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Shielding effect and emission criterion of a screw dislocation near an interfacial blunt crack*

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Shielding effect and emission criterion of a screw dislocation near an interfacial blunt crack are dealt with in this paper. Utilizing the conformal mapping technique, the closed-form solutions are derived for complex potentials and stress fields due to a screw dislocation located near the interfacial blunt crack. The stress intensity factor on the crack tips and the critical stress intensity factor for dislocation emission are also calculated. The influence of the orientation of the dislocation and the morphology of the blunt crack as well as the material elastic dissimilarity on the shielding effect and the emission criterion is discussed in detail. The results show that positive screw dislocations can reduce the stress intensity factor of the interfacial blunt crack tip (shielding effect). The shielding effect increases with the increase of the shear modulus of the lower half-plane, but it decreases with the increase of the dislocation azimuth angle. The critical loads at infinity for dislocation emission increases with the increase of emission angle and curvature radius of blunt crack tip, and the most probable angle for screw dislocation emission is zero. The present solutions contain previous results as special cases.

Keywords: screw dislocation, interfacial blunt crack, shielding effect, emission criterion
PACC: 6170G, 6430N, 6170J

1. Introduction

The behaviour of materials near the crack tip will ultimately form the basis for all fracture theories. In order to build up a fracture theory which describes the plastic deformation based on the behaviour of dislocations near the crack tip, the elastic interaction between dislocation and crack has been studied extensively.[1−5] Once dislocations are emitted, they move out of the crack tip area, leaving behind a dislocation-free zone. An internal back stress due to the dislocations emitted from crack tip accommodates the stress intensity due to applied load, causing an increase in fracture toughness of materials.[6] Rice and Thomson[7] obtained a quantitative criterion for ductile versus brittle behaviour. Ohr *et al*[8] modified the Rice and Thomson's model by taking friction stress into account and indicated that when the crack was blunted to a certain extent, the emission of dislocations could be interrupted, and so the blunted crack would not propagate until the nucleation of a new sharp crack appeared. Wang[9] analysed the dislocation nucleation and the emission with a slightly modified Rice framework. Lung and Wang,[10] and Lee[11] calculated the image force on the dislocation near a blunt crack tip. Huang and Li[12] obtained the criterion for an edge dislocation emission from a blunt crack tip.

On the other hand, the behaviour of dislocations near the interface plays an important role in determining the mechanical properties of composite materials and poly-crystals. Head[13] evaluated the image forces on a screw dislocation due to a grain boundary, or a free surface with an oxide film. Chu[14] studied the stress field of a screw dislocation in a two-phase isotropic thin film by using the conformal mapping technique. Shiue *et al*[15,16] investigated the elastic interaction between screw dislocation and interfacial surface crack, and Lee and Huang[17] studied the elastic interaction between screw dislocation and interfacial crack emanating from a semi-elliptic hole. However, the shielding effect and the emission criterion of a screw dislocation near an interfacial blunt crack have not been reported on yet.

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In the present paper, the shielding effect and the emission criterion of a screw dislocation near an interfacial blunt crack under loads at infinity are investigated. Complex potentials and stress fields are derived. The stress intensity factor on the crack tips and the critical stress intensity factor for dislocation emission are also calculated. The influence of the orientation of dislocation and the morphology of blunt crack as well as the material elastic dissimilarity on shielding effect and emission criterion is discussed in detail.

2. Solution

Consider a two-phase material with shear modulus μ_1 for phase 1 and shear modulus μ_2 for phase 2. The interface (real axis) contains a sharp crack of length 2, and the origin located at the centre of the crack. A positive screw dislocation of Burgers vector b_z is located at w_0 in phase 1. The dislocation extends infinitely in the ζ-direction, which is perpendicular to the w-plane. Under the action of a mode III stress Γ, the complex potential in the w-plane is given by[18]

$$\begin{aligned}\frac{\mathrm{d}f_1(w)}{\mathrm{d}w} &= \frac{\mu_1 b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)}\left(\frac{1}{w-w_0}-\frac{1}{w-\bar{w}_0}\right)\\ &+\frac{\mu_2 b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)}\left[\frac{\sqrt{(w_0+1)(w_0-1)}}{w-w_0}+\frac{\sqrt{(\bar{w}_0+1)(\bar{w}_0-1)}}{w-\bar{w}_0}+2\right]\frac{1}{\sqrt{(w+1)(w-1)}}\\ &+\frac{\mu_1}{\mu_1+\mu_2}(\Gamma+\bar{\Gamma})+\frac{\mu_2\Gamma-\mu_1\bar{\Gamma}}{\mu_1+\mu_2}\frac{w}{\sqrt{(w+1)(w-1)}},\end{aligned} \tag{1}$$

where $\bar{w}_0$ is the complex conjugate to w_0 and $\Gamma = -\mathrm{i}\sigma^{\infty}_{yz}$.

