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Stochastic resonance driven by time-modulated correlated coloured noise sources in a single-mode laser

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This paper investigates the phenomenon of stochastic resonance in a single-mode laser driven by time-modulated correlated coloured noise sources. The power spectrum and signal-to-noise ratio R of the laser intensity are calculated by the linear approximation. The effects caused by noise self-correlation time τ_1, τ_2 and cross-correlated time τ_3 for stochastic resonance are analysed in two ways: τ_1, τ_2 and τ_3 are taken to be the independent variables and the parameters respectively. The effects of the gain coefficient Γ and loss coefficient K on the stochastic resonance are also discussed. It is found that besides the presence of the standard form and the broad sense of stochastic resonance, the number of extrema in the curve of R versus K is reduced with the increase of the gain coefficient Γ.

Keywords: coloured noise, coloured correlation, time-periodic modulation, signal-to-noise ratio

PACC: 0540, 4260K

1. Introduction

In recent years, the phenomenon of stochastic resonance (SR) in a bistable nonlinear system with coupling between additive noise and multiplicative noise[1,2] as well as time-delayed feedback[3,4] has attracted a lot of attention. In general, adiabatic elimination theory and eigenvalues theory are adopted to research SR. However, in this paper the linear approximation approach is used to calculate signal-to-noise ratio (SNR) because it is easy to solve and obtain a precise result. What is more, the linear approximation can be used for Markovian and non-Markovian processes as well. Recently the linear approximation approach has been adopted to research SR in the single-mode laser. Han *et al* researched SR for bias signal-modulated noise in a single-mode laser.[5,6] Zhang *et al* further studied SR for amplitude modulated and frequency modulated signals in a single-mode laser,[7,8] which contributed to the furtherance of laser communication. But in all this work, the noise itself was modulated, the noise correlation was assumed to be fixed between $-1 \leq \lambda \leq 1$. Time-modulation correlated noise sources were first introduced to SR by Tessone *et al* in 2000,[9] in which a lot of novel phenomena were found. We have investigated time-modulated correlation in the case of white noise sources with white correlation,[10] then the case of coloured noise sources with white correlation.[11] In this paper, we study the most general case, which is the case of coloured noise sources with coloured correlation. It approaches the real facts most closely. In the following the SNR is calculated, then a discussion is made, and finally a conclusion is drawn.

2. The model and the output SNR

We consider a periodically driven gain laser model described by the following Langevin equation:

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -2KI + \frac{2\Gamma I}{1+\beta I} + D + \frac{2I}{1+\beta I}\xi(t) + 2\sqrt{I}\eta(t) + A\cos(\Omega t), \tag{1}$$

where I is the laser intensity, $\beta = \tilde{A}/\Gamma$, $\tilde{A}$ and Γ stand for the self-saturation and gain coefficients, respectively, K is the loss coefficient. A and Ω represent the amplitude and the frequency of the periodic signal, respectively. D is the intensity of the quantum noise. $\xi(t)$ and $\eta(t)$ are zero-mean Gauss noises, whose sta-

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tistical properties are

$$\begin{aligned}
&\langle\xi(t)\rangle = \langle\eta(t)\rangle = 0,\\
&\langle\xi(t)\xi(t')\rangle = \frac{Q}{2\tau_1}\exp\left[-\frac{|t-t'|}{\tau_1}\right],\\
&\langle\eta(t)\eta(t')\rangle = \frac{D}{2\tau_2}\exp\left[-\frac{|t-t'|}{\tau_2}\right],\\
&\langle\xi(t)\eta(t')\rangle = \langle\eta(t)\xi(t')\rangle = \frac{\lambda\sqrt{QD}\cos(\Omega_\lambda t)}{2\tau_3}\\
&\qquad\times\exp\left[-\frac{|t-t'|}{\tau_3}\right],
\end{aligned}\tag{2}$$

where λ is the noise correlation strength, $-1\le\lambda\le 1$. λ is periodically modulated with frequency Ω_λ. Q is the intensity of the pump noise. τ_1 is the self-correlation time of the pump noise, τ_2 is the self-correlation time of the quantum noise, τ_3 is the cross-correlated time of the noise.

