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Teleportation of tripartite entangled coherent states*

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This paper proposes a feasible scheme for the quantum teleportation of tripartite entangled coherent states by using linear optical devices such as beam splitters, phase shifters and photo detectors. The scheme is based on the bipartite maximally entangled coherent state and the tripartite entangled coherent state with bipartite maximal entanglement as quantum channels. It shows that when the mean number of photons is equal to 2, the total minimum of the average fidelity for an arbitrary tripartite entangled state is $1 - 0.67 \times 10^{-3}$.

Keywords: teleportation, entangled coherent state, transformation
PACC: 0365, 4250

1. Introduction

Quantum teleportation, first proposed by Bennett *et al*,[1] is a method of transmitting complete information about an unknown state from Alice (sender) to Bob (receiver) in a distant place with the help of long-range EPR correlation.[2,3] The original proposals for quantum teleportation concerned teleporting quantum states of a system in a finite-dimensional (discrete variable) Hilbert space.[4−8] Recently, quantum teleportation with continuous variable (CV) in infinite-dimensional space has attracted much attention from both theorists[9−11] and experimentalists.[12−17] CV promises to be not only efficient and secure, but also easy in generation as well as detection.

In Ref.[18], van Enk discussed how to teleport a Schrödinger cat state[19] $\varepsilon_+ |\alpha\rangle + \varepsilon_- |-\alpha\rangle$ with maximally bipartite entangled coherent states (ECS) which play the role of the quantum channel, and concluded that the success probability is $\frac{1}{2}$. In Ref.[20], Wang proposed a teleportation scheme of bipartite ECS $\varepsilon_+ |\alpha,\alpha\rangle_{12} + \varepsilon_- |-\alpha,-\alpha\rangle_{12}$ with maximally tripartite ECS as the quantum channel and concluded that the probability for successful teleportation is only $\frac{1}{2}$. In Ref.[21], Zheng *et al* proposed a scheme for teleportation of tripartite ECS $\varepsilon_+ |\alpha,\alpha,\alpha\rangle_{123} + \varepsilon_- |-\alpha,-\alpha,-\alpha\rangle_{123}$ by using bipartite maximally ECS and tripartite ECS with bipartite maximal entanglement as the quantum channels, and calculated the probability of successful teleportation, which is 0.4998 for a mean number of photons equal to 2. In Ref.[22], Prabash *et al* improved Wang's scheme by making small changes in the Bell-state measurement, and calculated the minimum average fidelity for successful teleportation. The calculated value is $1 - 0.98 \times 10^{-4}$ for a mean number of photons equal to 2. In this paper, we improve Prabash's scheme by making small change in phase shifters and using bipartite maximally ECS and tripartite ECS with bipartite maximal entanglement as the quantum channels, and propose a quantum teleportation scheme for an arbitrary tripartite ECS $\varepsilon_+ |\alpha,\alpha,\alpha\rangle_{123} + \varepsilon_- |-\alpha,-\alpha,-\alpha\rangle_{123}$. It will be shown that the total minimum of the average fidelity for successful teleportation is $1 - 0.67 \times 10^{-3}$ for a mean number of photons equal to 2. We shall also show that our scheme is not only feasible with the current technologies, but also effective for the quantum information processing.

2. Teleportation of a tripartite entangled coherent state

According to the conditional methods,[20] if the tripartite ECS is to be teleported, at least a four-particle entangled state is required as the quantum channel. Considering particularly the experimental achievement, it is too difficult to generate the multi-

