustrates that the proposed time-varying estimation scheme consistently outperforms the others and has a smaller error than that of the time-invariant method. Obviously, the performance of the time-invariant method is almost insensitive to the change in SNR, maintaining a low value of the inverse estimation variance around a low level of 30 dB even if SNR is much higher. There is a large

gap between the actual performance and the CRLB, especially at high SNRs, which indicates that the common time-invariant scheme is incapable of estimating the unknown initial conditions for CMLs. However, it is interesting to find out that the time-varying algorithm is sufficient for estimating the initial conditions in most cases. The inverse estimation variance obtained by using the time-varying method increases linearly between 20 dB and 40 dB. For SNRs above 40 dB, the inverse estimation variance stays almost unchanged at a high level of near 75 dB, which is much closer to the CRLB than that obtained by using the original time-invariant algorithm.

It is worth pointing out that the estimation error obtained from the proposed algorithm is mainly caused by both the symbol error $s_i(n)$ when changing the sampled time series into symbol sequences and $\gamma_i(n)$ when changing the effect of the coupling term into symbols. If the sampled data are not appropriately coded due to the effect of the noise, we may use totally different inverse mapping to recover the initial condition according to expression (14). When the SNR is small, the symbol error becomes very large and hence leads to bad performance. However, the probability of the symbol error will be significantly decreased with the increase of SNR. When it arrives at a certain value, the estimation error caused by the wrong symbol $s_i(n)$ can be ignored and only the symbol error from the wrong $\gamma_i(n)$ is contained and cannot be avoided. So, the performance of the proposed algorithm does not yield CRLB for CMLs, but it still provides a good behaviour for estimating the initial condition of the CMLs for a wide range of SNRs.

5. Conclusions

In this paper, a novel method using time-varying control parameter is proposed to estimate the initial condition of CMLs generated by iterating a logistic map. Based on the theory of symbolic dynamics, a computationally efficient algorithm of estimating the unknown initial conditions of a multi-dimensional spatiotemporal chaotic signal is developed. In the presented method, time-varying information regarding the coupling term of CMLs is taken into account to obtain good estimation behaviours for noiseless and noisy environments separately. The performance of the presented time-varying algorithm and the common time-invariant method are also analysed and compared with each other. It is found that the proposed method cannot produce a consistent estimation due to the effect of the coupling term, but the presented time-varying symbolic dynamic approach can effectively reduce the influence of the noise and approximately approach the CRLB at high SNRs. The computed results show that the proposed method can indeed present a more accurate estimation of the initial conditions of the CMLs than the conventional approach. Both theoretical and experimental analyses suggest that the proposed approach is suitable to carry out the inverse problem of the spatiotemporal coupled system, which has a wide application in many engineering contexts.

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