Equation (1) is linearized around the deterministic steady-state intensity $I_0 = (\Gamma - K)/\beta K$. Let $I = I_0 + \varepsilon(t)$, $\varepsilon(t)$ is a small disturbance around the deterministic steady-state intensity, then we have

$$\begin{aligned}
\frac{\mathrm{d}\varepsilon(t)}{\mathrm{d}t} =& -\gamma\varepsilon(t) + D + \frac{2I_0}{1+\beta I_0}\xi(t)\\
&+ 2\sqrt{I_0}\eta(t) + A\cos(\Omega t),
\end{aligned}\tag{3}$$

in which $\gamma = 2K(\Gamma - K)/\Gamma$. By straightforward integration of Eq.(3), $\varepsilon(t')$ is obtained directly and $\langle\varepsilon(t')\rangle$ is given by

$$\begin{aligned}
\langle\varepsilon(t)\rangle =& \mathrm{e}^{-\gamma t'}\left[\frac{D}{\gamma}(\mathrm{e}^{\gamma t'}-1) + \frac{A\mathrm{e}^{\gamma t'}\cos(\Omega t'-\theta)}{\sqrt{\gamma^2+\Omega^2}}\right.\\
&\left.-\frac{A\gamma}{\gamma^2+\Omega^2}\right],
\end{aligned}\tag{4}$$

where

$$\begin{aligned}
\sin\theta &= \frac{\Omega}{\sqrt{\gamma^2+\Omega^2}},\\
\cos\theta &= \frac{\gamma}{\sqrt{\gamma^2+\Omega^2}}.
\end{aligned}\tag{5}$$

The intensity correlation function is calculated and given by

$$\begin{aligned}
c(t) =& \lim_{t'\to\infty}\overline{\langle I(t')I\,(t+t')\rangle} = \frac{\Omega}{2\pi}\int_0^{\frac{2\pi}{\Omega}}\lim_{t'\to\infty}\langle I(t')I(t+t')\rangle\mathrm{d}t'\\
=&\left(I_0+\frac{D}{\gamma}\right)^2 + \left(\frac{2I_0}{1+\beta I_0}\right)^2\frac{Q}{2\tau_1}\frac{\gamma\exp\left(-\frac{|t|}{\tau_1}\right)-\tau_1^{-1}\exp(-\gamma\,|t|)}{\gamma(\gamma^2-\tau_1^{-2})} + \frac{A^2\cos(\Omega t)}{2(\gamma^2+\Omega^2)}\\
&+4I_0\frac{D}{2\tau_2}\frac{\gamma\exp\left(-\frac{|t|}{\tau_2}\right)-\tau_2^{-1}\exp(-\gamma\,|t|)}{\gamma(\gamma^2-\tau_2^{-2})} + \frac{4\lambda\sqrt{I_0^3QD}}{\tau_3(1+\beta I_0)}\frac{\Omega\sin\left(\frac{\Omega_\lambda}{\Omega}\pi\right)}{\pi\Omega_\lambda}\\
&\times\left\{\frac{\exp\left(-\frac{|t|}{\tau_3}\right)\times\left[\cos\left(\frac{\Omega_\lambda}{\Omega}\pi-\theta_2\right)\cos(\Omega_\lambda t)-\sin\left(\frac{\Omega_\lambda}{\Omega}\pi-\theta_2\right)\sin(\Omega_\lambda t)\right]}{(\gamma+\tau_3^{-1})\sqrt{(\gamma-\tau_3^{-1})^2+\Omega_\lambda^2}}\right.\\
&\left.-\frac{\exp(-\gamma\,|t|)\times\left[4\gamma\Omega_\lambda\sin\left(\frac{\Omega_\lambda}{\Omega}\pi-\theta_3\right)+2(\gamma^2-\tau_3^{-2}-\Omega_\lambda^2)\cos\left(\frac{\Omega_\lambda}{\Omega}\pi-\theta_3\right)\right]}{\tau_3[(\gamma+\tau_1^{-3})^2+\Omega_\lambda^2][(\gamma-\tau_1^{-3})^2+\Omega_\lambda^2]\sqrt{4\gamma^2+\Omega_\lambda^2}}\right\}.
\end{aligned}\tag{6}$$

