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Analysis of thermal conductivity in tree-like branched networks*

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Asymmetric tree-like branched networks are explored by geometric algorithms. Based on the network, an analysis of the thermal conductivity is presented. The relationship between effective thermal conductivity and geometric structures is obtained by using the thermal-electrical analogy technique. In all studied cases, a clear behaviour is observed, where angle (δ, θ) among parent branching extended lines, branches and parameter of the geometric structures have stronger effects on the effective thermal conductivity. When the angle δ is fixed, the optical diameter ratio β^* is dependent on angle θ. Moreover, γ and m are not related to β^*. The longer the branch is, the smaller the effective thermal conductivity will be. It is also found that when the angle $\theta < \delta/2$, the higher the iteration m is, the lower the thermal conductivity will be and it tends to zero, otherwise, it is bigger than zero. When the diameter ratio $\beta_1 < 0.707$ and angle δ is bigger, the optimal k of the perfect ratio increases with the increase of the angle δ; when $\beta_1 > 0.707$, the optimal k decreases. In addition, the effective thermal conductivity is always less than that of single channel material. The present results also show that the effective thermal conductivity of the asymmetric tree-like branched networks does not obey Murray's law.

Keywords: effective thermal conductivity, asymmetric tree-like branched networks, geometric parameters

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1. Introduction

Tree-like networks widely employed in physics, chemical engineering, biology, microelectronic engineering and energy sources recovery were proposed by Murray[1] in 1926. This is because the tree-like networks widely exists in nature such as vascular system, plant branches, and other relevant systems. It has been shown that natural tree network systems often give a minimal resistance and an optimal vascular diameter for driving blood in mammals and water in plants. Recently, the design, the optimization and the application of the tree-like networks have received considerable attention,[2−9] and these mechanisms are applied to the design of energy transport systems, cooling systems of electronic chips, etc. Many researchers also use this tree-like networks to explain or explore the mechanisms of different fields.

Bejan and Cobin formulated the principle as the 'constructal law' for generating flow configurations in Refs.[10, 11, 12] in 1996. It was shown that the high-conductivity material formed a tree-like network in an optimal design, and this structure had a minimum thermal resistance and minimum entropy generation if the total heat flow rate and the volume fraction of high conductivity material were both fixed. In later work, the tree-like branched networks were explored with self-similar H-shaped, T-shaped, equilateral triangles, networks with a circle connected with its centre, three-dimensional structures, etc.[13−15] Moreover, a capillary network was described with allometric scaling and fractal replication based on the Murray's law given by West *et al.*[16] Mandelbrot[17] pointed out that tree-like networks were fractal, and fractal-tree networks were optimal. Chen and Cheng[18] studied the convective heat transfer and pressure drop in fractal branched networks of rectangular shape, and compared the network with the conventional parallel channels. Yu and Li,[19] and Xu *et al*[20] used the thermal-electrical analogy technique to analyse the thermal conductivity of fractal-like networks. They found that the fractal-like tree networks played an important role

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and can significantly affect the thermal conductivity, i.e. they could produce a lower thermal conductivity. In addition, they found that the thermal conductivity of the fractal-like tree networks did not obey Murray's law. Recently, Xu *et al*[21] also analysed the transport properties including electrical conductive, heat conduction, convective heat transfer, laminar flow and turbulent flow in the tree-like networks.

Above all, the symmetric tree-like branched networks have been extensively studied. In reality, the asymmetric tree-like branched networks play a unique role in vascular systems, plant branches, etc. Unfortunately, so far there has been found very limited information in this aspect, especially, about thermal conductivity of the asymmetric tree-like branched networks.

In the present paper, we explore tree-like branched networks for different diameters of daughters and compare them with a general model. The effective thermal conductivity is studied by using the thermal-electrical analogy technique and the constructal theory. In addition, the relationship between thermal conductivity and network geometric parameters is discussed. Those results might be helpful for understanding the natural systems, the constructal theory, the designing of superconductor, etc.