Now, we introduce the transform function

$$\begin{aligned}&w\\ &= G(z)\\ &= \frac{1}{2}\left[\frac{z\left[1+\sqrt{1-\left(\frac{c}{z}\right)^2}\right]}{Rc}+\frac{Rc}{z\left[1+\sqrt{1-\left(\frac{c}{z}\right)^2}\right]}\right],\end{aligned} \tag{2}$$

which maps the sharp crack in the w-plane ($-1 \le w \le 1$) onto the elliptic contour $x^2/a^2+y^2/b^2=1$ in the z-plane (Fig.1). Here, $R=\sqrt{(a+b)/(a-b)}$, $c=\sqrt{a^2-b^2}$, and a and b are, respectively, the lengths of the major semi-axis and the minor semi-axis of the ellipse. The interface is the real-axis in both planes.

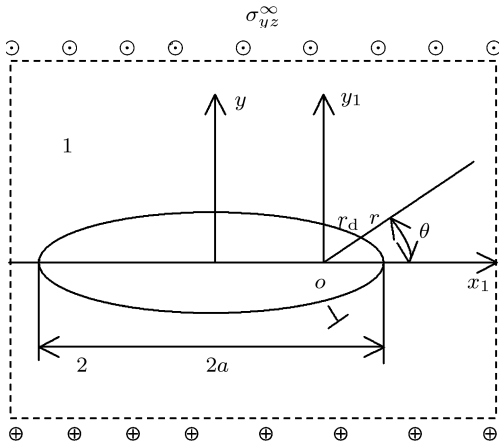


Fig.1. A screw dislocation around an interfacial blunt crack.

From Eq.(1) and expression (2), the complex potential of a screw dislocation interacting with an interfacial blunt crack can be derived as

$$\begin{aligned}\frac{\mathrm{d}f_1(z)}{\mathrm{d}w} &= \frac{\mu_1 b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)}\left[\frac{1}{G(z)-G(z_0)}-\frac{1}{G(z)-\overline{G(z_0)}}\right]\\ &+\frac{\mu_2 b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)}\left[\frac{\sqrt{G^2(z_0)-1}}{G(z)-G(z_0)}+\frac{\sqrt{\overline{G^2(z_0)}-1}}{G(z)-\overline{G(z_0)}}+2\right]\frac{1}{\sqrt{[G(z)+1][G(z)-1]}}\\ &+k\frac{\mu_2\Gamma-\mu_1\bar{\Gamma}}{\mu_1+\mu_2}\frac{G(z)}{\sqrt{G^2(z)-1}}.\end{aligned} \tag{3}$$

Letting $b = 0$, the elliptical hole reduces to a sharp crack, and if $a = 1$, we have $w = G(z) = z$, and equation (3) is in agreement with Eq.(1).

The total stress field produced by the screw dislocation and the mode III load is

$$\begin{aligned}
&\sigma_{xz} - \mathrm{i}\sigma_{yz} \\
&= \frac{\mathrm{d}\mu_1 f_1(z)}{\mathrm{d}z} = \frac{\mathrm{d}\mu_1 f_1(z)}{\mathrm{d}w}\frac{\mathrm{d}w}{\mathrm{d}z} \\
&= \left[\frac{1+\sqrt{1-\left(\frac{c}{z}\right)^2}}{2Rc\sqrt{1-\left(\frac{c}{z}\right)^2}} - \frac{Rc}{2z^2\sqrt{1-\left(\frac{c}{z}\right)^2}\left[1+\sqrt{1-\left(\frac{c}{z}\right)^2}\right]}\right] \times \left\{\frac{\mu_1^2 b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)}\left[\frac{1}{G(z)-G(z_0)} - \frac{1}{G(z)-\overline{G(z_0)}}\right]\right. \\
&\quad + \frac{\mu_1\mu_2 b_z}{2\pi\mathrm{i}(\mu_1+\mu_2)}\left[\frac{\sqrt{G^2(z_0)-1}}{G(z)-G(z_0)} + \frac{\sqrt{\overline{G^2(z_0)}-1}}{G(z)-\overline{G(z_0)}} + 2\right]\frac{1}{\sqrt{[G(z)+1][G(z)-1]}} \\
&\quad \left. + k\frac{\mu_2\Gamma - \mu_1\bar{\Gamma}}{\mu_1+\mu_2}\frac{G(z)}{\sqrt{G^2(z)-1}}\right\}. \qquad (4)
\end{aligned}$$