By the Fourier transform of Eq.(6), the power spectrum reads

$$S(\Omega) = \int_{-\infty}^{\infty}\lim_{t'\to\infty}\overline{\langle I(t+t')I(t')\rangle}\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}t = S_1(\omega) + S_2(\omega),\tag{7}$$

where $S_1(\omega)$ is the power spectrum of the output signal,

$$S_1(\omega) = \frac{A^2}{8(\gamma^2+\Omega^2)}[2\pi\delta(\omega-\Omega) + 2\pi\delta(\omega+\Omega)],\tag{8}$$

$S_2(\omega)$ is the output power spectrum of the noise

$$
\begin{aligned}
S_2(\omega) =& \frac{1}{2}\left(I_0+\frac{D}{\gamma}\right)^2 2\pi\delta(\omega)+\left(\frac{I_0}{1+\beta I_0}\right)^2\frac{2Q\tau_1^{-2}}{(\tau_1^{-2}+\omega^2)(\gamma^2+\omega^2)} \\
&+\frac{2I_0D\tau_2^{-2}}{(\tau_2^{-2}+\omega^2)(\gamma^2+\omega^2)}+\frac{\lambda\Omega\sqrt{I_0^3QD}}{1+\beta I_0}\sin\left(\frac{\Omega_\lambda}{\Omega}2\pi\right) \\
&\times\left\{\frac{2\cos\left(\frac{\Omega_\lambda}{\Omega}2\pi-\theta_2\right)}{\pi\Omega_\lambda(\gamma+\tau_3^{-1})\sqrt{(\gamma-\tau_3^{-1})^2+\Omega_\lambda^2}}\left[\frac{1}{1+\tau_3^2(\omega-\Omega_\lambda)^2}+\frac{1}{1+\tau_3^2(\omega+\Omega_\lambda)^2}\right]\right. \\
&-\frac{2\sin\left(\frac{\Omega_\lambda}{\Omega}2\pi-\theta_2\right)}{\pi\Omega_\lambda(\gamma+\tau_3^{-1})\sqrt{(\gamma-\tau_3^{-1})^2+\Omega_\lambda^2}}\left[\frac{\tau_3(\omega+\Omega_\lambda)}{1+\tau_3^2(\omega+\Omega_\lambda)^2}-\frac{\tau_3(\omega-\Omega_\lambda)}{1+\tau_3^2(\omega-\Omega_\lambda)^2}\right] \\
&\left.-\frac{8\gamma\tau_3^{-2}\left[2\gamma\sin\left(\frac{\Omega_\lambda}{\Omega}2\pi-\theta_3\right)+\frac{\gamma^2-\tau_3^{-2}-\Omega_\lambda^2}{\Omega_\lambda}\cos\left(\frac{\Omega_\lambda}{\Omega}2\pi-\theta_3\right)\right]}{\pi(\gamma^2+\omega^2)[(\gamma+\tau_3^{-1})^2+\Omega_\lambda^2][(\gamma-\tau_3^{-1})^2+\Omega_\lambda^2]\sqrt{4\gamma^2+\Omega_\lambda^2}}\right\}. \qquad (9)
\end{aligned}
$$

The output power of the signal reads

$$
P_1=\int_0^\infty S_1(\omega)\mathrm{d}\omega=\frac{A^2}{8(\gamma^2+\Omega^2)}. \qquad (10)
$$

