Vol 18 No 4, April 2009
1674-1056/2009/18(04)/1707-04

Chinese Physics B

© 2009 Chin. Phys. Soc.
and IOP Publishing Ltd

Interference phase of mass neutrino in CM space-time*

Chen Xia(陈 霞) and Wang Yong-Jiu(王永久)†

Institute of Physics, Hunan Normal University, Changsha 410081*, China*

(Received 31 March 2008; revised manuscript received 23 August 2008)

In the gravitational field of central mass with electric and magnetic charges and magnetic moment (CM space-time), this paper calculates the interference phase of mass neutrino along geodesic in the radial direction, and discusses the contribution of the electric and magnetic charges and magnetic moment of the central mass to the phase.

Keywords: interference phase, CM space-time, mass neutrino
PACC: 9760L, 0460, 0470

1. Introduction

With the highly confident atmospheric neutrino experiment of Super-Kamiokande to assure the neutrino mass,[1] the mass neutrino oscillation has been probed with intense theoretical and experimental vigour, especially its characteristics in curved space-time. Taking the gravitational background into account, we will extend the physics related to the neutrino oscillation in Minkowski space-time with Lorentz invariant to Riemannian space-time with general coordinate transformation. Since the neutrino is massive, Ahuwalia and Burgard claimed that they had discovered the interference phase of mass neutrino induced by gravitational field and found that the new effect may have profound consequences for type-II supernova evolution.[2] A number of authors paid much attention to calculating this interference phase of mass neutrino in different gravitational fields and used various treatments, including plane wave approximation, wave packet treatment and standard treatment.[3−5] In 1996, Fornengo and his co-workers studied the interference phase of relativistic neutrinos in the case of radial and non-radial propagation, using plane wave analysis, in Schwarzschild space-time.[4] In 2001, Zhang and Beesham used a unified description of the phases for both high energy particle and low energy particle to calculate the gravitationally induced interference phase of mass neutrino.[6]

In 1931, Dirac predicted the existence of magnetic monopoles. According to the RC model,[7] the magnetic charges absorbed by neutron star will catalyse the disintegration of proton, so it is recognized that the neutron star might have a large number of magnetic charges, moreover, it is well known that neutron star has sufficient magnetic moment ($p \sim 10^{30}$ (Gauss unit)),[8,9] so it is significant to study the characteristics of gravitational field of central mass with electric and magnetic charges and magnetic moment, which is CM space-time. In this paper, we will discuss the interference phase of the mass neutrino along the geodesic in CM space-time. We neglect the spin-dependent terms for the present, and for the reason of simplicity, we confine the treatment of interference phase to the case of radial propagation of the neutrino.

The outline of this paper is as follows. In Section 2, we obtain the general expression of the interference phase of mass neutrino when it propagates along geodesic in CM space-time. In Section 3, we discuss the phase in special case of radial direction and compare it with that in the Schwarzschild field. Furthermore, conclusion of this paper is given in the last section. We set $G = c = \hbar = 1$ throughout this paper.

2. Geodesic phase of mass neutrino in CM space-time

At first, in the curved space-time, the neutrino phase factor can be written in a covariant form[10]

$$\Phi = \int m\mathrm{d}s = \int P_\nu \mathrm{d}x^\nu = \int g_{\mu\nu} P^\mu \mathrm{d}x^\nu, \qquad (1)$$

*Project supported by National Basic Research Program of China (Grant No 2003CB716300), the National Natural Science Foundation of China (Grant No 10873004), the Scientific Research Fund of Hunan Provincial Education Department (Grant No 08B051) and the Scientific Research Fund of Hunan Normal University.

†Corresponding author. E-mail: wyj@hunnu.edu.cn

where m is the mass of neutrino, P_ν is the canonical momentum conjugate to coordinates x^ν, and $g_{\mu\nu}$ and $\mathrm{d}s$ are the space-time tensor and line element respectively, Φ is called covariant phase. Now we use this covariant phase to deal with the phase of mass neutrino along the geodesic in CM space-time, in which the line element can be written as[8]

$$\mathrm{d}s^2 = g_{00}\mathrm{d}t^2 + g_{11}\mathrm{d}r^2 + g_{22}\mathrm{d}\theta^2 + g_{33}\mathrm{d}\varphi^2, \tag{2}$$

where

$$g_{00} = 1 - \frac{2M}{r} + \frac{e^2+q^2}{r^2} + \frac{a^2p^2\cos^2\theta}{r^4}, \qquad g_{11} = -\left\{1 - \frac{2M}{r} + \frac{e^2+q^2}{r^2} - \frac{a^2p^2}{2r^4}(2\cos^2\theta - 1)\right\}^{-1},$$
$$g_{22} = -r^2\left(1 - \frac{a^2p^2\cos^2\theta}{2r^4}\right), \qquad g_{33} = -r^2\left(1 - \frac{a^2p^2\cos^2\theta}{2r^4}\right)\sin^2\theta, \tag{3}$$

in Eq.(3) M is the gravitational mass of the star, e and q are electric and magnetic charge respectively, p is the magnetic moment of star.

