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Control of stochastic resonance in bistable systems by using periodic signals*

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According to the characteristic structure of double wells in bistable systems, this paper analyses stochastic fluctuations in the single potential well and probability transitions between the two potential wells and proposes a method of controlling stochastic resonance by using a periodic signal. Results of theoretical analysis and numerical simulation show that the phenomenon of stochastic resonance happens when the time scales of the periodic signal and the noise-induced probability transitions between the two potential wells achieve stochastic synchronization. By adding a bistable system with a controllable periodic signal, fluctuations in the single potential well can be effectively controlled, thus affecting the probability transitions between the two potential wells. In this way, an effective control can be achieved which allows one to either enhance or realize stochastic resonance.

Keywords: stochastic resonance, bistable system, control
PACC: 0250, 0545, 0540

1. Introduction

Since the concept of stochastic resonance (SR) was originally put forward by Benzi *et al*[1] in 1981, the law of SR and its performance in the various fields of natural science have been extensively and profoundly understood by people, and SR is now considered as a technology of application to be researched.[2−7] However, these studies on SR are done more from the perspective of observations rather than of the impacts. To increase the output power of a bistable system at the input signal frequency, Gammaitoni *et al*[8] first proposed the method of controlling SR, and then Löcher *et al*[9] and other groups[10−13] proposed a variety of SR control methods in different systems. Controlling SR has provided technical methods for the application of SR, and further promoted the in-depth study of SR theory.

SR is a nonlinear phenomenon caused by the interactions between weak periodic signals and bistable systems with the medium of noise. Specifically, for a certain noise, a nonlinear bistable system has a strong output response to the weak periodic signal and there exists a transfer mechanism of the noise energy to signal energy. The bistable system, a typical model of SR, is an important milestone in the history of SR, the in-depth research of which has undoubtedly greatly promoted the development of SR theory. Under the actions of signal and noise, the behaviours of two potential wells in a bistable system are no longer independent. The system within one potential well will drop into another with different probabilities in different times. There are two time scales, one determined by the periodic signal and another induced by noise, existing in the bistable system. When the two time scales achieve a random sync, SR happens. For an input periodic signal with a given frequency, the time scale of the evolution of system can be changed by regulating the noise intensity or the parameters of the bistable system to realize SR. However, in practical problems, the noise intensity is not arbitrarily controllable, and the system parameters are often objectively given and can not be altered or require a huge price to be changed.

In this paper, stochastic fluctuations in the single potential well and probability transitions between the two potential wells were analysed based on the double-well structure of the bistable system and the conditions for sync. By adding the bistable system with a controllable periodic signal, fluctuations in the single potential well can be effectively controlled, thus affecting the probability transitions between the two

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potential wells. In this way, an effective control can be achieved which allows us to either enhance or realize the SR.

2. Controlling SR with a periodic signal

2.1. Movements intrawell and between wells of a bistable system

As the classical model of SR, we considered the following one-dimensional dynamic system:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -V'(x) + A\cos(\omega_0 t) + N(t), \tag{1}$$

where $V(x) = -\frac{1}{2}ax^2 + \frac{x^4}{4}$ is the double-well function, $N(t)$ denotes a zero-mean, Gaussian white noise with autocorrelation function $\langle N(t)N(0)\rangle = 2D\delta(t)$ and intensity D. Equation (1) describes the overdamped motion of a particle in a bistable potential in the presence of noise and periodic force. When $A = D = 0$, the system has two stable states at $x_m = \pm\sqrt{a}$.

In the absence of periodic force ($A = 0$) and in the presence of noise ($D \neq 0$) with small enough intensity D, the dynamic equation Eq.(1) can be simplified as the following forms of the one order system:[14]

$$\dot{x} = -2a(x - x_m) + N(t). \tag{2}$$

Then $T_\mathrm{I} = 1/2a$, which has the time dimension, is the characteristic time scale of the stochastic fluctuations in the single potential well in a bistable system.