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particle entangled system, especially when $N > 3$. Alternatively, we employ the bipartite maximally entangled state

$$|\phi\rangle_{45} = N_{45}\left(|\alpha\rangle_4|\alpha\rangle_5 - |-\alpha\rangle_4|-\alpha\rangle_5\right),$$
$$N_{45} = \frac{1}{\sqrt{2\left(1-x^4\right)}},$$
$$x = \mathrm{e}^{-|\alpha|^2}, \tag{1}$$

and the tripartite entangled state with bipartite maximal entanglement

$$|\phi\rangle_{678} = N_{678}(|\sqrt{2}a\rangle|\alpha\rangle_7|\alpha\rangle_8 - |-\sqrt{2}\alpha\rangle_6|-\alpha\rangle_7|-\alpha\rangle_8),$$
$$N_{678} = \frac{1}{\sqrt{2\left(1-x^8\right)}} \tag{2}$$

as quantum channels to teleport the tripartite ECS

$$|\phi\rangle_{123} = \varepsilon_+|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3 + \varepsilon_-|-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3 \tag{3}$$

with the normalization condition $|\varepsilon_+|^2 + |\varepsilon_-|^2 + 2\mathrm{e}^{-6|\alpha|^2}\mathrm{Re}\left(\varepsilon_+^*\varepsilon_-\right) = 1$.

We firstly introduce a key operation $\mathcal{B}_{ij} = P_jB_{ij}P_j$ which equips the beam splitter B_{ij} by a pair of $-\frac{\pi}{2}$ phase shifter P_j, and transforms the coherent state $|\alpha\rangle_i|\beta\rangle_j$ as

$$\mathcal{B}_{ij}|\alpha\rangle_i|\beta\rangle_j = \left|\frac{(\alpha+\beta)}{\sqrt{2}}\right\rangle_i\left|\frac{(\alpha-\beta)}{\sqrt{2}}\right\rangle_j. \tag{4}$$

Now Alice wishes to teleport the ECS $|\phi\rangle_{123}$ to a remote partner Bob by sharing $|\phi\rangle_{45}$ and $|\phi\rangle_{678}$. Systems 1, 2, 3, 4 and 6 are at Alice's side, systems 5, 7 and 8 are at Bob's side. The initial state of the whole system is then given by $|\phi\rangle_{12345678} = |\phi\rangle_{123}|\phi\rangle_{45}|\phi\rangle_{678}$. First we disentangle the system by applying the operator $\mathcal{B}_{12}$ to $|\phi\rangle_{12345678}$

$$\begin{aligned}&\mathcal{B}_{12}|\phi\rangle_{12345678}\\ =&|0\rangle_1(\varepsilon_+|\sqrt{2}\alpha\rangle_2|\sqrt{2}\alpha\rangle_3 + \varepsilon_-|-\sqrt{2}\alpha\rangle_2|-\sqrt{2}\alpha\rangle_3)|\phi\rangle_{45}|\phi\rangle_{678} = |0\rangle_1|\phi'\rangle_{2345678}.\end{aligned} \tag{5}$$

Then, by applying the second transformation $\mathcal{B}_{34}$ to $|\phi'\rangle_{2345678}$, we obtain

$$\begin{aligned}|\phi''\rangle_{2345678} = N_{45}[&\varepsilon_+(|\sqrt{2}\alpha\rangle_2|\sqrt{2}\alpha\rangle_3|0\rangle_4|\alpha\rangle_5 - |\sqrt{2}\alpha\rangle_2|0\rangle_3|\sqrt{2}\alpha\rangle_4|-\alpha\rangle_5)\\ &+\varepsilon_-(|-\sqrt{2}\alpha\rangle_2|0\rangle_3|-\sqrt{2}\alpha\rangle_4|\alpha\rangle_5 - |-\sqrt{2}\alpha\rangle_2|-\sqrt{2}\alpha\rangle_3|0\rangle_4|-\alpha\rangle_5)]|\phi\rangle_{678}.\end{aligned} \tag{6}$$

After these two transformations, Alice performs a two-mode photon number measurement on modes 3 and 4. The probability of finding k and l photons in modes 3 and 4 is given by

$$P(k,l) = |{}_3\langle k|{}_4\langle l|\phi''\rangle_{2345678}|^2. \tag{7}$$

Hence in order to have nonzero probability one of the two integers k and l must be zero. From Eqs.(6) and (7), the probabilities $P(k,0)$ and $P(0,l)$ for k and l are obtained as

$$P(k,0) = \frac{x^4|2\alpha|^{2k}}{4k!(1-x^8)(1-x^4)},$$
$$P(0,l) = \frac{x^4|2\alpha|^{2l}}{4l!(1-x^8)(1-x^4)}. \tag{8}$$

We find that the result of photon counting in outputs in modes 3 and 4 is either (i) odd in mode 3 and zero in mode 4; (ii) zero in mode 3 and odd in mode 4; (iii) zero in mode 3 and even in mode 4; (iv) even in mode 3 and zero in mode 4.