We only consider the phase of the mass neutrino in CM space-time by means of the geodesic equation of the particle:

$$\frac{\mathrm{d}^2x^\lambda}{\mathrm{d}s^2} + \Gamma^\lambda_{\mu\nu}\frac{\mathrm{d}x^\mu}{\mathrm{d}s}\frac{\mathrm{d}x^\nu}{\mathrm{d}s} = 0, \tag{4}$$

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\lambda\tau}(g_{\mu\tau,\nu} + g_{\tau\nu,\mu} - g_{\nu\mu,\tau}), \tag{5}$$

along the radial direction, $\mathrm{d}\theta = \mathrm{d}\varphi = 0$. The nontrivial timelike equation is

$$\frac{\mathrm{d}^2t}{\mathrm{d}s^2} + 2\Gamma^0_{01}\frac{\mathrm{d}t}{\mathrm{d}s}\frac{\mathrm{d}r}{\mathrm{d}s} = 0, \tag{6}$$

$$\Gamma^0_{01} = \frac{1}{2}g^{00}\frac{\mathrm{d}g_{00}}{\mathrm{d}r}. \tag{7}$$

Substituting Eq.(7) into Eq.(6), we obtain

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(g_{00}\frac{\mathrm{d}t}{\mathrm{d}s}\right) = 0. \tag{8}$$

Equation (8) gives

$$g_{00}\frac{\mathrm{d}t}{\mathrm{d}s} = \text{const} \quad \text{or} \quad E = P_0 = mg_{00}\frac{\mathrm{d}t}{\mathrm{d}s} = \text{const}. \tag{9}$$

Equation (9) represents that the covariant energy is the constant of motion along the geodesic, by which the calculation of the phase is proceeded. It is stressed that the covariant energy P_0 (not P^0) is the constant of motion, and the value of this constant E is the asymptotic energy of the neutrino at $r = \infty$.[11]

The mass shell condition can be written in CM space-time, which is not independent and can be deducted from the geodesic equations:

$$g_{\mu\nu}\frac{\mathrm{d}x^\mu}{\mathrm{d}s}\frac{\mathrm{d}x^\nu}{\mathrm{d}s} = 1, \tag{10}$$

substituting Eq.(9) into Eq.(10) with $\dfrac{\mathrm{d}\varphi}{\mathrm{d}s} = 0$ and $\dfrac{\mathrm{d}\theta}{\mathrm{d}s} = 0$, which means the radial motion, we obtain

$$\frac{\mathrm{d}s}{\mathrm{d}r} = \left[-g_{00}g_{11}/\left(\frac{P_0^2}{m^2} - g_{00}\right)\right]^{1/2} = \left[-g_{00}g_{11}/(k + 1 - g_{00})\right]^{1/2}, \tag{11}$$

where $k = \left(\dfrac{P_0}{m}\right)^2 - 1$, then

$$\Phi_{(\mathrm{geod})} = \int m\frac{\mathrm{d}s}{\mathrm{d}r}\mathrm{d}r = m\int \frac{1 + \dfrac{4a^2p^2\cos^2\theta - a^2p^2}{2r^4 - 4Mr^3 + 2(e^2+q^2)r^2 - a^2p^2(2\cos^2\theta - 1)}}{k - (e^2r^2 + q^2r^2 - 2Mr^3 + a^2p^2\cos^2\theta)/r^4}\mathrm{d}r, \tag{12}$$

where the integral is performed from r_0 to r. This is the exact phase of mass neutrino when the neutrino propagates along geodesic in CM space-time. For simplicity and convenience to see its physical meaning, we confine our treatment to the case of $\theta = 0$.

3. Phase in the case of $\theta = 0$

According to Eq.(12), we calculate the interference phase in the radial direction with $\theta = 0$, thus we have

$$\Phi_{(\mathrm{geod})} = m \int \frac{1 + 3a^2p^2/(2r^4 - 4Mr^3 + 2e^2r^2 + 2q^2r^2 - a^2p^2)}{k - (e^2r^2 + q^2r^2 - 2Mr^3 + a^2p^2)/r^4} \mathrm{d}r, \tag{13}$$

for the reason that it is complicated to calculate the direct integral, we expand the integral factor to the second order in p, then we obtain

$$\begin{aligned}\Phi_{(\mathrm{geod})} = m \int \Bigg[& \left(k + \frac{2M}{r} - \frac{e^2+q^2}{r^2}\right)^{-1/2} + \frac{1}{2}\left(k + \frac{2M}{r} - \frac{e^2+q^2}{r^2}\right)^{-3/2} a^2p^2/r^4 \\ & + \frac{3}{4}\left(k + \frac{2M}{r} - \frac{e^2+q^2}{r^2}\right)^{-1/2} a^2p^2/(-2mr^3 + e^2r^2 + q^2r^2 + r^4) + o[p^3] \Bigg] \mathrm{d}r. \end{aligned} \tag{14}$$