According to the Kramers escape theory, the escape rate of a particle between the two potential wells is

$$r_\mathrm{k} = \frac{a}{\sqrt{2}\pi}\exp\left(-\frac{a^2}{4D}\right), \tag{3}$$

and the characteristic time scale of probability transitions between the two potential wells is

$$T_\mathrm{K} = 2/r_\mathrm{k} = \frac{2\sqrt{2}\pi\exp(a^2/4D)}{a}. \tag{4}$$

Therefore, there are two different forms of movements in the bistable system with the action of noise, fluctuations within the well and the transitions between the two wells, and they have different time scales. The traditional theory of SR only considers transitions between the two potential wells, and uses stochastic synchronization between T_K and $2\pi/\omega_0$ as the matching condition for SR, neglecting the influence of stochastic fluctuations in the single potential well, where T_K is a random variable. When a control signal was introduced into the bistable system with the bistable potential function and the fixed intensity of noise, it can effectively lead the intrawell random fluctuations and thus effectively control the time scale of the transitions between two potential wells to achieve the synchronization between T_K and $2\pi/\omega_0$ artificially. In this way, an effective control can be achieved which allows us to either enhance or realize the SR.

2.2. Controlling SR with a periodic signal

The movement of a bistable system consists of the intrawell random fluctuations and the transition between the two potential wells. The intrawell fluctuations occupy a longer time and have associations with the characteristic time scale T_I. In fact, the bistable system can be divided into macro level and micro level according to levels involved in the movements of the system. The probability transition between the two potential wells is the movement at macro level, and the random fluctuation within a single potential well is relatively the micro-movement. These two levels not only have different time scales and unique rules, but also have close relations with each other. So adding the system with a control periodic signal to lead the intrawell random fluctuations will surely have an influence on the transitions between the two potential wells. If the stochastic synchronization between the periodic signal and the transitions between the two potential wells was achieved artificially, SR can be realized.

As shown in Fig.1, a bistable system with the control signal was constructed. The corresponding dynamic equation is

$$\frac{\mathrm{d}x}{\mathrm{d}t} = ax - x^3 + A\cos(\omega_0 t) + B\cos(\Omega t) + N(t), \tag{5}$$

where $A\cos(\omega_0 t)$ is the input periodic signal. When $B = 0$, Eq.(5) is the extensively studied equation of typical SR. In this paper, the periodic signal $B\cos(\Omega t)$ was added on the right side of Eq.(5). The amplitude B and frequency Ω are controllable variables, and the amplitude of the power spectrum at frequency ω_0 of the bistable system output $x(t)$ can be also considered as a variable.

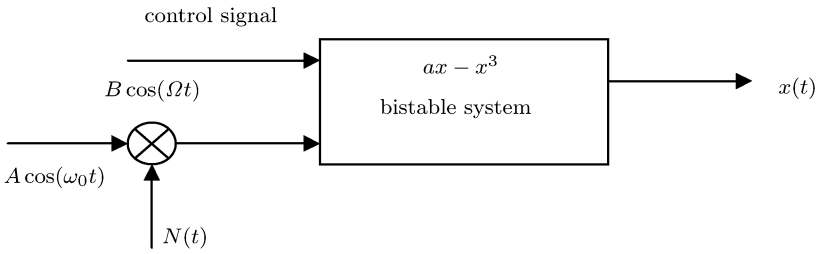


Fig.1. Controlling SR with a periodic signal.

The control periodic signal is directly added to the equation as a driving force. Its macro function is to control the power spectrum value of the system output $x(t)$ at frequency ω_0, and the mechanism of micro function is leading the intrawell random fluctuations. Because the time-scale of the macro-level is greater than the time-scale of the micro-level, that is, $T_{\mathrm{K}} > T_{\mathrm{I}}$, the frequency of the control signal must satisfy $\omega_0 < \Omega < 2\pi/T_{\mathrm{I}}$ to make the periodic control signal able to lead the micro-movement.

3. Numerical simulation results and analysis

Figure 1 shows the SR of a bistable system with a controllable periodic signal, which is determined by the corresponding dynamic equation Eq.(5). The output response $x(t)$ of the bistable system has relationships with parameter a of the bistable system, parameters A and ω_0 of the input signal $A\cos(\omega_0 t)$, parameters B and Ω of control signal $B\cos(\Omega t)$, and intensity D of the noise $N(t)$. With the parameter $a = 0.9$ fixed for the bistable system, the roles of the control signal for SR were analysed with different amplitudes of input signal and intensities of noise.