The output power of the noise at $\omega=\Omega$ is

$$
\begin{aligned}
P_2 =& S_2(\omega=\Omega)=\left(\frac{I_0}{1+\beta I_0}\right)^2\frac{2Q\tau_1^{-2}}{(\tau_1^{-2}+\Omega^2)(\gamma^2+\Omega^2)} \\
&+\frac{2I_0D\tau_2^{-2}}{(\tau_2^{-2}+\Omega^2)(\gamma^2+\Omega^2)}+\frac{\lambda\Omega\sqrt{I_0^3QD}}{1+\beta I_0}\sin\left(\frac{\Omega_\lambda}{\Omega}2\pi\right) \\
&\times\left\{\frac{2\cos\left(\frac{\Omega_\lambda}{\Omega}2\pi-\theta_2\right)}{\pi\Omega_\lambda(\gamma+\tau_3^{-1})\sqrt{(\gamma-\tau_3^{-1})^2+\Omega_\lambda^2}}\left[\frac{1}{1+\tau_3^2(\Omega-\Omega_\lambda)^2}+\frac{1}{1+\tau_3^2(\Omega+\Omega_\lambda)^2}\right]\right. \\
&-\frac{2\sin\left(\frac{\Omega_\lambda}{\Omega}2\pi-\theta_2\right)}{\pi\Omega_\lambda(\gamma+\tau_3^{-1})\sqrt{(\gamma-\tau_3^{-1})^2+\Omega_\lambda^2}}\left[\frac{\tau_3(\Omega+\Omega_\lambda)}{1+\tau_3^2(\Omega+\Omega_\lambda)^2}-\frac{\tau_3(\Omega-\Omega_\lambda)}{1+\tau_3^2(\Omega-\Omega_\lambda)^2}\right] \\
&\left.-\frac{8\gamma\tau_3^{-2}\left[2\gamma\sin\left(\frac{\Omega_\lambda}{\Omega}2\pi-\theta_3\right)+\frac{\gamma^2-\tau_3^{-2}-\Omega_\lambda^2}{\Omega_\lambda}\cos\left(\frac{\Omega_\lambda}{\Omega}2\pi-\theta_3\right)\right]}{\pi(\gamma^2+\Omega^2)[(\gamma+\tau_3^{-1})^2+\Omega_\lambda^2][(\gamma-\tau_3^{-1})^2+\Omega_\lambda^2]\sqrt{4\gamma^2+\Omega_\lambda^2}}\right\}, \qquad (11)
\end{aligned}
$$

where

$$
\begin{aligned}
\cos\theta_2 &= \frac{\gamma-\tau_3^{-1}}{\sqrt{(\gamma-\tau_3^{-1})^2+\Omega_\lambda^2}}, \\
\sin\theta_2 &= \frac{\Omega_\lambda}{\sqrt{(\gamma-\tau_3^{-1})^2+\Omega_\lambda^2}}, \\
\cos\theta_3 &= \frac{2\gamma}{\sqrt{4\gamma^2+\Omega_\lambda^2}}, \\
\sin\theta_3 &= \frac{\Omega_\lambda}{\sqrt{4\gamma^2+\Omega_\lambda^2}}.
\end{aligned}
$$

The SNR defined as $R=P_1/P_2$ can be got from Eqs.(10) and (11).

3. Discussion

The linearization is valid when the relative fluctuation of the laser intensity is far below 1,[12] so the parameters taken for curves in this paper will guarantee the relative fluctuation of the laser intensity to

be less than 0.01. In what follows the effects of correlation time τ_1, τ_2 and τ_3 on SR are discussed in two aspects: (1) τ_1, τ_2 and τ_3 are taken to be the independent values of SNR; (2) τ_1, τ_2 and τ_3 are taken to be the parameters for different SNR curves.

3.1. The curves of R versus τ_1, τ_2 and τ_3

The SNR as a function of the self-correlation time τ_1 of the pump noise is plotted in Fig.1. It is seen that the $R-\tau_1$ curve increases monotonously and tends to be constant. When the other parameters are adjusted, the tendency of increase remains unchanged except the curve slope and the ultimate constant have a slight change. The behaviour of the curve of $R-\tau_2$ is similar to the curve of $R-\tau_1$, for brevity, the $R-\tau_2$ curve is not presented here.