If we let $p = 0$, the result reduces to

$$\Phi_{(\mathrm{geod})} = m \int \left(k + \frac{2M}{r} - \frac{e^2+q^2}{r^2}\right)^{-1/2} \mathrm{d}r. \tag{15}$$

This is the exact interference phase of mass neutrino along geodesic in radial propagation in Reissner–Nordström space-time.[12,13]

We proceed to expand each integral factor to the second order in $\sqrt{e^2+q^2}$, thus

$$\begin{aligned}\Phi_{(\mathrm{geod})} = m \int \Bigg\{ & \left(k + \frac{2M}{r}\right)^{-1/2} + \frac{1}{2}\left(k + \frac{2M}{r}\right)^{-3/2} (e^2+q^2)/r^2 + o\left[\left(e^2+q^2\right)^{3/2}\right] \\ & + \frac{1}{4}\left[\left(k + \frac{2M}{r}\right)^{-3/2} /r^4 + \frac{3}{8}\left(k + \frac{2M}{r}\right)^{-5/2} \left(e^2+q^2\right)/r^6 + o\left[\left(e^2+q^2\right)^{3/2}\right]\right] a^2p^2 \\ & + \frac{3}{8}\left(k + \frac{2M}{r}\right)^{-1/2} a^2p^2/\left(-2Mr^3 + r^4\right) - \frac{3}{16}\left(k + \frac{2M}{r}\right)^{-1/2} r^2\left(e^2+q^2\right)a^2p^2/\left(-2Mr^3 + r^4\right)^2 \\ & + \left[\frac{3}{8}\left(k + \frac{2M}{r}\right)^{-3/2} \left(e^2+q^2\right)/8r^2(-2Mr^3 + r^4) + o\left[\left(e^2+q^2\right)^{3/2}\right]\right] a^2p^2 \Bigg\} \mathrm{d}r + o\left[p^3\right]. \end{aligned} \tag{16}$$

In the region far from the gravitational source, we can think $\dfrac{2M}{kr}$ as small quantity, then we can make the following approximation:

$$\begin{aligned} \left(1 + \frac{2M}{kr}\right)^{1/2} &= 1 + \frac{M}{kr}, \\ \left(1 + \frac{2M}{kr}\right)^{-1/2} &= 1 - \frac{M}{kr}, \\ \left(1 + \frac{2M}{kr}\right)^{-3/2} &= 1 - 3\frac{M}{kr}, \\ \left(1 + \frac{2M}{kr}\right)^{-5/2} &= 1 - 5\frac{M}{kr}. \end{aligned} \tag{17}$$

Substituting the above approximate condition into Eq.(16) and integrating it, we get

$$\begin{aligned} & \Phi_{(\mathrm{geod})} \\ \doteq\ & \frac{m}{\sqrt{k}}(r - r_0) - \frac{2Mm}{k^{3/2}} \ln\left(\frac{r}{r_0}\right) \\ & + \frac{e^2+q^2}{2k^{3/2}} m\left(\frac{1}{r_0} - \frac{1}{r}\right) - \frac{\sqrt{k}m}{24M^2}\left(\frac{1}{r_0} - \frac{1}{r}\right) a^2p^2 \\ & + \frac{5k^{3/2}m}{8M^4}\left(\frac{1}{r_0} - \frac{1}{r}\right)(e^2+q^2)a^2p^2 + o\left[p^3\right], \end{aligned} \tag{18}$$

for electron neutrino, $k \sim 10^{12} \gg 1$, so equation (18)

can be written as

$$\begin{aligned}&\Phi_{(\text{geod})}\\ \doteq\ &\frac{m^2}{P_0}(r-r_0)-\frac{2Mm^4}{P_0^3}\ln\left(\frac{r}{r_0}\right)\\ &+\frac{e^2+q^2}{2P_0^3}m^4\left(\frac{1}{r_0}-\frac{1}{r}\right)-\frac{P_0}{24M^2}\left(\frac{1}{r_0}-\frac{1}{r}\right)a^2p^2\\ &+\frac{5P_0^3}{8M^4m^2}\left(\frac{1}{r_0}-\frac{1}{r}\right)(e^2+q^2)a^2p^2+o\left[p^3\right]. \quad (19)\end{aligned}$$

From Eq.(19), we can see that the first two terms are the interference phase of mass neutrino which is the same as that in Schwarzschild field,[4] and the last three terms are the correction that induced by CM space-time, which represents the contribution of the electric and magnetic charges and magnetic moment to the neutrino oscillation phase. In case of $k \sim 10^{12} \gg 1$ and $r \gg M$ as well as the conditions $m \sim 1\,\text{eV}$ and the energy $P \sim 1\,\text{MeV}$[14,15] for the electron neutrino, which are really small quantities, we can see that the phase induced by electric and magnetic charges and magnetic moment is little when compared with the first two terms, but as to special celestial body like neutron star, which has sufficient magnetic charges and magnetic moment, the correction may be obvious.