With the probabilities of number measurement the result is

$$\begin{aligned}P_{\mathrm{odd}}(k,0) = \sum_{\mathrm{oddk}}P(k,0) = P_{\mathrm{odd}}(0,l) = \sum_{\mathrm{oddl}}P(0,l)\\ = P_{\mathrm{even}}(k,0) = \sum_{\mathrm{evenk}}P(k,0) = P_{\mathrm{even}}(0,l)\\ = \sum_{\mathrm{evenl}}P(0,l) = \frac{1}{4}.\end{aligned} \tag{9}$$

Let us suppose $k \neq 0$ when $l = 0$, in this case $|\phi''\rangle_{2345678}$ collapses into

$$\begin{aligned}|\phi''_{k0}\rangle_{25678} = N_{45}N_{678}[&\varepsilon_+(|\sqrt{2}\alpha\rangle_2|\alpha\rangle_5|\sqrt{2}\alpha\rangle_6|\alpha\rangle_7|\alpha\rangle_8 - |\sqrt{2}\alpha\rangle_2|\alpha\rangle_5|-\sqrt{2}\alpha\rangle_6|-\alpha\rangle_7|-\alpha\rangle_8)\\ &-(-1)^k\varepsilon_-(|-\sqrt{2}\alpha\rangle_2|-\alpha\rangle_5|\sqrt{2}\alpha\rangle_6|\alpha\rangle_7|\alpha\rangle_8 - |-\sqrt{2}\alpha\rangle_2|-\alpha\rangle_5|-\sqrt{2}\alpha\rangle_6|-\alpha\rangle_7|-\alpha\rangle_8)].\end{aligned} \tag{10}$$

Alice applies the third transformation $\mathcal{B}_{26}$, we obtain

$$|\phi'''_{k0}\rangle_{25678} = N_{45}N_{678}[\varepsilon_+(|2\alpha\rangle_2|\alpha\rangle_5|0\rangle_6|\alpha\rangle_7|\alpha\rangle_8 - |0\rangle_2|\alpha\rangle_5|2\alpha\rangle_6|\sqrt{2}\alpha\rangle_7|-\alpha\rangle_8) - (-1)^k\varepsilon_-(|0\rangle_2|-\alpha\rangle_5|-2\alpha\rangle_6|\alpha\rangle_7|\alpha\rangle_8 - |-2\alpha\rangle_2|-\alpha\rangle_5|0\rangle_6|-\alpha\rangle_7|-\alpha\rangle_8)]. \quad (11)$$

From Eq.(11), we can draw a conclusion that k may get odd or even numbers.

Let us first consider the result when k is odd and l is zero. In this case the state $|\phi'''_{k0}\rangle_{25678}$ becomes

$$|\phi'''_{k_{\rm odd}0}\rangle_{14578} = N_{45}N_{678}[\varepsilon_+(|2\alpha\rangle_2|\alpha\rangle_5|0\rangle_6|\alpha\rangle_7|\alpha\rangle_8 - |0\rangle_2|\alpha\rangle_5|2\alpha\rangle_6|-\alpha\rangle_7|-\alpha\rangle_8) + \varepsilon_-(|0\rangle_2|-\alpha\rangle_5|-2\alpha\rangle_6|\alpha\rangle_7|\alpha\rangle_8 - |-2\alpha\rangle_2|-\alpha\rangle_5|0\rangle_6|-\alpha\rangle_7|-\alpha\rangle_8)]. \quad (12)$$

