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Entropy of a rotating and charged black string to all orders in the Planck length*

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By using the entanglement entropy method, this paper calculates the statistical entropy of the Bose and Fermi fields in thin films, and derives the Bekenstein–Hawking entropy and its correction term on the background of a rotating and charged black string. Here, the quantum field is entangled with quantum states in the black string and thin film to the event horizon from outside the rotating and charged black string. Taking into account the effect of the generalized uncertainty principle on quantum state density, it removes the difficulty of the divergence of state density near the event horizon in the brick-wall model. These calculations and discussions imply that high density quantum states near the event horizon of a black string are strongly correlated with the quantum states in a black string and that black string entropy is a quantum effect. The ultraviolet cut-off in the brick-wall model is not reasonable. The generalized uncertainty principle should be considered in the high energy quantum field near the event horizon. From the viewpoint of quantum statistical mechanics, the correction value of Bekenstein–Hawking entropy is obtained. This allows the fundamental recognition of the correction value of black string entropy at nonspherical coordinates.

Keywords: quantum statistics, nonspherical symmetry spacetime, generalized uncertainty principle, entropy of black string

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1. Introduction

A black hole has Bekenstein–Hawking (B–H) entropy which is proportional to the area of the event horizon;[1−3] the standard formula for B–H entropy is

$$S_{\rm BH} = \frac{A}{4l_{\rm p}^2}, \tag{1}$$

where A is the area of the black hole event horizon and $l_{\rm p}$ is the Planck length.

The entropy given by Eq.(1) is obtained by comparing parameters of the black hole with general thermodynamic quantities. It is well-known that general thermodynamic systems can be described by statistical mechanics. For a black hole as a thermodynamic system, we certainly have an interest in its statistical mechanical background. The research on the statistical origin of the black hole's entropy has been extraordinarily active. Many methods have been provided to calculate the entropy.[4−8] First, based on statistical mechanics, any macroscopical entropy has its microcosmic statistical explanation. This is a quantity which scales the microcosmic degrees of freedom of the system. However, from a statistical point of view, how can we understand the origin of black hole entropy? This is not a problem which can be solved by curved spacetime quantum field theory. Because in this range, only the matter field is quantized, the spacetime background is still classical. Comprehending black hole entropy needs further understanding of the microcosmic degrees of freedom of spacetime. So it is a key problem which needs to be solved in quantum gravity.

Subsystems of a multiplex system which is in an entanglement state are present in a mixed states. The entropy observed by a subsystem is entanglement entropy. According to the theory of entanglement entropy, black hole entropy results from entanglement entropy due to entanglement from the degrees of freedom inside and outside the black hole event horizon.[4] Hooft interpreted black hole entropy as the entropy of quantum states outside the horizon.[5] Its essence is to make the quantum inside and outside the black hole event horizon constitute an entanglement state. Further, by calculating the entropy of the degrees of

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freedom of a subsystem outside the black hole, the Bekenstein–Hawking (B–H) entropy of the black hole is obtained. Because the quantum field inside and outside the black hole event horizon is at an entanglement state, an observer outside the black hole can only observe the degrees of freedom outside the event horizon; the density matrix of a mixed state measures the information loss in the whole black hole. Therefore the entanglement entropy of the degrees of freedom outside the black hole is the entropy of the degrees of freedom in the black hole, and it is the B–H entropy of the black hole. In this framework, the brick-wall model[5] presents a method for calculating the black hole entanglement entropy. This method has been used for calculations on the black hole,[5−12] promoting the understanding of the origin of black hole entropy. However, the quantum state density is divergent near the black hole horizon surface. In order to derive the B–H entropy, we need to introduce an ultraviolet cut-off, which is unnatural. It is found that the black hole entropy is mostly the contribution of the quantum state near the black hole horizon surface. The brick-wall model is improved and the membrane model is provided.[13−15] The membrane model only considers the quantum state in the thin film near the horizon. The infrared cut-off and litter mass approximation in the original brick-wall model are removed. However, the ultraviolet cut-off still exists.

String theory and noncommutative geometry (NCG) correct the Heisenberg uncertainty principle so it becomes the generalized uncertainty principle (GUP).[16−17] By using the GUP, the black hole entanglement entropy is calculated.[18−23] The results show that near the horizon quantum state density and its statistic entropy are finite. The ultraviolet cut-off in the brick-wall model can be removed.