2. Description and features of tree-like networks

Figure 1 depicts a sketch of asymmetric tree-like branched network with the k-th level. Two daughter lengths are identical, while the diameters are dissimilar in the network. In the present work, we consider that every stem is divided into two branches, each branch of the network is of a smooth cylinder, the thickness of the tube wall is ignored and all the ducts are sufficiently slender. We also assume that each branched boundary is kept adiabatic and the heat loss of cross branches is ignored. A typical branch at some intermediate levels k's ($k = 1, 2, 3, \ldots$) has length l_k and diameter d_k correspondingly. To characterize the branching structures, we introduce scale factors i.e. $\beta_{1(L)} = d_{k+1(L)}/d_k$, $\beta_{2(R)} = d_{k+1(R)}/d_k$, and $\gamma = l_{k+1}/l_k$ (β_1, β_2 repress $\beta_{1(L)}$, $\beta_{2(R)}$ in this paper), and thus they lead to $d_k = d_0\beta^k$, and $l_k = l_0\gamma^k$. In order to research the asymmetric tree-like branched network, it is considered that two daughter lengths are not identical, their diameters satisfy the relation of triangle as shown in bottom-right corner of Fig.1. According to geometric algorithms (sine theorem), the relationship between diameter ratios can be obtained as $d_{k+1(L)}/\sin(\delta-\theta) = d_{k+1(R)}/\sin\theta$. According to geometric algorithms, their relationship can be attained as a diameter ration $\beta_{1(L)}/\beta_{2(R)} = \sin(\delta-\theta)/\sin\theta$, where l_0 and d_0 are the diameter and length of the 0-th branching level, subscripts L and R represent the left-hand daughter and the right-hand daughter with the same parent, respectively, θ is an angle between the parent branching extended line and a branching, and δ is the angle between two branches as shown in Fig.1.

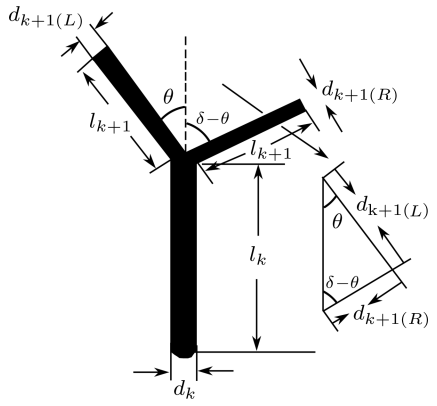


Fig.1. Sketch of asymmetric tree-like branched network with k level.

3. The thermal conductivity for tree-like networks

Thermal conductivity has attracted considerable attention[22−25] and the thermal-electrical analogy technique was used for the thermal conductivity of composites by Dul'nev *et al*[26] and Verma *et al.*[27] The thermal-electrical analogy technique was successfully applied to the solution of a number of complex problems and satisfactory results were obtained.[19−21,28−30] Therefore, the thermal-electrical analogy technique is used in the present work. Because under some assumptions one-dimensional heat flow model can also be applied in the present work. Figure 2 displays an equivalent thermal resistance network to an asymmetric tree-like network $m = 2$, the inferior shaded parts represent the resultant k-th level effective resistance R_k. According to the thermal-electrical analogy technique, the thermal resistance of a single channel of the kth level can be expressed as $R_{ki} = l_k/\lambda A_{ki}$, where λ is thermal conductivity of the thermal resistance, A_{ki} is an area of

the ith channel of k-th level. According to the resistance algorithms, we obtain relationship of the kth equivalent thermal resistance as

$$\frac{1}{R_k} = \frac{1}{R_{k1}} + \frac{1}{R_{k2}} + \frac{1}{R_{k3}} \cdots \frac{1}{R_{k2^k}}. \tag{1}$$

Therefore, the total thermal resistance R of the tree-like network is expressed as follows:

$$R = \sum_0^m R_k = \frac{4l_0}{\lambda\pi d_0^2} \frac{1 - \left(\dfrac{\gamma}{\beta_1^2 + \beta_2^2}\right)^{m+1}}{1 - \dfrac{\gamma}{\beta_1^2 + \beta_2^2}}. \tag{2}$$

where m ($m = 0, 1, 2, 3, \ldots$) is the total number of levels of the networks.