We can expand the state $|\pm 2\alpha\rangle$ in terms of vacuum non-zero even (NZE) and odd number states as[23]

$$|\pm 2\alpha\rangle = x^2|0\rangle + \frac{1}{\sqrt{2}}(1-x^4)|{\rm NZE}, 2\alpha\rangle \pm \sqrt{\frac{1}{2}(1-x^8)}|{\rm ODD}, 2\alpha\rangle. \quad (13)$$

Using Eq.(13) for $|\pm 2\alpha\rangle$, we can write the output state as

$$\begin{aligned}|\phi'''_{k_{\rm odd}0}\rangle_{25678} = {} & \frac{x^2}{2\sqrt{(1-x^4)(1-x^8)}}|0\rangle_2|0\rangle_6[\varepsilon_+(|\alpha\rangle_5|\alpha\rangle_7|\alpha\rangle_8 - |\alpha\rangle_5|-\alpha\rangle_7|-\alpha\rangle_8) \\ & + \varepsilon_-(|-\alpha\rangle_5|\alpha\rangle_7|\alpha\rangle_8 - |-\alpha\rangle_1|-\alpha\rangle_7|-\alpha\rangle_5)] \\ & + \frac{1}{2\sqrt{2}\sqrt{1+x^4}}[|{\rm NZE}, 2\alpha\rangle_2|0\rangle_6(\varepsilon_+|\alpha\rangle_5|\alpha\rangle_7|\alpha\rangle_8 - \varepsilon_-|\alpha\rangle_5|\alpha\rangle_7|\alpha\rangle_8) \\ & + |0\rangle_2|{\rm NZE}, 2\alpha\rangle_6(-\varepsilon_+|\alpha\rangle_5|-\alpha\rangle_7|-\alpha\rangle_8 + \varepsilon_-|-\alpha\rangle_5|\alpha\rangle_7|\alpha\rangle_8)] \\ & + \frac{1}{2\sqrt{2(1-x^4)}}[|{\rm ODD}, 2\alpha\rangle_2|0\rangle_6(\varepsilon_+|\alpha\rangle_5|\alpha\rangle_7|\alpha\rangle_8 + \varepsilon_-|-\alpha\rangle_5|-\alpha\rangle_7|\alpha\rangle_8) \\ & + |0\rangle_2|{\rm ODD}, \alpha\rangle_6(-\varepsilon_+|\alpha\rangle_5|\alpha\rangle_7|-\alpha\rangle_8 - \varepsilon_-|-\alpha\rangle_5|-\alpha\rangle_7|\alpha\rangle_8)]. \end{aligned} \quad (14)$$

Now Alice performs a two-mode photon number measurement on the modes 2 and 6 and conveys this result to Bob through some classical channel. After receiving this information Bob makes some unitary transformation on states in modes 5, 7 and 8 (if necessary) and then retrieves the original information.

The results of the photon counting done on modes 2 and 6 may be one of I result (zero, zero) , II result (nonzero even, 0) , III result (0, nonzero even), IV result (odd, 0) and V result (0, odd).

In the result $(0, 0)$, the teleportation fails. However the probability of getting this result is very small. For the case of outcome (0, odd) happening, Bob makes a phase shifter $-\widehat{P_7}(\pi)\widehat{P_8}(\pi)$ on modes 7, 8, and the teleportation is perfectly successful. When the outcome (odd, 0) appears, it can be seen that Bob obtains a perfect replica of original unknown state without any operation.