3.1. Small amplitude of input signal and intensity of the noise

The function $x(t)$ can only fluctuate in the single potential well with a small amplitude of the input signal and intensity of noise. As shown in Fig.2, when the amplitude and frequency of the input periodic signal are $A = 0.2$ and $\omega_0 = 0.01\pi$, intensity $D = 0.3$, output response $x(t)$ of the bistable system cannot cross the barrier into another well, and thus the phenomenon of SR will not happen, and the corresponding output power spectrum is very small.

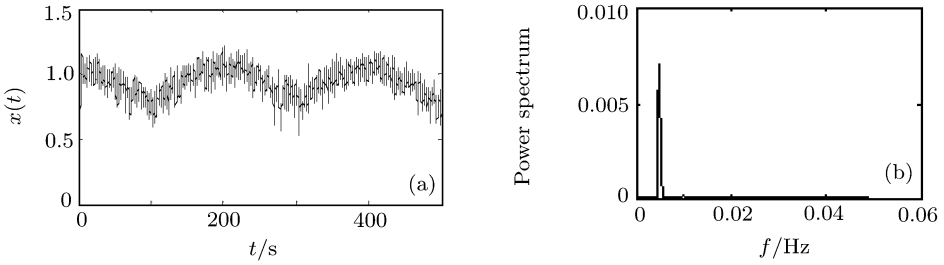


Fig.2. Output response $x(t)$ with $A = 0.2$ and $D = 0.3$. (a) Waveforms, (b) power spectrum.

Now we introduce the control signal into system. When the parameters of the control signal are $B = 0.3$ and $\Omega = 0.06\pi$, $x(t)$ can cross the barrier and jump between the potential wells. The obvious phenomenon of SR occurs in the bistable system, and the output power spectrum at the frequency ω_0 is

$P_s(\omega_0) = 0.7428 \gg 0.0065$, as shown in Fig.3. The control signal determines the intrawell fluctuation frequency of $x(t)$ and also the transition rate between the two potential wells, so it has the function of causing the system which does not meet the conditions of SR to make it.

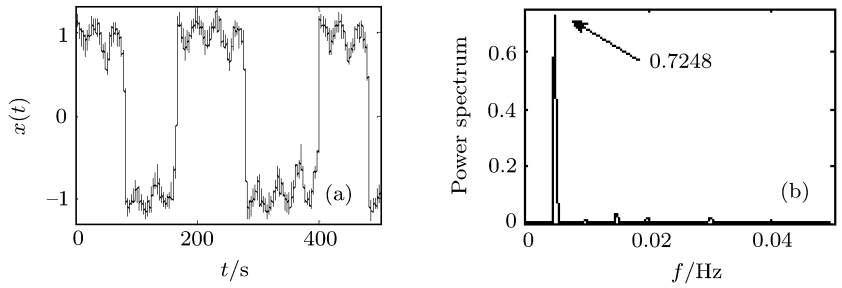


Fig.3. SR with parameters $B = 0.3$ and $\Omega = 0.06\pi$ of control signal. (a) Waveforms, (b) power spectrum.

If the same intensity $D = 0.3$ of the noise was kept and the amplitude of the input signal was reduced to $A = 0.15$, the amplitude B of the control signal must be increased to make the system realize SR. As shown in Fig.4, the amplitude $B = 0.36$ was used.

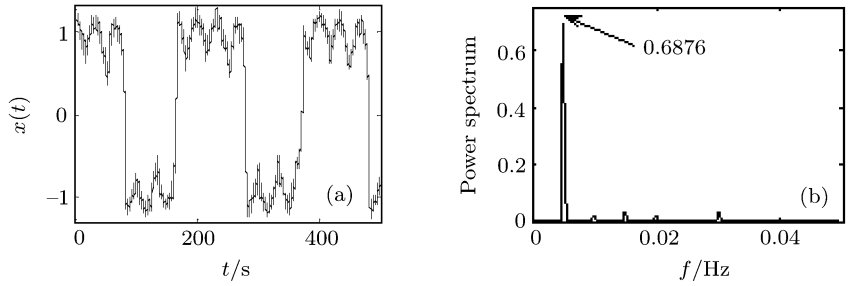


Fig.4. SR with parameters $A = 0.15$ of input signal and $B = 0.36$ of control signal. (a) Waveforms, (b) power spectrum.