It is worth while to point out that the constant $k = Rc$ in Eq.(3) and expression (4) is added in order to satisfy the boundary condition, i.e. the stress field is equal to the applied stress, as y approaches infinity.

Using the relationship $z_1 = z - a_1$, $z_{10} = z_0 - a_1$ and $a_1 = a - \rho/2$, the stress field in the x–y coordinate system is converted into that in the x_1–y_1 coordinate system, whose origin is $\rho/2$ away from the right tip of the elliptical hole, where $\rho = b^2/a$ is the curvature radius of the elliptical hole.

3. Shielding effect

According to Ref.[19], a generalized mode III stress intensity factor, K_{III}, can be defined assuming that the elliptical hole is replaced by a sharp crack with a translation of the crack tip in the curvature centre of the elliptical hole. The stress intensity factor on the right tip can be obtained from the stress field determined by expression (4) as follows:

$$\begin{aligned}
K_{\mathrm{III}} &= \lim_{z_1\to\rho/2}\sqrt{2\pi z_1}\sigma_{yz} \\
&= \mathrm{Re}\left[\frac{\mu_1\mu_2 b_z}{2\sqrt{\pi a}(\mu_1+\mu_2)}\left\{\frac{\sqrt{[G(z_{10}-a_1)+1][G(z_{10}-a_1)-1]}}{G(a)-G(z_{10}-a_1)}\right.\right. \\
&\quad \left.\left. + \frac{\sqrt{[\overline{G(z_{10}-a_1)}+1][\overline{G(z_{10}-a_1)}-1]}}{G(a)-\overline{G(z_{10}-a_1)}} + 2 + \sqrt{\pi}k\sigma_{yz}^{\infty}\frac{G(a)}{\sqrt{a}}\right\}\right]. \qquad (5)
\end{aligned}$$

Substituting expression (2) into expression (5) yields

$$\begin{aligned}
&K_{\mathrm{III}} \\
&= \frac{\mu_1\mu_2 b_z}{\sqrt{\pi a}(\mu_1+\mu_2)}\mathrm{Re}\left[\frac{a+b}{b_1 - z_{10} - \sqrt{z_{10}^2 + 2a_1 z_{10} + \rho^2/4}} + \frac{a+b}{b_1 - \bar{z}_{10} - \sqrt{\bar{z}_{10}^2 + 2a_1\bar{z}_{10} + \rho^2/4}}\right] + \frac{\sqrt{\pi}k\sigma_{yz}^{\infty}}{\sqrt{a}}, \qquad (6)
\end{aligned}$$

where $b_1 = b + \rho/2$. The first and second terms in the right-hand side of expression (6) are the stress intensity factors induced by the screw dislocation and the applied stress respectively. It is found that the stress intensity factor induced by the screw dislocation is just the same as that in the single-phase case if the shear modulus of the single-phase medium, μ, is replaced by the effective shear modulus $2\mu_1\mu_2/(\mu_1+\mu_2)$. With $\sigma_{yz}^{\infty} = 0$ and $\mu_1 = \mu_2$, the stress intensity factor expressed by expression (6) is in agreement with that given in Ref.[10].

Expression (6) is an exact solution for the case of a screw dislocation near an interfacial elliptical hole. After some tedious calculations by substituting $z_{10} = r\,\mathrm{e}^{\mathrm{i}\theta}$ into expression (6) and ignoring some high order

terms of r/a and ρ/a we obtain an approximate solution for K_{III} in the explicit form, i.e.