We can write $|\phi\rangle_{123}$ as

$$|\phi\rangle_{123} = A_+|{\rm EVEN}, \alpha, \alpha, \alpha\rangle_{123} + A_-|{\rm ODD}, \alpha, \alpha, \alpha\rangle_{123}, \quad (15)$$

where

$$|{\rm EVEN}, \alpha, \alpha, \alpha\rangle_{123} = \frac{|\alpha, \alpha, \alpha\rangle_{123} + |-\alpha, -\alpha, -\alpha\rangle_{123}}{\sqrt{2(1+x^6)}} \quad (16)$$

and

$$|{\rm ODD}, \alpha, \alpha, \alpha\rangle_{123} = \frac{|\alpha, \alpha, \alpha\rangle_{123} - |-\alpha, -\alpha, -\alpha\rangle_{123}}{\sqrt{2(1-x^6)}} \quad (17)$$

are orthogonal to each other. Constants $A_\pm$ are related to $\varepsilon_\pm$ by

$$\begin{aligned} A_\pm &= (\varepsilon_+ \pm \varepsilon_-)\sqrt{\frac{(1 \pm x^6)}{2}}, \\ \varepsilon_\pm &= [2(1+x^6)]^{-\frac{1}{2}}A_+ \pm [2(1-x^6)]^{-\frac{1}{2}}A_-. \end{aligned} \quad (18)$$

Corresponding to the outcome (nonzero even , 0), Bob obtains the states

$$|\psi_{k_{\rm odd},0}\rangle^1 = \varepsilon_+|\alpha\rangle_5|\alpha\rangle_7|\alpha\rangle_8 - \varepsilon_-|-\alpha\rangle_5|-\alpha\rangle_7|-\alpha\rangle_8. \tag{19}$$

Using Eqs.(15), (16) and (17), we can write $|\psi_{k_{\rm odd},0}\rangle^1$ as

$$|\psi_{k_{\rm odd},0}\rangle^1 = A_-\sqrt{\frac{1+x^6}{1-x^6}}|{\rm EVEN},\alpha,\alpha,\alpha\rangle_{578} + A_+\sqrt{\frac{1-x^6}{1+x^6}}|{\rm ODD},\alpha,\alpha,\alpha\rangle_{578}. \tag{20}$$

Then Bob makes the unitary transformations with

$$U^1_{k_{\rm odd},0} = |{\rm EVEN},\alpha,\alpha,\alpha\rangle_{578578}\langle{\rm ODD},\alpha,\alpha,\alpha| + |{\rm ODD},\alpha,\alpha,\alpha\rangle_{578578}\langle{\rm EVEN},\alpha,\alpha,\alpha|, \tag{21}$$

which changes $|\psi_{k_{\rm odd},0}\rangle^1$ to

$$|\psi'_{k_{\rm odd},0}\rangle^1 \sim \sqrt{\frac{1-x^6}{1+x^6}}A_+|{\rm EVEN},\alpha,\alpha,\alpha\rangle_{578} + \sqrt{\frac{1+x^6}{1-x^6}}A_-|{\rm ODD},\alpha,\alpha,\alpha\rangle_{578}. \tag{22}$$

When (0, nonzero even) appears, Bob makes a phase shifter $\widehat{P_7}(\pi)\widehat{P_8}(\pi)$ on modes 7 and 8. We obtain

$$|\psi_{k_{\rm odd},0}\rangle^2 = -\sqrt{\frac{1+x^6}{1-x^6}}A_-|{\rm EVEN},\alpha,\alpha,\alpha\rangle_{578} - \sqrt{\frac{1-x^6}{1+x^6}}A_+|{\rm ODD},\alpha,\alpha,\alpha\rangle_{578}. \tag{23}$$

Then Bob makes the unitary transformation with

$$\begin{aligned} U^2_{k_{\rm odd},0} = &-|{\rm EVEN},\alpha,\alpha,\alpha\rangle_{578578}\langle ODD,\alpha,\alpha,\alpha| \\ &- |{\rm ODD},\alpha,\alpha,\alpha\rangle_{578578}\langle{\rm EVEN},\alpha,\alpha,\alpha|, \end{aligned} \tag{24}$$

which changes $|\psi_{k_{\rm odd},0}\rangle^2$ to

$$|\psi'_{k_{\rm odd},0}\rangle^2 \sim \sqrt{\frac{1-x^6}{1+x^6}}A_+|{\rm EVEN},\alpha,\alpha,\alpha\rangle_{578} + \sqrt{\frac{1+x^6}{1-x^6}}A_-|{\rm ODD},\alpha,\alpha,\alpha\rangle_{578}. \tag{25}$$