4. Conclusion

On the phase of the mass neutrino in CM space-time, we exploit the geodesic route and radial direction to calculate it. We have also calculated the interference phase of mass neutrino in Reissner–Nordström space-time. If we let $p = 0$, $e = 0$, our result for neutrino oscillation relative phase is in agreement with that in Ref.[6]. We obtain the formula which describes the contribution of electric charge e, magnetic charge q and magnetic moment p to neutrino oscillation phase. Here, we neglect the effect of interaction between spin (of neutrino) and magnetic field which is created by central mass.

References

[1] Fukuda Y, Hayakawa T, Ichihara E, Inoue K, Ishihara K, Ishino H, Itow Y, Kajita T, Kameda J, Kasuga S, Kobayashi K, Kobayashi Y, Koshio Y, Martens K, Miura M, Nakahata M, Nakayama S, Okada A, Oketa M, Okumura K, Ota M, Sakurai N, Shiozawa M, Suzuki Y, Takeuchi Y, Totsuka Y, Yamada S, Earl M, Habig A, Hong J T, Kearns E, Kim S B, Masuzawa M, Messier M D, Scholberg K, Stone J L, Sulak L R, Walter C W, Goldhaber M, Barszczak T, Gajewski W, Halverson P G, Hsu J, Kropp W R, Price L R, Reines F, Sobel H W, Vagins M R, Ganezer K S, Keig W E, Ellsworth R W, Tasaka S, Flanagan J W, Kibayashi A, Learned J G, Matsuno S, Stenger V, Takemori D, Ishii T, Kanzaki J, Kobayashi T, Nakamura K, Nishikawa K, Oyama Y, Sakai A, Sakuda M, Sasaki O, Echigo S, Kohama M, Suzuki A T, Haines T J, Blaufuss E, Sanford R, Svoboda R, Chen M L, Conner Z, Goodman J A, Sullivan G W, Mori M, Hill J, Jung C K, Mauger C, McGrew C, Sharkey E, Viren B, Yanagisawa C, Doki W, Ishizuka T, Kitaguchi Y, Koga H, Miyano K, Okazawa H, Saji C, Takahata M, Kusano A, Nagashima Y, Takita M, Yamaguchi T, Yoshida M, Etoh M, Fujita K, Hasegawa A, Hasegawa T, Hatakeyama S, Iwamoto T, Kinebuchi T, Koga M, Maruyama T, Ogawa H, Suzuki A, Tsushima F, Koshiba M, Nemoto M, Nishijima K, Futagami T, Hayato Y, Kanaya Y, Kaneyuki K, Watanabe Y, Kielczewska D, Doyle R, George J, Stachyra A, Wai L, Wilkes J and Young K 1998 *Phys. Rev. Lett.* **81** 1158 1998 *Phys. Rev. Lett.* **81** 1158

[2] Ahluwalia D V and Burgard C 1996 *Gen. Rel. Grav.* **28** 1161

[3] Giunit C, Kim C W and Lee U W 1991 *Phys. Rev.* D **44** 3635

[4] Fornengo N, Giunti C, Kim C W and Song J 1997 *Phys. Rev.* D **56** 1895

[5] Huang X J and Wang Y J 2003 *Commun. Theor. Phys.* **40** 742

[6] Zhang C M and Beesham A 2001 *Gen. Rel. Grav.* **33** 1011

[7] Rubakov A 1981 *Gen. Rel. Grav.* **33** 644

[8] Wang Y J and Tang Z M 2002 *Astrophysics and Space Science* **28** 363

[9] Wang Y J and Peng Q H 1984 *Sci. Chin.* **28** 422

[10] Stodolsky L 1979 *Gen. Rel. Grav.* **11** 391

[11] Bhattacharya T, Habib S and Mottola E 1996 gr-qc/9605074

[12] Huang X J and Wang Y J 2004 *Chin. Phys.* **13** 1588

[13] Huang X J, Li Z J and Wang Y J 2006 *Chin. Phys.* **15** 229

[14] Boehm F and Vogel P 1992 *Massive Neutrinos* (Cambridge: Cambridge University Press) p34

[15] Bahcall J N 1994 *Neutrino Astrophysics* (Cambridge: Cambridge University Press) p147