3.2. Suitable amplitude of input signal and intensity of the noise

With the parameters $A = 0.30$, $\omega_0 = 0.01\pi$ and $D = 2.2$, the system exhibits the phenomenon of traditional SR, as shown in Fig.5, and the output power spectrum is $P_s(\omega_0) = 0.6881$.

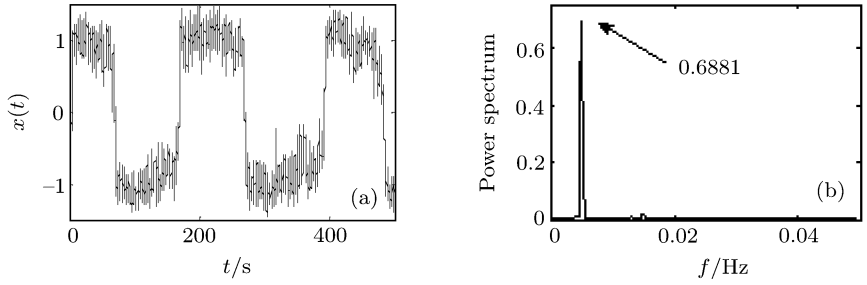


Fig.5. Traditional SR with parameters $A = 0.3$, $D = 2.2$. (a) Waveforms, (b) power spectrum.

When introducing the control signal with the parameters $B = 0.05$, $\Omega = 0.06\pi$ into the system, the SR effect of the bistable system was significantly enhanced and the output power spectrum at the frequency ω_0 was $P_s(\omega_0) = 0.7407 > 0.6881$, as shown in Fig.6. It was the control signal that enhanced the effect of SR. And the small power of the control periodic signal indicates that the control function was realized within the potential well through resonance.

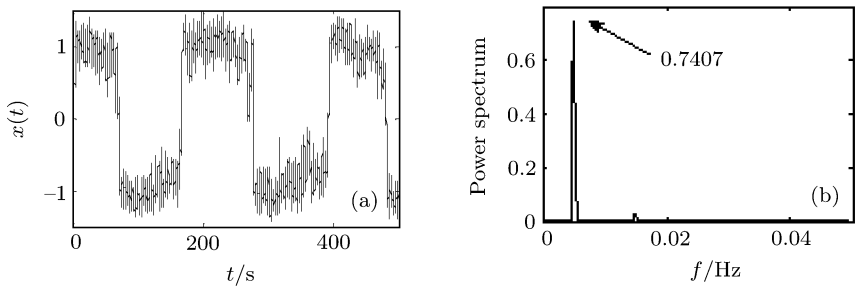


Fig.6. SR with parameters $B = 0.05$, $\Omega = 0.06\pi$ of control signal. (a) Waveforms, (b) power spectrum.

3.3. Large amplitude of input signal and intensity of the noise

The function $x(t)$ can easily transit between the potential wells with large amplitude of input signal and intensity of noise, but the transition rate does not match the frequency of the input signal and SR can not be realized. As shown in Fig.7, with the paramenters $A = 0.2$, $\omega_0 = 0.01\pi$, $D = 3.2$, $x(t)$ transits between the potential wells, and the output power spectrum is $P_s(\omega_0) = 0.4407$.

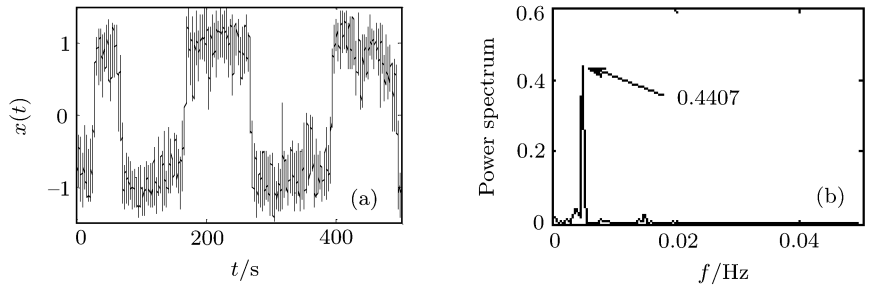


Fig.7. Output response $x(t)$ with $A = 0.2$ and $D = 0.32$. (a) Waveforms, (b) power spectrum.