Utilizing the simplest GUP,

$$\Delta x \Delta p \geq \frac{1}{2}\left(1+\lambda(\Delta p)^2\right) \tag{2}$$

and correcting the state density

$$\mathrm{d}n = \frac{\mathrm{d}^3x\mathrm{d}^3p}{(2\pi)^3(1+\lambda p^2)^3}, \tag{3}$$

Ref.[24] calculated the statistic entropy of a scalar field under the background of a Reissner–Nordstrom black hole and obtained the series expression of field entropy near the black hole horizon. However, in their results the higher order term of entropy is divergent. Reference [25] again discussed the statistic entropy of a scalar field under the background of a Reissner–Nordstrom black hole and obtained the series expression of entropy near the black hole horizon. Recently, Ref.[26] proposed a correction to state density due to GUP

$$\mathrm{d}n = \frac{\mathrm{d}^3x\mathrm{d}^3p}{(2\pi)^3}\mathrm{e}^{-\lambda p^2}, \tag{4}$$

where $p^2 = p^i p_i$, λ, which has the Planck area quantity, and is a correction constant of the Heisenberg uncertainty principle due to gravitation.

At present, gravitation theory and quantum theory face a tremendous challenge. Though there is no completely successful quantum gravity, applying the semi-classical method to study the correction to the thermodynamic quantity of a black hole may provide a valuable reference for the final formation of quantum gravity. A great deal of attention has been paid to the correction to the thermodynamic quantity of a black hole.[26−34] The methods include GUP, modified dispersion relations, NCG and noncommutative quantum field theory. All discussions about the black hole entropy correction are based on the B–H entropy of the black hole. However, the statistical origin of black hole entropy has still not been solved successfully.

We generalize the method of Ref.[25] and investigate the entropy of a charged and rotating black string. By utilizing the quantum statistical method,[19,35] the difficulty of solving the wave equation of various spacetimes is removed. Since we calculate the quantum statistic entropy of the black hole by GUP, we only calculate the statistic entropy of the Bose and Fermi fields in a thin film with the Planck scale near the horizon. Applying the entanglement entropy method, the statistical entropy of the Bose and Fermi fields in a thin film is calculated. Here, the quantum field is entangled with the quantum states in the black string. It is derived that the quantum statistical entropy of the black string is proportional to the horizon area. The obtained entropy is the number of quantum state in the Planck scale near the horizon. It is a quantum effect, and is an intrinsic characteristic of spacetime. The calculation implies that the quantum states near the event horizon of the black string are strongly correlated with the degrees of freedom in the black string. The ultraviolet cut-off in the brick-wall model is not reasonable. In the quantum fields inside and outside the brick-wall, the degrees of freedom contributing to the black string entropy are just those correlating with the degrees of freedom in the

black string. By calculation, the convergent series expression of the black string entropy is obtained. When we take the constant λ (λ is introduced in the GUP and is related to the dimension of the space) as a fixed value, the B–H entropy of a rotating and charged black string and its correction term are derived. From the viewpoint of quantum statistical mechanics, the B–H entropy of the black string and its correction value are obtained. This semi-classical method provides a valuable reference for recognizing the quantum effect of black string thermodynamics. It allows the fundamental recognition of the entropy of a black string and its correction value at nonspherical coordinates. In this paper, we take the simplest function form of the temperature ($c = G = K_{\rm B} = 1$).

2. Rotating and charged black string of anti-de Sitter space–time

The line element of a rotating and charged anti-de Sitter black string is as follows:[36,37]

$$\begin{aligned} {\rm d}s^2 =& -\Xi^2\left(f(r)-\frac{a^2r^2}{\Xi^2l^4}\right){\rm d}t^2+\frac{1}{f(r)}{\rm d}r^2 \\ &+\frac{r^2}{l^2}{\rm d}z^2-2\frac{a\Xi l}{r}\left(b-\frac{l}{r}\gamma^2\right){\rm d}t{\rm d}\varphi \\ &+\left[\Xi^2r^2-a^2f(r)\right]{\rm d}\varphi^2, \end{aligned} \tag{5}$$

$$A_\mu = -\Xi\frac{l\gamma}{r}\left(\delta^0_\mu-\frac{a}{\Xi}\delta^2_\mu\right), \tag{6}$$

where

$$f(r)=\frac{r^2}{l^2}-\frac{bl}{r}+\frac{\gamma^2l^2}{r^2},\quad \Xi^2=1+\frac{a^2}{l^2}, \tag{7}$$

in which a, b and γ are constants. On the z axis, mass, charge and angular momentum per unit height are respectively

$$M=\frac{1}{8}\left(3\Xi^2-1\right)b,\quad Q=\frac{\Xi\gamma}{2},\quad J=\frac{3}{8}\Xi ab. \tag{8}$$