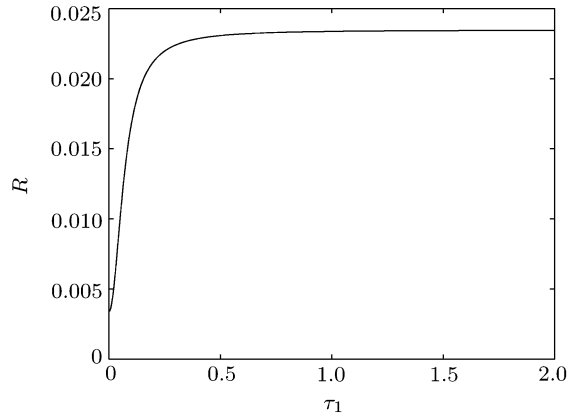


Fig.1. The SNR R as a function of the self-correlation time τ_1 of the pump noise. The values of other parameters are $A=3$, $K=1$, $\Gamma=9$, $\Omega=37$, $I_0=1$, $D=0.1$, $Q=0.6$, $\lambda=0.9$, $\Omega_\lambda=437$, $\tau_2=0.001$ and $\tau_3=0.01$.

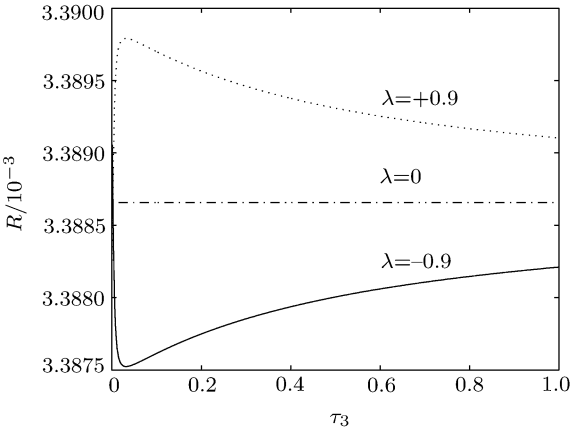


Fig.2. The SNR R as a function of the cross-correlation time τ_3 between the noise terms. The values of other parameters are $A=3$, $K=1$, $\Gamma=9$, $\Omega=37$, $I_0=0.1$, $D=0.1$, $Q=0.6$, $\Omega_\lambda=237$, $\tau_1=0.001$ and $\tau_2=0.001$.

The SNR versus the correlation time τ_3 of the coupling between two noise terms is plotted in Fig.2. Seen from Fig.2, when $\lambda>0$, the $R-\tau_3$ curve has a resonance peak; when $\lambda<0$, the $R-\tau_3$ curve has a minimum, namely the curve exhibits an intense suppression; when $\lambda=0$, the $R-\tau_3$ curve is a horizontal line. Finally these three curves approach unity.

3.2. The curve of R versus D

The SNR as a function of the noise intensity D of quantum noise is plotted in Figs.3–5 for different values of the pump noise self-correlation time τ_1, the quantum noise self-correlation time τ_2, and the cross-correlation time τ_3 between two noise terms, respectively. Seen from Figs.3–5, all the $R-D$ curves exhibit

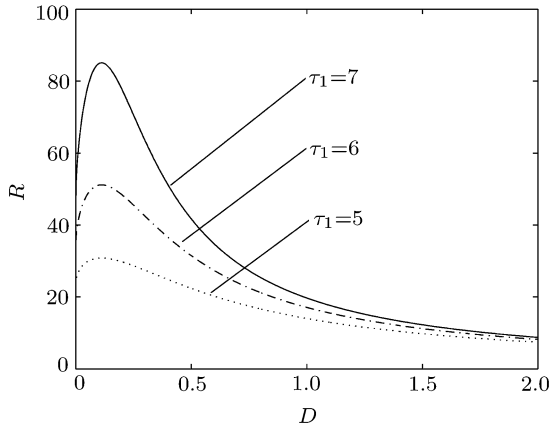


Fig.3. The SNR R as a function of the noise intensity D of quamtum noise for different values of the pump noise correlation time τ_1. The values of other parameters are $A=30$, $K=9$, $\Gamma=9$, $\Omega=33$, $I_0=1$, $Q=0.6$, $\Omega_\lambda=237$, $\lambda=-0.9$, $\tau_2=1$ and $\tau_3=0.6$.