In conclusion, the teleportation is perfect successful with fidelity 1 for results (odd, 0) and (odd, 0) while it fails for result (0, 0). For results (0, nonzero even) and (nonzero even, 0) the fidelity is

$$F = \frac{[1-x^6(|A_+|^2-|A_-|^2)]^2}{1+x^{12}-2x^6(|A_+|^2-|A_-|^2)}. \tag{26}$$

If we write $\dfrac{A_+}{A_-} = \tan\dfrac{\theta}{2}{\rm e}^{{\rm i}\theta}$, the fidelity of Eq.(26) is found to have two maxima at $\theta = 0$ and π, and a minimum at $\theta = \cos^{-1}(x^6)$ and $F_{\rm min} = (1-x^{12})$. This is quite close to unity for an appreciable value of $|\alpha|^2$.

In Ref.[22], Prakash *et al* define minimum average fidelity (MAF) as the sum of the product of probability for occurrence of a case and the corresponding minimum fidelity. In the case that k is odd when l is 0 they showed that the MAF is

$$F_{\rm av.min.}{}_{k_{\rm odd},0} = \frac{-x^4-x^{-2}+x^{-4}+2x^{-6}-x^{-10}-2x^{-12}+2x^{-16}}{2-2x^{-2}+2x^{-4}+4x^{-6}-2x^{-10}+2x^{-12}+2x^{-16}}. \tag{27}$$

Now we consider the result in the case that k is even when l is zero. In this case the state $|\phi'''_{k0}\rangle_{25678}$ becomes

$$\begin{aligned} |\phi'''_{k_{\rm odd}0}\rangle_{25678} = N_{45}N_{678}[&\varepsilon_+(|2\alpha\rangle_2|\alpha\rangle_5|0\rangle_6|\alpha\rangle_7|\alpha\rangle_8 - |0\rangle_2|\alpha\rangle_5|2\alpha\rangle_6|-\alpha\rangle_7|-\alpha\rangle_8) \\ &-\varepsilon_-(|0\rangle_2|-\alpha\rangle_5|-2\alpha\rangle_6|\alpha\rangle_7|\alpha\rangle_8 - |-2\alpha\rangle_2|-\alpha\rangle_5|0\rangle_6|-\alpha\rangle_7|-\alpha\rangle_8)]. \end{aligned} \tag{28}$$

Following the same steps as before, Bob makes some unitary transformation corresponding to the result of the two-mode number measurement on modes 3 and 4 which is sent by Alice through some classical channel, and then retrieves the original information.

For the result (0, 0), the teleportation fails. Corresponding to the outcome (nonzero even, 0), it can be seen that the teleportation is perfectly successful.

When (0, nonzero even) appears, Bob makes a phase shifter $-\widehat{P_7}(\pi)\widehat{P_8}(\pi)$ on modes 7 and 8, we obtain

$$|\psi'_{k_{\rm even},0}\rangle^1 = \varepsilon_+|\alpha\rangle_5|\alpha\rangle_7|\alpha\rangle_8 + \varepsilon_-|-\alpha\rangle_5|-\alpha\rangle_7|-\alpha\rangle_8, \tag{29}$$

which is a perfect replica of the information state.