The Hawking radiation temperature and angular velocity are respectively

$$T_+=\frac{f'(r_+)}{4\pi\Xi}=\frac{3r_+^4-\gamma^2l^4}{4\pi\Xi l^2r_+^3},\quad \Omega_+=\frac{a}{\Xi l^2}, \tag{9}$$

where r_+ is the horizon location of the black string and satisfies equation $f(r_+)=0$. On the z axis, the horizon area per unit height is

$$A_+=\frac{2\pi r_+^2}{l}. \tag{10}$$

According to Ref.[38], the radiation temperature obtained by an observer at rest at an infinite distance is as follows:

$$T=\frac{T_+}{\sqrt{-\tilde{g}_{tt}}}, \tag{11}$$

where

$$-\tilde{g}_{tt}=-\frac{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}{g_{\varphi\varphi}}=\frac{r^2f(r)}{g_{\varphi\varphi}}. \tag{12}$$

3. Entropy of the Bose field

For the Bose system, in unit volume the partition function Z_0 satisfies

$$\ln Z_0=-\sum_i g_i\ln(1-{\rm e}^{-\beta\varepsilon_i}). \tag{13}$$

For spacetime Eq.(5), at arbitrary point r, on the z axis, the area per unit height is

$$A(r)=2\pi\frac{r}{l}\sqrt{g_{\varphi\varphi}}. \tag{14}$$

Outside the horizon, at arbitrary point r, on the z axis, the string shell volume element per unit height is

$${\rm d}V=A(r)\sqrt{g_{rr}}{\rm d}r. \tag{15}$$

So outside the horizon, at arbitrary point r, the partition function of a system in the string shell with arbitrary thickness is

$$\begin{aligned} \ln Z =& -\int A(r)\sqrt{g_{rr}}{\rm d}r\sum_i g_i\ln(1-{\rm e}^{-\beta\varepsilon_i}) \\ =& -\int\frac{A(r)\sqrt{g_{rr}}{\rm d}r}{2\pi^2}\int_0^\infty p^2{\rm d}p{\rm e}^{-\lambda p^2}\ln(1-{\rm e}^{-\beta\omega_0}) \\ \approx& \int A(r)\sqrt{g_{rr}}{\rm d}r\int_{m\sqrt{-\tilde{g}_{tt}}}^\infty \\ &\times\frac{\beta_0}{6\pi^2({\rm e}^{\beta\omega_0}-1)}p^3{\rm e}^{-\lambda p^2}{\rm d}\omega, \end{aligned} \tag{16}$$

where $\beta=\beta_0\sqrt{-\tilde{g}_{tt}}$. The free energy of the system is

$$\begin{aligned} F =& -\frac{1}{\beta_0}\ln Z \\ =& -\int A(r)\sqrt{g_{rr}}{\rm d}r\int_{m\sqrt{-\tilde{g}_{tt}}}^\infty \\ &\times\frac{1}{6\pi^2({\rm e}^{\beta\omega_0}-1)}p^3{\rm e}^{-\lambda p^2}{\rm d}\omega. \end{aligned} \tag{17}$$