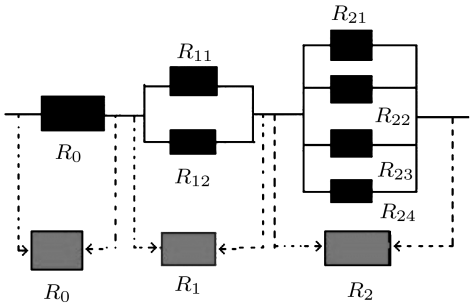


Fig.2. Equivalent thermal resistance network with an asymmetric tree-like network m=2, where the inferior shaded parts represent the resultant k-th level effective thermal resistance R_k.

In order to compare the effective thermal conductivity of the network with that of single channel material, let the network be equivalent to a single channel, and define the effective thermal conductivity of the network as the thermal conductivity of the single channel.[20] The thermal resistance of the equivalent single channel can be expressed as

$$R = \frac{L}{\lambda_e A}, \tag{3}$$

where L, λ_e, and A are the equivalent length, the effective thermal conductivity and the cross-sectional area of the single channel.

The total volume is expressed as

$$\begin{aligned} V &= \frac{\pi}{4}\left[d_0^2 l_0 + (d_{11}^2 l_1 + d_{12}^2 l_1)\right. \\ &\quad \left. + \ldots (d_{k1}^2 l_k + d_{k2}^2 l_k + \ldots d_{k2^N} l_k)\right] \\ &= \frac{\pi}{4} d_0^2 l_0 \frac{1 - [\gamma(\beta_1^2 + \beta_2^2)]^{m+1}}{1 - \gamma(\beta_1^2 + \beta_2^2)}. \end{aligned} \tag{4}$$

The length of the equivalent single channel is

$$L = \sum_0^m l_k = \frac{l_0(1 - \gamma^{m+1})}{1 - \gamma}. \tag{5}$$

According to the relation between total volume and effective length, i.e. $V = AL$, we can obtain the effective cross-sectional area as follows:

$$A = \frac{V}{L} = \frac{\pi d_0^2}{4} \frac{1 - \gamma}{1 - \gamma^{m+1}} \frac{1 - [\gamma(\beta_1^2 + \beta_2^2)]^{m+1}}{1 - \gamma(\beta_1^2 + \beta_2^2)}. \tag{6}$$

Substituting expressions (2), (5) and (6) into expression (3), the effective thermal conductivity of the network can be expressed as

$$\begin{aligned} \lambda_e = \lambda &\left(\frac{1 - \gamma^{m+1}}{1 - \gamma}\right)^2 \frac{1 - \gamma(\beta_1^2 + \beta_2^2)}{1 - [\gamma(\beta_1^2 + \beta_2^2)]^{m+1}} \\ &\times \frac{1 - \dfrac{\gamma}{\beta_1^2 + \beta_2^2}}{1 - \left(\dfrac{\gamma}{\beta_1^2 + \beta_2^2}\right)^{m+1}}. \end{aligned} \tag{7}$$

Using expression (7), we can obtain the dimensionless effective thermal conductivity λ^+ as follows:

$$\begin{aligned} \lambda^+ = \frac{\lambda_e}{\lambda} &= \left(\frac{1 - \gamma^{m+1}}{1 - \gamma}\right)^2 \frac{1 - \gamma(\beta_1^2 + \beta_2^2)}{1 - [\gamma(\beta_1^2 + \beta_2^2)]^{m+1}} \\ &\times \frac{1 - \dfrac{\gamma}{\beta_1^2 + \beta_2^2}}{1 - \left(\dfrac{\gamma}{\beta_1^2 + \beta_2^2}\right)^{m+1}}. \end{aligned} \tag{8}$$