For the case of outcome (0, odd), Bob makes a phase shifter $\widehat{P_7}(\pi)\widehat{P_8}(\pi)$ on modes 7 and 8, we obtain

$$|\psi_{k_{\rm even},0}\rangle^2 = -\sqrt{\frac{1+x^6}{1-x^6}}A_-|{\rm EVEN},\alpha,\alpha,\alpha\rangle_{578} - \sqrt{\frac{1-x^6}{1+x^6}}A_+|{\rm ODD},\alpha,\alpha,\alpha\rangle_{578}. \tag{30}$$

Then Bob makes the unitary transformations with

$$U^2_{k_{\rm even},0} = -|{\rm EVEN},\alpha,\alpha,\alpha\rangle_{578578}\langle{\rm ODD},\alpha,\alpha,\alpha| - |{\rm ODD},\alpha,\alpha,\alpha\rangle_{578578}\langle{\rm EVEN},\alpha,\alpha,\alpha|, \tag{31}$$

which changes $|\psi_{k_{\rm even},0}\rangle^2$ to

$$|\psi'_{k_{\rm even},0}\rangle^2 \sim \sqrt{\frac{1-x^6}{1+x^6}}A_+|{\rm EVEN},\alpha,\alpha,\alpha\rangle_{578} + \sqrt{\frac{1+x^6}{1-x^6}}A_-|{\rm ODD},\alpha,\alpha,\alpha\rangle_{578}. \tag{32}$$

When the outcome (odd, 0) appears, Bob obtains the states

$$|\psi_{k_{\rm even},0}\rangle^3 = \sqrt{\frac{1+x^6}{1-x^6}}A_-|{\rm EVEN},\alpha,\alpha,\alpha\rangle_{578} + \sqrt{\frac{1-x^6}{1+x^6}}A_+|{\rm ODD},\alpha,\alpha,\alpha\rangle_{578}. \tag{33}$$

Then Bob makes the unitary transformations with

$$U^3_{k_{\rm even},0} = |{\rm EVEN},\alpha,\alpha,\alpha\rangle_{578578}\langle{\rm ODD},\alpha,\alpha,\alpha| + |{\rm ODD},\alpha,\alpha,\alpha\rangle_{578578}\langle{\rm EVEN},\alpha,\alpha,\alpha|, \tag{34}$$

which change $|\psi_{k_{\rm even},0}\rangle^3$ to

$$|\psi'_{k_{\rm even},0}\rangle^3 \sim \sqrt{\frac{1-x^6}{1+x^6}}A_+|{\rm EVEN},\alpha,\alpha,\alpha\rangle_{578} + \sqrt{\frac{1+x^6}{1-x^6}}A_-|{\rm ODD},\alpha,\alpha,\alpha\rangle_{578}. \tag{35}$$

From Eqs.(29), (33) and (37), the teleportation is perfectly successful with fidelity 1 for result (0, nonzero even) and (nonzero even, 0) while it fails for result (0, 0). For results (odd, 0) and (0, odd) the fidelity is

$$F = \frac{[1-x^6(|A_+|^2-|A_-|^2)]^2}{1+x^{12}-2x^6(|A_+|^2-|A_-|^2)}. \tag{36}$$

The MAF for the case in which k is even when l is 0 is

$$F_{\rm av.\,min.k_{even},0} = \frac{-3+x^4+x^{-2}+4x^{-4}-x^{-6}-4x^{-8}+2x^{-12}}{2+2x^{-2}-2x^{-6}+2x^{-12}}. \tag{37}$$

As seen from Fig.1, the $F_{\rm av.min.k_{odd},0}$ is different from the $F_{\rm av.min.k_{even},0}$. The reason is that the outcomes (odd, 0) and (0, odd) for the case in which k is odd when l is zero are perfectly successful with fidelity 1, but for the case in which k is even when l is 0 is almost perfect. This reflects the action of the π phase shifter.

Following similar steps as before, we can obtain the MAF for the case in which l is even when k is 0 is $F_{\rm av.min.0,l_{even}} = F_{\rm av.min.k_{even},0}$ and the MAF for the case in which l is odd when k is 0 is $F_{\rm av.min.0,l_{odd}} = F_{\rm av.min.k_{odd},0}$.