Entropy of the system is

$$
\begin{aligned}
S_{\rm B} &= \beta_0^2\frac{\partial F}{\partial\beta_0}\\
&= \beta_0^2\int A(r)\sqrt{g_{rr}}{\rm d}r\int_{m\sqrt{-\tilde g_{tt}}}^{\infty}\frac{\omega{\rm e}^{\beta\omega_0}}{6\pi^2({\rm e}^{\beta\omega_0}-1)^2}p^3{\rm e}^{-\lambda p^2}{\rm d}\omega\\
&= \beta_0^2\int A(r)\sqrt{g_{rr}}{\rm d}r\int_{m\sqrt{-\tilde g_{tt}}}^{\infty}\frac{\omega{\rm e}^{\beta\frac{\omega}{\sqrt{-\tilde g_{tt}}}}}{6\pi^2({\rm e}^{\beta\frac{\omega}{\sqrt{-\tilde g_{tt}}}}-1)^2}{\rm e}^{-\lambda\left(\frac{\omega^2}{-\tilde g_{tt}}-m^2\right)}\left(\frac{\omega^2}{-\tilde g_{tt}}-m^2\right)^{3/2}{\rm d}\omega\\
&= \frac{1}{6\pi^2}\int A(r)\sqrt{g_{rr}}{\rm d}r\int_{m\beta}^{\infty}\frac{x{\rm e}^x}{({\rm e}^x-1)^2}{\rm e}^{-\lambda\left(\frac{x^2}{\beta^2}-m^2\right)}\left(\frac{x^2}{\beta^2}-m^2\right)^{3/2}{\rm d}x. \qquad (18)
\end{aligned}
$$

The relation between energy, momentum and mass of the particle $\frac{\omega^2}{-\tilde g_{tt}}=p^2+m^2$ has been used in Eq.(18). m is the static mass of the particle. We integrate Eq.(18) with respect to r near the horizon. Near the horizon, $\tilde g_{tt}(r_+)\to 0$, so Eq.(18) can be reduced to

$$
\begin{aligned}
&S_{\rm B}\\
&= \frac{1}{6\pi^2}\int A(r)\sqrt{g_{rr}}{\rm d}r\int_0^{\infty}\frac{x^4{\rm e}^x}{\beta^3({\rm e}^x-1)^2}{\rm e}^{-\lambda\frac{x^2}{\beta^2}}{\rm d}x\\
&= \frac{1}{3\pi\beta_0^3 l}\int\frac{\sqrt{g_{\varphi\varphi}g_{rr}}r{\rm d}r}{(-\tilde g_{tt})^{3/2}}\int_0^{\infty}\frac{x^4{\rm e}^x}{({\rm e}^x-1)^2}{\rm e}^{-\lambda\frac{x^2}{\beta^2}}{\rm d}x\\
&= \frac{1}{3\pi\beta_0^3 l}\int_0^{\infty}\frac{{\rm d}x}{4\sinh^2\left(\frac{x}{2}\right)}\int\frac{\sqrt{g_{\varphi\varphi}g_{rr}}}{(-\tilde g_{tt})^{3/2}}x^4{\rm e}^{-\lambda\frac{x^2}{\beta^2}}r{\rm d}r\\
&= \frac{1}{3\pi\beta_0^3 l}\int_0^{\infty}\frac{{\rm d}x}{4\sinh^2\left(\frac{x}{2}\right)}I(x,\varepsilon), \qquad (19)
\end{aligned}
$$

where

$$
\begin{aligned}
I(x,\varepsilon) &= \int\frac{\sqrt{g_{\varphi\varphi}g_{rr}}}{(-\tilde g_{tt})^{3/2}}x^4{\rm e}^{-\lambda\frac{x^2}{\beta^2}}r{\rm d}r\\
&= \beta_0^4\int\sqrt{-g_{\varphi\varphi}g_{rr}\tilde g_{tt}}\frac{\partial^2}{\partial\lambda^2}{\rm e}^{-\lambda\frac{x^2}{\beta^2}}r{\rm d}r\\
&= \beta_0^4\int\frac{\partial^2}{\partial\lambda^2}{\rm e}^{-\lambda\frac{x^2}{\beta^2}}r^2{\rm d}r. \qquad (20)
\end{aligned}
$$