Defining $\beta^2 = \beta_1^2 + \beta_2^2$, and substituting $\beta_{1(L)}/\beta_{2(R)} = \sin(\delta - \theta)/\sin\theta$ into β^2, we can obtain

$$\beta^2 = \beta_1^2 \left[\frac{\sin^2\theta + \sin^2(\delta - \theta)}{\sin^2(\delta - \theta)}\right]. \tag{9}$$

The dimensionless effective thermal conductivity is written as

$$\begin{aligned} \lambda^+ = &\left(\frac{1 - \gamma^{m+1}}{1 - \gamma}\right)^2 \frac{1 - \gamma\beta^2}{1 - [\gamma\beta^2]^{m+1}} \\ &\times \frac{1 - \dfrac{\gamma}{\beta^2}}{1 - \left(\dfrac{\gamma}{\beta^2}\right)^{m+1}}. \end{aligned} \tag{10}$$

For a symmetric tree-like branched network, $\beta_1 = \beta_2$, expression (8) can be reduced to

$$\begin{aligned} \lambda^+ = &\left(\frac{1 - \gamma^{m+1}}{1 - \gamma}\right)^2 \frac{1 - 2\gamma\beta_1^2}{1 - (2\gamma\beta_1^2)^{m+1}} \\ &\times \frac{1 - \dfrac{\gamma}{2\beta_1^2}}{1 - \left(\dfrac{\gamma}{2\beta_1^2}\right)^{m+1}}. \end{aligned} \tag{11}$$

From expressions (8) and (9), it can been shown that the effective thermal conductivity of the asymmetric tree-like branched network is a function of γ,

θ, δ, m and β_1 or β_2. Comparing expression (10) with expression (11), it can be seen that expression (11) is a special case of expression (10) ($\beta_1 = \beta_2$). Expression (11) is a function of parameters of the symmetric tree-like branched network, and it has been studied by Xu *et al*.[20]

4. Results and discussion

Figure 3 presents the curves for dimensionless effective thermal conductivity λ^+ versus diameter ratio β_1. It is found that the dimensionless effective thermal conductivity is always smaller than that of the single channel. Figures 3(a) and 3(b) show that the dimensionless effective thermal conductivity of the network decreases as the increase of m at the fixed values of γ, δ and θ, and the effective thermal conductivity decreases as the increase of γ at the fixed values of m, δ and θ. This is expected because with m increasing, the network becomes densely filled with much slenderer branches, leading to the increase of thermal resistance and to the decrease of thermal conductivity. A bigger value of γ implies longer branches, leading to a higher resistance and thus a lower thermal conductivity.[19] From Figs.3(a), 3(b) and 3(c) it can be seen that the values of thermal conductivity λ^+ all increase with diameter ratio β_1 and they each arrive at a maximum value when diameter ratio β_1 increases up to a certain value.