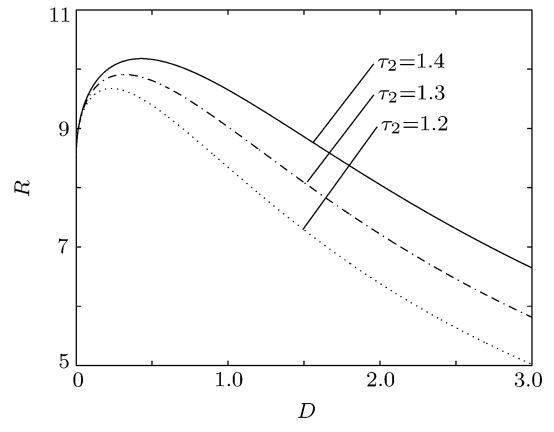


Fig.4. The SNR R as a function of the noise intensity D of quantum noise for different values of the quantum noise correlation time τ_2. The values of other parameters are $A=30$, $K=3$, $\Gamma=9$, $\Omega=33$, $I_0=1$, $\lambda=-0.9$, $Q=0.6$, $\Omega_\lambda=237$, $\tau_1=3$ and $\tau_3=0.6$.

a resonance peak which is the standard form of SR. Figures 3 and 4 show, when τ_1 or τ_2 increases, the peak of $R-D$ curve becomes higher. However, Fig.5 shows that when τ_3 increases, the peak of the $R-D$ curve becomes lower, which is contrary to the cases of τ_1 and τ_2.

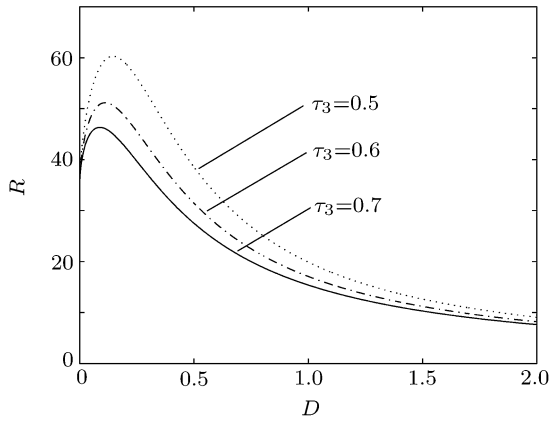


Fig.5. The SNR R as a function of the noise intensity D of quamtum noise for different values of the correlation time τ_3 of the coupling between two noise terms. The values of other parameters are $A=30$, $K=3$, $\Gamma=9$, $\Omega=33$, $I_0=0.1$, $Q=0.6$, $\Omega_\lambda=237$, $\lambda=-0.9$, $\tau_1=6$ and $\tau_2=1$.

3.3. The curve of R versus Q

The behaviour of R against Q for different values of τ_1, τ_2 and τ_3 is similar to that of R against D, for brevity, $R\sim Q$ curves are not presented in this paper.

3.4. The curve of R versus Ω_λ

The SNR versus the periodically modulated frequency Ω_λ of λ is plotted in Figs.6–8 for different values of the pump noise self-correlation time τ_1, the quantum noise self-correlation time τ_2, and the cross-correlation time τ_3 between two noise terms, respectively. The $R-\Omega_\lambda$ curves exhibit SR of periodic oscillation form. It is seen that the peak value escalates at first, then decreases, and finally approaches a steady-state. Seen from Fig.6, as the value of parameter τ_1 increases, the whole $R-\Omega_\lambda$ curve moves upward, and the resonant amplitude variation slightly increases. When the value of parameter τ_2 increases, the whole $R-\Omega_\lambda$ curve still moves upward, but the resonant amplitude variation obviously increases, which is shown in Fig.7. Seen from Fig.8, the resonant amplitude variation is obviously affected by the value of τ_3, but τ_3 has no effect on the ultimate steady-state.