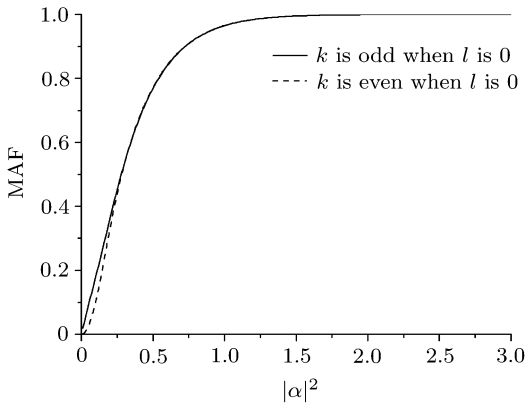


Fig.1. Variation of MAF against different values of $|\alpha|^2$.

Then, we define the total MAF(TMAF) as the sum of a case (i.e. k is even when l is zero, or k is odd when l is zero, or l is even when k is zero, or l is odd when k is zero) and the corresponding MAF.

The TMAF of teleportation tripartite ECS $|\phi\rangle_{123}$ is given by

$$F_{\text{av.total.min}} = \frac{-1-x^{-8}+3x^{-16}-4x^{-32}+3x^{-40}+6x^{-48}-6x^{-56}-8x^{-64}+4x^{-72}+7x^{-80}-3x^{-96}-2x^{-104}+2x^{-120}}{2x^{-8}+4x^{-32}+6x^{-40}-4x^{-48}-2x^{-56}+6x^{-72}+4x^{-80}-4x^{-96}+2x^{-104}+2x^{-120}}. \tag{38}$$

For $|\alpha|^2 = 1, 2$ and 3, the value of $F_{\text{av.total.min}}$ are $1-0.35\times10^{-1}$, $1-0.67\times10^{-3}$, $1-1.2\times10^{-5}$ respectively.

It is seen from Fig.2 that the TMAF of teleportation tripartite ECS $|\phi\rangle_{123}$ approaches unity for a mean number of photons $|\alpha|^2 > 2$. Hence, we conclude that, the proposed scheme can obtain nearly perfect teleportation for an appreciable mean number of photons.

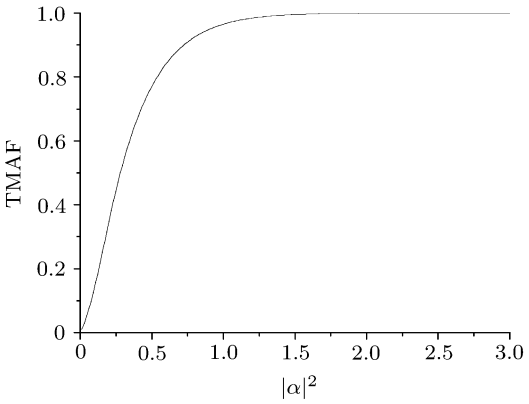


Fig.2. Variation of TMAF against different values of $|\alpha|^2$.

3. Conclusion

In conclusion, we have proposed a feasible scheme of quantum teleportation by using linear optical devices transport for an arbitrary tripartite ECS. In the scheme, we improve Prabash's scheme by making a small change in phase shifters and using the bipartite maximally ECS and the tripartite ECS with bipartite maximal entanglement as the quantum channels. We show that with this setup the TMAF increases with the increase of the mean number of photons $|\alpha|^2$, which can approach unity in the limit $|\alpha|^2 \to \infty$ and less than unity by a quantity of the order of $1-0.67\times10^{-3}$ for an appreciable mean number of photons equal to 2. Compared to the previously presented teleportation schemes,[18,20,21,22] the present version has some advantages. Firstly, it is easier to generate bipartite maximal ECS and tripartite ECS with bipartite maximal entanglement than multipartite maximal ECS with bipartite maximal entanglement, so our scheme is more experimentally realizable with the current technology. Secondly, it is shown that the proposed scheme can obtain a nearly perfect teleportation for an appreciable mean number of photons. Both the measurement and preparation of the quantum channel can be implemented in the experiments by the present techniques.[20] We expect that the present scheme can be used in the experiments to demonstrate the quantum teleportation of the tripartite ECS.

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