$$K_{\mathrm{III}} \simeq \frac{\mu_1\mu_2 b_z}{\sqrt{\pi a}(\mu_1+\mu_2)}\left[\frac{2(a+b)\left(b_1 - r\cos\theta - \sqrt{2a_1 r}\cos\dfrac{\theta}{2}\right)}{r^2 + 2\sqrt{2a_1}r^{3/2}\cos\dfrac{\theta}{2} + 2(a_1 - b_1\cos\theta)r - 2b_1\sqrt{2a_1 r}\cos\dfrac{\theta}{2} + b_1^2}\right] + \frac{\sqrt{\pi}k\sigma_{yz}^{\infty}}{\sqrt{a}}. \qquad (7)$$

It should be noted that expressions (6) and (7) are applicable for the case of $r \geq r_{\mathrm{d}}$, where

$$r_{\mathrm{d}} = \frac{\sqrt{a^2 - 0.25b^2\sin^2\alpha} - (a-\rho/2)\cos\alpha}{\rho\cos^2\alpha + a\sin^2\alpha}\rho$$

is the distance from the original point to the surface of the blunt crack.

Figure 2 shows the normalized stress intensity factor (SIF) $K_{\mathrm{III}}/\mu_1\sqrt{b}$ versus $(r-r_{\mathrm{d}})/b_z$ with different values of μ_2/μ_1 at $\sigma_{yz}^{\infty}=0$, $\theta=\pi/6$, $a=10^{-5}$, and $\rho=10^{-9}$. It is seen that the value of K_{III}/μ_1 is always negative, which means that positive screw dislocations can reduce the SIF of the interfacial blunt crack (shielding effect). The shielding effect increases with the value of μ_2/μ_1 increasing, but it quickly decreases with the value of $(r-r_{\mathrm{d}})/b_z$ increasing.

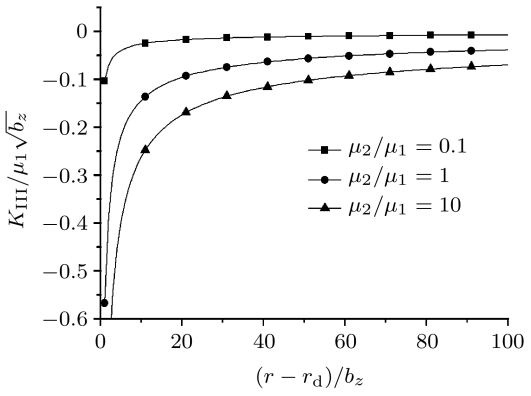


Fig.2. Curves for normalized SIF $K_{\mathrm{III}}/\mu_1\sqrt{b_z}$ versus dislocation position $(r-r_{\mathrm{d}})/b_z$ with different values of μ_2/μ_1 at $\sigma_{yz}^{\infty}=0$, $\theta=\pi/6$, $a=10^{-5}$, and $\rho=10^{-9}$.

The variations of $K_{\mathrm{III}}/\mu_1\sqrt{b}$ with the value of θ for different values of μ_2/μ_1 are shown in Fig.3 for the case of $\sigma_{yz}^{\infty}=0$, $a=10^{-5}$, $\rho=10^{-9}$, and $(r-r_{\mathrm{d}})/b_z=1$. It is seen that the shielding effect increases with the value of μ_2/μ_1 increasing, but decreases with the value of θ increasing.

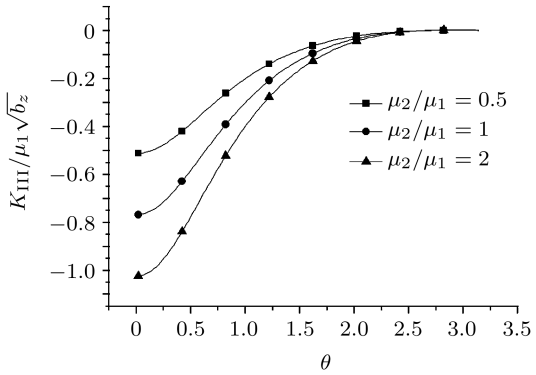


Fig.3. Curves for normalized SIF $K_{\mathrm{III}}/\mu_1\sqrt{b_z}$ versus dislocation orientation θ with different values of μ_2/μ_1 at $\sigma_{yz}^{\infty}=0$, $(r-r_{\mathrm{d}})/b_z=1$, $a=10^{-5}$, and $\rho=10^{-9}$.

4. Dislocation emission

The image force acting on the screw dislocation can be written as

$$F = F_x - \mathrm{i}F_y = \mathrm{i}b_z[\tilde{\sigma}_{xz}(z_0) - \mathrm{i}\tilde{\sigma}_{yz}(z_0)], \qquad (8)$$

where $\tilde{\sigma}_{xz}(z_0)$ and $\tilde{\sigma}_{yz}(z_0)$ are the components of the perturbation stress.