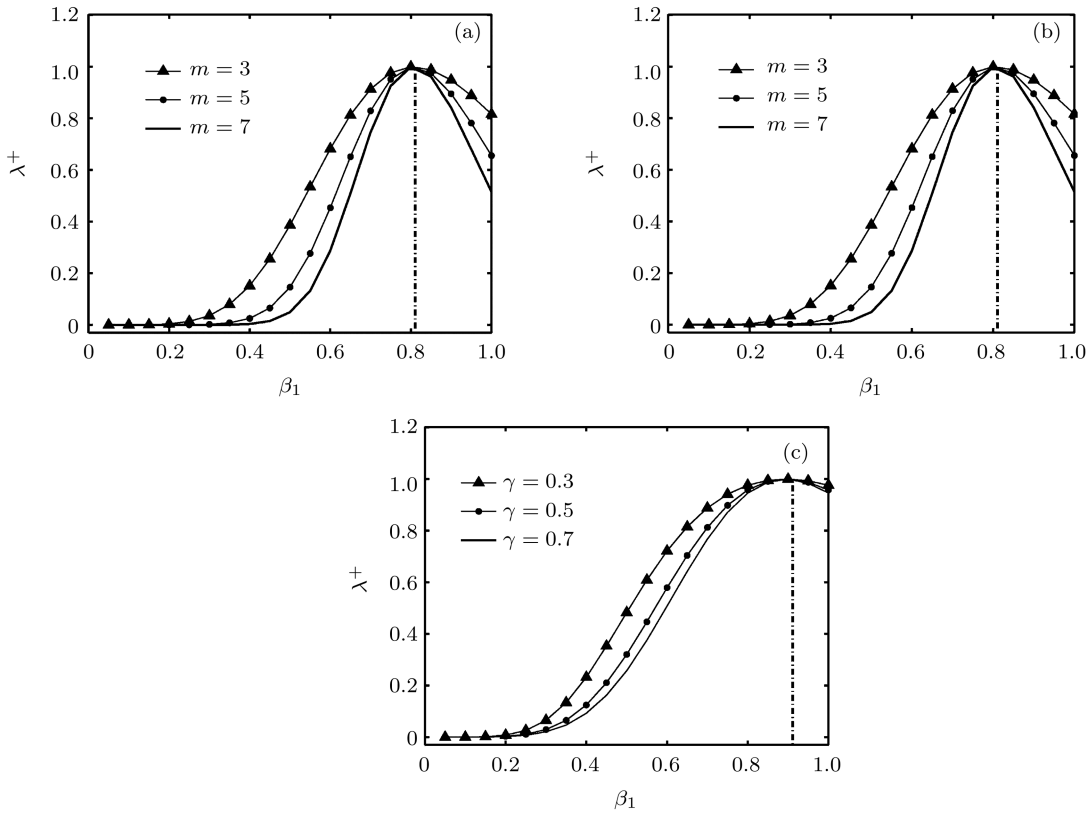


Fig.3. Curves for dimensionless effective thermal conductivity λ^+ versus diameter ratios β_1 (a) for different values of m at $\gamma = 0.4$, $\delta = 2\pi/3$, and $\theta = \pi/4$; (b) for different values of m at $\gamma = 0.7$, $\delta = 2\pi/3$, and $\theta = \pi/4$ (c) for different values of γ at $m = 3$, $\delta = 2\pi/3$, and $\theta = \pi/6$.

The maximum values of thermal conductivity in Figs.3(a) and 3(b) both occur at the same optimal diameter ratio β^* with the same values of γ, δ, and m but different values of angle θ. Figure 3(a) shows the case where $\gamma = 0.4$, $\delta = 2\pi/3$, and $\theta = \pi/4$ for different values of m; Figure 3(b) indicates the case where $\gamma = 0.7$, $\delta = 2\pi/3$ and $\theta = \pi/4$ for different values of m. However, a different value of angle θ can lead to a different value of optimal diameter ratios β^* where a maximum of effective thermal conductivity λ^+ occurs as shown in Fig.3(c)

where the maximum values of effective thermal conductivity λ^+ all occur at a diameter ratio different from that in Figs.3(a) and 3(b), with fixed values of $m = 3$, $\delta = 2\pi/3$, $\theta = \pi/6$, and different values of γ. This can be explained as follows: β^* is the maximum possible diameter ratio for preserving area at a fixed angle θ, leading to a minimum thermal resistance and thus to a maximum thermal conductivity. It is also found from Fig.3 that when $\beta_1 \leq \beta^*$ the effective thermal conductivity λ^+ increases with the increase of the diameter ratio β_1, but when $\beta_1 \geq \beta^*$ the effective thermal conductivity λ^+ decreases with the increase of the diameter ratio β_1. Figure 3 also reveals that the maximum dimensionless effective thermal conductivity is always 1 and equal to that of single channel material, which is independent of the network geometric parameters (m, γ, δ, θ).