When introducing a control signal with the parameters $B = 0.1$, $\Omega = 0.04\pi$ into the system, the obvious phenomenon of SR was exhibited and the output power spectrum at the frequency ω_0 was $P_s(\omega_0) = 0.5558 > 0.4407$, as shown in Fig.8. The simulation results show that the control signal can effectively lead the intrawell fluctuations, making the transitions between two potential wells be more in order and the energy of output signal be focused at the frequency ω_0.

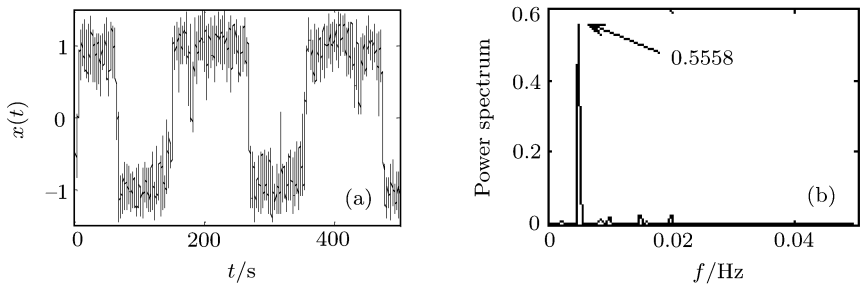


Fig.8. SR with parameters $B = 0.1$, $\Omega = 0.04\pi$ of control signal. (a) Waveforms, (b) power spectrum.

4. Conclusion

Fluctuations in the single potential well and transitions between the two potential wells exist in the double-well bistable system. There are many forms of movements in the bistable system under the actions of the input signal and noise with different parameters; stochastic resonance is only one of them.

In this paper, the characteristic structure of a double well in a bistable system and the synchronization conditions for SR were analysed. According to the time relativity of the intrawell random fluctuations, a method of controlling SR was proposed. It uses a periodic signal as a control signal to lead the fluctuations in the single potential well and thus affects the transitions between two potential wells; finally, an effective control can be achieved which allows us to either enhance or realize SR.

As the periodic signal is easy to generate, using a periodic signal as a control signal is the simplest and most easily achievable control method. In particular, the method reveals the relationship between the movements of the bistable system at two levels, which deepens the understanding of the micro mechanism of stochastic resonance.

References

[1] Benzi R, Sutera A and Vulpiana A 1981 *Phys.* A **14** L453

[2] Gammaitoni L, Hanggi P, Jung P and Marchesoni F 1998 *Rev. Mod. Phys.* **70** 223

[3] Babinec P 1997 *Phys. Lett.* A **225** 179

[4] Nozaki D, Mar D J, Grigg P and Collins J J 1999 *Phys. Rev. Lett.* **82** 2402

[5] Qin G R, Gong D C, Hu G and Wen X D 1992 *Acta Phys. Sin.* **41** 360 (in Chinese)

[6] Zhu H J, Li R and Wen X D 2003 *Acta Phys. Sin.* **52** 2404 (in Chinese)

[7] Lin M and Huang Y M 2006 *Acta Phys. Sin.* **55** 3277 (in Chinese)

[8] Gammaitoni L, Löcher M, Bulsara A R, Hänggi P, Neff J, Wiesenfeld K, Ditto W L and Inchiosa M E 1999 *Phys. Rev. Lett.* **82** 4574

[9] Löcher M, Inchiosa M E, Neff J, Bulsara A R, Wiesenfeld K, Gammaitoni L, Hänggi P and Ditto W L 2000 *Phys. Rev.* E **62** 317

[10] Schmid G and Hanggi P 2005 *Physica* A **351** 95

[11] Lin M, Mao Q M, Zheng Y J and Li D S 2007 *Acta Phys. Sin.* **56** 5021 (in Chinese)

[12] Lin M and Huang Y M 2007 *Acta Phys. Sin.* **56** 6173 (in Chinese)

[13] Lin M, Fang L M and Zhu R G 2008 *Acta Phys. Sin.* **57** 2638 (in Chinese)

[14] Hu G 1994 *Stochastic Forces and Nonlinear Systems* (Shanghai: Shanghai Scientific and Technological Education Publishing House) (in Chinese)