Since we only consider the quantum field near the black hole horizon, we take $[r_+, r_+ + \varepsilon]$ as the integral interval with respect to r. ε is determined by the smallest length given by GUP[39]

$$
\begin{aligned}
\sqrt{\frac{e\lambda}{2}} &= \int_{r_+}^{r_++\varepsilon}\sqrt{g_{rr}(r)}{\rm d}r\\
&= \int_{r_+}^{r_++\varepsilon}\sqrt{\frac{1}{f(r)}}{\rm d}r\\
&\approx \frac{2}{\sqrt{f'(r_+)}}\sqrt{\varepsilon}, \qquad (21)
\end{aligned}
$$

then

$$
\begin{aligned}
&I_1(x,\varepsilon)\\
&= \beta_0^4\int_{r_+}^{r_++\varepsilon}\frac{\partial^2}{\partial\lambda^2}{\rm e}^{-\lambda\frac{x^2}{\beta^2}}r^2{\rm d}r\\
&= \beta_0^4\frac{\partial^2}{\partial\lambda^2}\int_{r_+}^{r_++\varepsilon}\frac{r^2{\rm d}r}{\sum_{n=0}\frac{1}{n!}\left(\frac{\lambda x^2}{-\beta_0^2\tilde g_{tt}(r)}\right)^n}\\
&\approx \beta_0^6\frac{\partial^2}{\partial\lambda^2}\int_{r_+}^{r_++\varepsilon}\\
&\quad -\frac{r_+^2}{\lambda x^2-\beta_0^2\tilde g'_{tt}(r_+)(r-r_+)+o((r-r_+)^2)}\\
&\quad \times\tilde g'_{tt}(r_+)(r-r_+){\rm d}r. \qquad (22)
\end{aligned}
$$

Neglecting the higher order term, we can rewrite Eq.(22) as

$$
\begin{aligned}
&I(x,\varepsilon)\\
&= \beta_0^4\frac{\partial^2}{\partial\lambda^2}\int_{r_+}^{r_++\varepsilon}\left[1-\frac{\lambda x^2}{\lambda x^2-\beta_0^2\tilde g'_{tt}(r_+)(r-r_+)}\right]r_+^2{\rm d}r\\
&= \beta_0^4 r_+^2\frac{\partial^2}{\partial\lambda^2}\left[\varepsilon+\frac{\lambda x^2}{B}\ln\left(\frac{\lambda x^2-B\varepsilon}{\lambda x^2}\right)\right]\\
&= -\beta_0^4 r_+^2\frac{\varepsilon^2x^2B}{\lambda(\lambda x^2-B\varepsilon)^2}\\
&= -\frac{\beta_0^4 r_+^2}{64\lambda}\frac{e^2x^2B(f'(r_+))^2}{(x^2-eBf'(r_+)/8)^2}\\
&= \frac{\beta_0^3 r_+^2}{\lambda}\frac{\pi^3e^2x^2\Xi}{(x^2+2e\pi^2)^2}, \qquad (23)
\end{aligned}
$$

where $B=\beta_0^2\tilde g'_{tt}(r_+)=-\beta_0^2\frac{r_+^2\Delta'(r_+)}{g_{\varphi\varphi}(r_+)}=-\frac{16\pi^2}{f'(r_+)}$. Substituting Eq.(23) into Eq.(19), we have

$$
S_{\rm B}=\frac{\pi A_+e^2}{48\lambda}\int_0^{\infty}\frac{x^2}{\sinh^2 x}\frac{{\rm d}x}{(x^2+e\pi^2/2)^2}. \qquad (24)
$$

Using a complex-valued function, we calculate the above integral and take

$$
f(z)=\frac{z^2}{\sinh^2 z(z^2+e\pi^2/2)^2}. \qquad (25)
$$

When $n\neq 0$ and is an integer, $z={\rm i}n\pi$ and $z={\rm i}\sqrt{\frac{e}{2}}\pi$ are two second order poles of $f(z)$, and the residues

are respectively

$$-\frac{\mathrm{i}}{2\sin^2(\sqrt{e/2}\pi)}\left[\cot(\sqrt{e/2}\pi)-\frac{1}{2\sqrt{e/2}\pi}\right]$$

and $-2in(n^2+e/2)/[\pi^3(n^2-e/2)^3]$. According to the residue theorem, we have

$$\begin{aligned}S_{\mathrm{B}} &= \frac{\pi A(r_+)e^2}{48\lambda}\left(\frac{\pi\cot(\sqrt{e/2}\pi)}{2\sin^2(\sqrt{e/2}\pi)}\right.\\ &\quad -\frac{\pi}{4\sqrt{e/2}\sin^2(\sqrt{e/2}\pi)}\\ &\quad \left.+\frac{2}{\pi^2}\sum_{n=1}\frac{n(n^2+e/2)}{(n^2-e/2)^3}\right).\end{aligned} \tag{26}$$