Substituting expression (4) into expression (8) yields

$$F = F_x - \mathrm{i}F_y = F_{\mathrm{imag}} + F_{\Gamma}, \qquad (9)$$

where

$$F_{\mathrm{imag}} = \left[\frac{1+\sqrt{1-\left(\dfrac{c}{z_0}\right)^2}}{2Rc\sqrt{1-\left(\dfrac{c}{z_0}\right)^2}} - \frac{Rc}{2z_0^2\sqrt{1-\left(\dfrac{c}{z_0}\right)^2}\left[1+\sqrt{1-\left(\dfrac{c}{z_0}\right)^2}\right]}\right]$$

$$\times\left\{-\frac{\mu_1^2 b_z^2}{2\pi(\mu_1+\mu_2)}\frac{1}{G(z_0)-\overline{G(z_0)}}+\frac{\mu_1\mu_2 b_z^2}{2\pi(\mu_1+\mu_2)}\left[\frac{\sqrt{\overline{G^2(z_0)-1}}}{G(z_0)-\overline{G(z_0)}}+2\right]\frac{1}{\sqrt{G^2(z_0)-1}}\right\}, \quad (10)$$

$$F_\Gamma = kb_z\sigma_{yz}^{\infty}\left[\frac{1+\sqrt{1-\left(\frac{c}{z_0}\right)^2}}{2Rc\sqrt{1-\left(\frac{c}{z_0}\right)^2}}-\frac{Rc}{2z_0^2\sqrt{1-\left(\frac{c}{z_0}\right)^2}\left[1+\sqrt{1-\left(\frac{c}{z_0}\right)^2}\right]}\right]\frac{G(z_0)}{\sqrt{G^2(z_0)-1}}. \quad (11)$$

For a screw dislocation slip on a plane denoted by angle θ, the slip force acting on the screw dislocation can be written as

$$F_{\text{emit}} = F_x\cos\theta + F_y\sin\theta = \text{Re}\,[F_{\text{imag}}+F_\Gamma]\cos\theta - \text{Im}[F_{\text{imag}}+F_\Gamma]\sin\theta. \quad (12)$$

A commonly accepted criterion for the emission of dislocation monopoles from a crack tip is, when the force acts on them, is equal to zero. Moreover, the dislocation distance to the crack surface must be equal to, or larger than, the dislocation core radius (see Ref.[7]). In the present work a similar assumption is held and $r = r_{\text{d}} + b$ is set to be the condition for the emission of screw dislocation. Combining expressions (9)–(12) yields the critical applied stress $\sigma_{yz}^{\infty\text{D}}$ and the critical stress intensity factor $K_{\text{III}}^{\text{D}}$ for the dislocation emission

$$\sigma_{yz}^{\infty\text{D}} = \frac{\sin\theta\ \text{Im}\,[F_{\text{imag}}(z_1)] - \cos\theta\ \text{Re}\,[F_{\text{imag}}(z_1)]}{kb_z(\cos\theta\ \text{Re}\,[H] - \sin\theta\ \text{Im}\,[H])} \quad (13)$$

and

$$K_{\text{III}}^{\text{D}} = \frac{\sqrt{\pi}k\sigma_{yz}^{\infty\text{D}}}{\sqrt{a}} = \frac{\sqrt{\pi}}{\sqrt{a}}\frac{\sin\theta\ \text{Im}\,[F_{\text{imag}}(z_1)] - \cos\theta\ \text{Re}\,[F_{\text{imag}}(z_1)]}{b_z(\cos\theta\ \text{Re}\,[H] - \sin\theta\ \text{Im}\,[H])}, \quad (14)$$

where

$$H = \left[\frac{1+\sqrt{1-\left(\frac{c}{z_1}\right)^2}}{2Rc\sqrt{1-\left(\frac{c}{z_1}\right)^2}}-\frac{Rc}{2z_1^2\sqrt{1-\left(\frac{c}{z_1}\right)^2}\left[1+\sqrt{1-\left(\frac{c}{z_1}\right)^2}\right]}\right]\frac{G(z_1)}{\sqrt{G^2(z_1)-1}},$$

and

$$z_1 = a - \frac{\rho}{2} + (r_{\text{d}} + b_z)(\cos\theta + \text{i}\sin\theta).$$