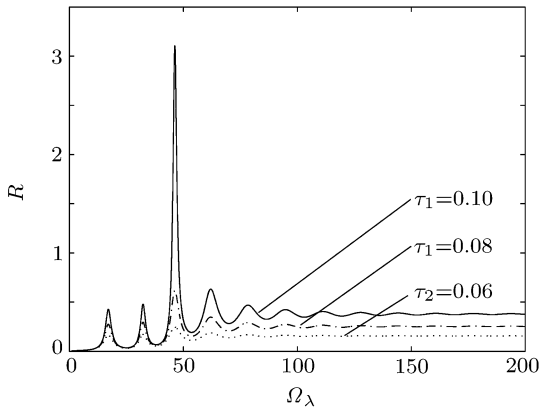


Fig.6. The SNR R as a function of the modulated frequency Ω_λ of λ for different values of the self-correlation time τ_1 of the pump noise. The values of other parameters are $A=30$, $K=1$, $\Gamma=8$, $\Omega=73$, $I_0=0.5$, $Q=0.6$, $D=0.8$, $\lambda=0.6$, $\tau_2=3$, and $\tau_3=0.1$.

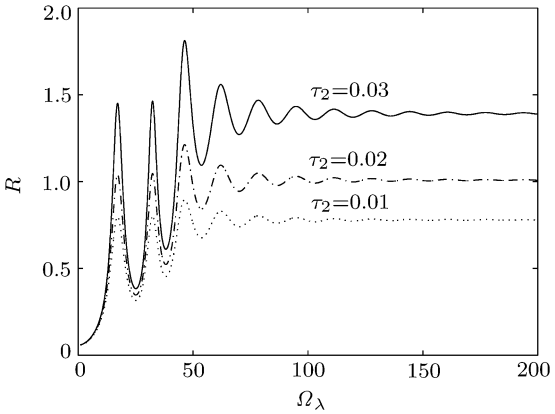


Fig.7. The SNR R as a function of the modulated frequency Ω_λ of λ for different values of the self-correlation time τ_2 of the quantum noise. The values of other parameters are $A=30$, $K=2$, $\Gamma=8$, $\Omega=33$, $I_0=1$, $Q=0.3$, $D=0.1$, $\lambda=0.6$, $\tau_1=8$, and $\tau_3=0.1$.

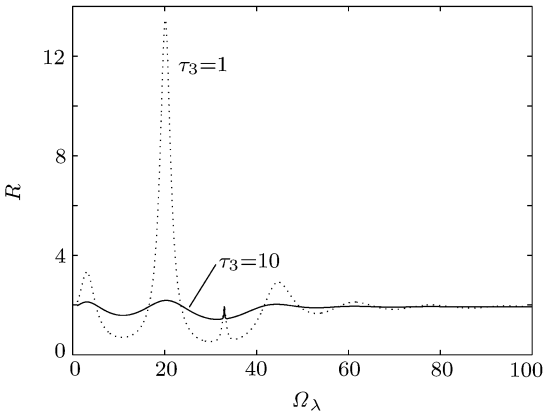


Fig.8. The SNR R as a function of the modulated frequency Ω_λ of λ for different values of the cross-correlation time τ_3 between two noise terms. The values of other parameters are $A=30$, $K=2$, $\Gamma=8$, $\Omega=33$, $I_0=1$, $Q=0.6$, $D=0.1$, $\lambda=0.6$, $\tau_1=8$, and $\tau_2=0.04$.

3.5. The curve of R versus K

The SNR as a function of the loss coefficient K is plotted in Figs.9(a)–9(c) for different values of gain coefficient Γ. It is seen that the curve of the SNR is changed from two peaks to one peak when Γ increases.