If we let the smallest length introduced in GUP be

$$\lambda=\frac{\pi^2e^2}{48}\frac{2\sqrt{e/2}\cot(\sqrt{e/2}\pi)-1}{\sqrt{e/2}\sin^2(\sqrt{e/2}\pi)}, \tag{27}$$

the leading term in the series expression of entropy Eq.(26) is a quarter of the black hole area. This satisfies B–H entropy. Recently, there has been much attention devoted to the correction value of the black hole entropy. Many researchers have expressed a vast interest in the correction value of the black hole entropy. They all derived valuable results. In this paper, by calculating the quantum statistical entropy of the Bose field near the black hole horizon, we derive the correction value of B–H entropy. The series expression of this correction value is convergent. From the view of quantum statistical mechanics, we propose the B–H entropy of the black hole and the correction value.

4. Entropy of the Fermi field

For the Fermi system, the partition function satisfies

$$\ln Z=\sum_i g_i\ln(1+\mathrm{e}^{-\beta\varepsilon_i}). \tag{28}$$

From Eq.(18), the entropy of the Fermi system is

$$\begin{aligned}S_{\mathrm{F}} &= \frac{1}{6\pi^2}\int A(r)\sqrt{g_{rr}}\mathrm{d}r\int_0^\infty\frac{x^4\mathrm{e}^x}{\beta^3(\mathrm{e}^x+1)^2}\mathrm{e}^{-\lambda\frac{x^2}{\beta^2}}\mathrm{d}x\\ &= \frac{1}{3\pi\beta_0^3 l}\int_0^\infty\frac{\mathrm{d}x}{4\cosh^2\left(\frac{x}{2}\right)}I(x,\varepsilon)\\ &= \frac{\pi A_+e^2}{48\lambda}\int_0^\infty\frac{x^2}{\cosh^2 x}\frac{\mathrm{d}x}{(x^2+e\pi^2/2)^2}.\end{aligned} \tag{29}$$

The entropy corresponding to the Fermi field near the black hole horizon is

$$\begin{aligned}S_{\mathrm{F}} &= \frac{\pi A(r_+)e^2}{48\lambda}\left(\frac{\pi\tan(\sqrt{e/2}\pi)}{2\cos^2(\sqrt{e/2}\pi)}\right.\\ &\quad -\frac{\pi}{4\sqrt{e/2}\cos^2(\sqrt{e/2}\pi)}\\ &\quad \left.+\frac{2}{\pi^2}\sum_{n=0}\frac{(n+1/2)((n+1/2)^2+e/2)}{((n+1/2)^2-e/2)^3}\right).\end{aligned} \tag{30}$$

If we let the smallest length introduced in GUP be

$$\lambda=\frac{\pi^2e^2}{48}\frac{2\sqrt{e/2}\tan(\sqrt{e/2}\pi)-1}{\sqrt{e/2}\cos^2(\sqrt{e/2}\pi)}, \tag{31}$$

the leading term in the series expression of Fermi entropy is proportional to the horizon area. This satisfies B–H entropy.

5. Conclusion

Gravitation quantization is an open problem in the world. Up to now there has not been a satisfactory method. In this paper, using the new equation of state density motivated by the generalized uncertainty relation, we calculate statistic entropies of Bose and Fermi fields near the black hole horizon. In our calculation, we need not introduce a cut-off and a small mass approximation. A series expression for the statistical entropy of a black string is derived. When we take the constant λ (λ is introduced in the GUP and is related to the dimension of the space) as a fixed value, B–H entropy and its correction term are derived on the background of a rotating and charged black string. From the viewpoint of quantum statistical mechanics, the B–H entropy of the black hole and its correction value are obtained. This semi-classical method provides a valuable reference for recognizing the quantum effect of black hole thermodynamics. This discussion helps to further research on the origin and natural properties of black string entanglement entropy. It allows fundamental recognition of the entropy and its correction value on the background of a black string at nonspherical coordinates.

Comparing λ value of Eqs.(26) and (30) with the value given in Ref.[40], we know that the λ value in rotating and charged black string spacetime is the same as in axially symmetric spacetime. Whether the λ value is only related to the dimensionality of spacetime or not, this problem will be investigated in future.

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