Figure 4 shows the curves for normalized critical load at infinity, $\sigma_{yz}^{\infty\text{D}}/\mu_1$, versus θ with different values of μ_2/μ_1 at $a = 10^{-5}$ and $\rho = 10^{-9}$. It can be found that the critical load at infinity $\sigma_{yz}^{\infty\text{D}}/\mu_1$ increases from a finite positive value to infinite with the increase of the emission angle θ. So the most probable angle for positive screw dislocation emission is always zero. When emission angle θ is fixed at a certain value, the value of $\sigma_{yz}^{\infty\text{D}}/\mu_1$ decreases with the value of μ_2/μ_1 increasing, which means the repulsion acting on the dislocation from phase 2 makes the dislocation emission take place more easily.

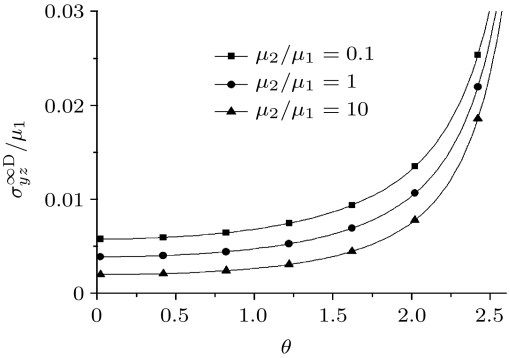


Fig.4. Curves for normalized critical load $\sigma_{yz}^{\infty\text{D}}/\mu_1$ versus emission angle θ with different values of μ_2/μ_1 at $a = 10^{-5}$ and $\rho = 10^{-9}$.

The curves for variations of $\sigma_{yz}^{\infty \mathrm{D}}/\mu_1$ with respect to θ with different values of ρ are shown in Fig.5 for the case of $a = 10^{-5}$ and $\mu_2/\mu_1 = 2$. It is seen that the critical load at infinity, $\sigma_{yz}^{\infty \mathrm{D}}/\mu_1$, increases from a finite positive value to infinite with the increase of the emission angle θ. When emission angle θ is fixed at a certain value, the value of $\sigma_{yz}^{\infty \mathrm{D}}/\mu_1$ increases with the value of ρ increasing, which means that the passivation of crack tip makes the dislocation emission take place more difficultly.

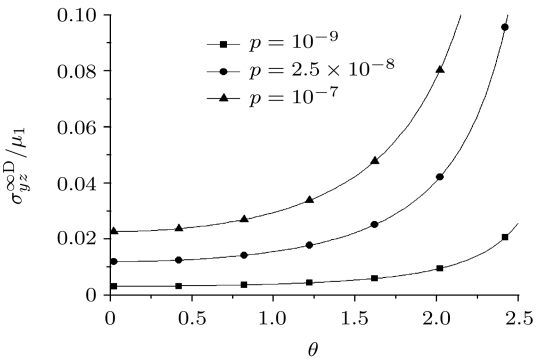


Fig.5. Curves for normalized critical load $\sigma_{yz}^{\infty \mathrm{D}}/\mu_1$ (normalized critical loads) versus emission angle θ with different values of ρ at $a = 10^{-5}$ and $\mu_2/\mu_1 = 2$.

5. Conclusions

(i) The shielding effect and the emission criterion of a screw dislocation near an interfacial blunt crack under loads at infinity are dealt with. Utilizing the conformal mapping technique, the closed-form solutions are derived for complex potentials and stress fields due to a screw dislocation located near the interfacial blunt crack. The stress intensity factor and the critical stress intensity factor for the dislocation emission are also calculated

(ii) Positive screw dislocations can reduce the SIF of the interfacial blunt crack (shielding effect). The shielding effect increases with the value of μ_2/μ_1increasing, but it decreases with the values of θ and $(r - r_{\mathrm{d}})/b_z$ increasing.

(iii) The critical load at infinity, $\sigma_{yz}^{\infty \mathrm{D}}/\mu_1$, increases with the values of the emission angle θ and ρ increasing, but it decreases with the value of μ_2/μ_1 increasing. The most probable angle for positive screw dislocation emission is zero. The repulsion acting on the dislocation from phase 2 makes the dislocation emission take place more easily, and the passivation of crack tip makes the dislocation emission occur more difficultly.

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