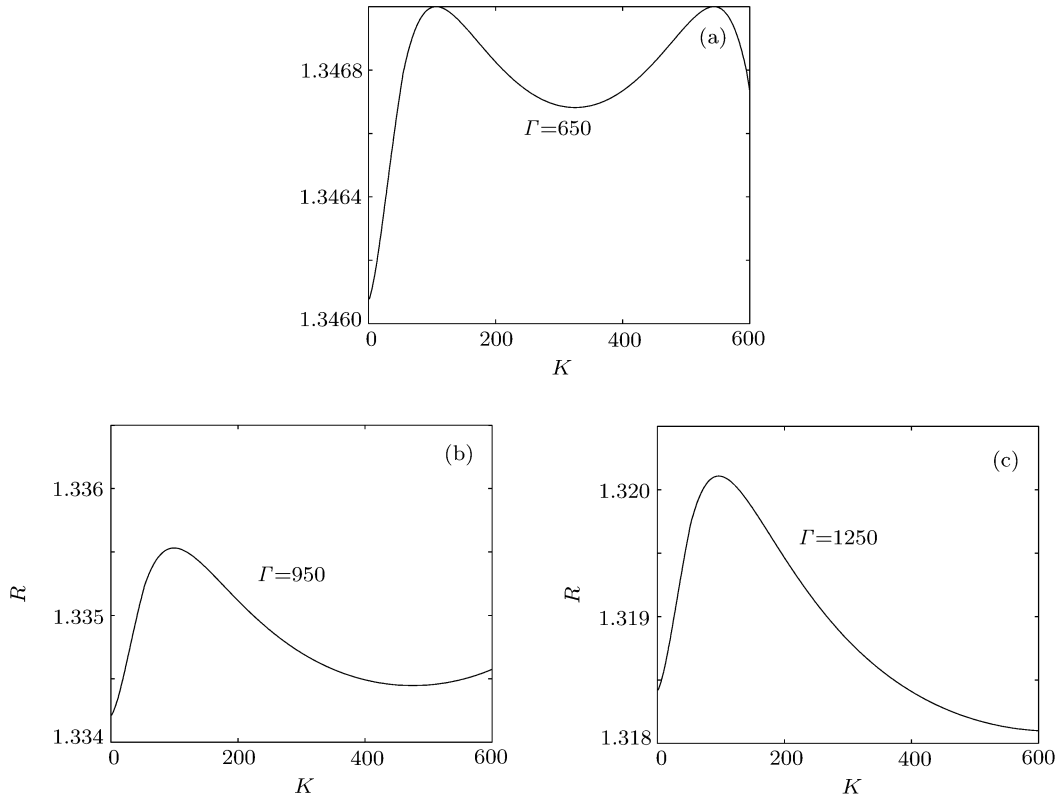


Fig.9. The SNR R as a function of the loss coefficient K for (a) $\Gamma = 650$; (b) $\Gamma = 950$; (c) $\Gamma = 1250$. The values of other parameters are $A = 800$, $\Omega = 55$, $\Omega_\lambda = 337$, $I_0 = 80$, $Q = 0.6$, $D = 0.6$, $\lambda = -0.9$, $\tau_1 = 0.01\tau_2 = 0.01$, and $\tau_3 = 0.01$.

4. Conclusions

The present study has investigated SR in a single-mode laser driven by time-modulated correlated coloured noise sources. It is found that besides the SR of a standard form, such as SR in the $R - D$ curve or $R - Q$ curve, the $R - \tau_3$ curve also has a resonance peak and the $R - \Omega_\lambda$ curve shows SR of a periodic oscillation form, which is SR in a broad sense. Multi-extrema exist in the $R - K$ curves for different values of gain coefficient Γ. When K is between $0 \leq K \leq 600$ and the value of Γ increases, the $R - K$ curve experiences the transition from possessing three extrema to possessing two extrema and lastly a single extremum.

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