Figure 4 shows the dimensionless effective thermal conductivity λ^+ versus diameter ratio β_1 for different values of θ at $m = 4$, $\gamma = 0.4$, $\delta = 2\pi/3$. From Figs.3 and 4 it can be found that the optimal diameter ratio β^* for the maximum effective thermal conductivity is related to angle between the parent branching extended line and a branching (δ, θ). For example, with $\theta = \pi/2$, optimal diameter ratio $\beta^* = 0.432$. Moreover, the diameter ratio β^* is independent of the network geometric parameters (m, γ). For the water flow in a botanical tree, the blood flow in a human cardiovascular system and the airflow in a bronchial tree,[31,32] their optimized diameter ratios are all close to the space-filling branching ratio 0.793, which is what Murray's law states.[1] But from Figs.3 and 4, we can find that there is a difference in diameter ratio β^* among different values of angles (δ, θ). Therefore, the effective thermal conductivity of

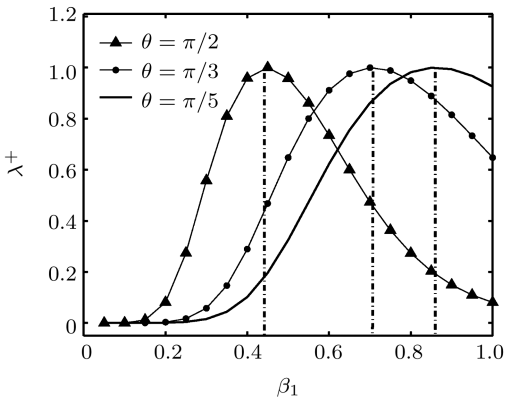


Fig.4. Curves for dimensionless effective thermal conductivity versus diameter ratio β_1 for different values of θ at $m = 4$, $\gamma = 0.4$, and $\delta = 2\pi/3$.

the asymmetric tree-like network does not obey Murray's law.[1]

In order to obtain the relation between effective thermal conductivity and angles (δ, θ), the relation between effective thermal conductivity and the ratio is investigated. Figure 5 presents the curves for dimensionless effective thermal conductivity λ^+ versus ratio θ/δ for different values of δ. Figure 5 reveals that there exists an optical ratio $k = \theta/\delta$ when effective thermal conductivity reaches a maximum value, for example, with $\beta_1 = 0.45$, $\delta = 2\pi/3$, and optical ratio $k = 0.721$. From Fig.5 we also find that when $\theta/\delta \leq k$, the effective thermal conductivity increases with the increase of the ratio θ/δ, while $\theta/\delta \geq k$, the effective thermal conductivity decreases with the increase of the ratio θ/δ. It is also found that the value of k is dependent on the values of δ and β_1. At $\beta_1 = 0.707$ as shown in Fig.5(b), all of coordinates of the ratio k and the maximum effective thermal conductivity converge to a point. It should be noted that the point is corresponding to a case of symmetric tree-like branched network and it has been studied in Ref.[20]. It is also proved that present model is feasible. When $\beta_1 \leq 0.707$, the maximum value of the effective thermal conductivity shifts toward smaller values of ratio θ/δ with the increase of the value of δ; when $\beta_1 \geq 0.707$, the maximum value of the effective thermal conductivity shifts toward the bigger values of ratio θ/δ with the increase of the value of δ as shown in Fig.5. It is also found that when $\theta/\delta < k$, the effective thermal conductivity increases with the increase of the value of β_1 and, interestingly, it is consistent with Fig.3.

Figure 6 presents the curves for dimensionless effective thermal conductivity λ^+ versus iteration m for different values of θ at $\beta_1 = 0.45$, $\gamma = 0.55$ and $\delta = 2\pi/3$. The results again interestingly show that the dimensionless thermal conductivity λ^+ decreases with the increase of the iteration m and approaches to an unconverted value. It can be explained as follows: as the value of m increases, the network becomes densely filled with slenderer branches, leading to the increase of the thermal resistance and to the decrease of thermal conductivity.[19] Note that when $\theta \leq \delta/2$, the unconverted value equals zero, but when $\theta \geq \delta/2$, the value does not equal zero. Moreover, the effective thermal conductivity is easy to approach to an unconverted value in smaller values of θ. This is consistent with the physical situation. With those properties, the network may be applied to the designing of temperature materials by changing the value of network θ.

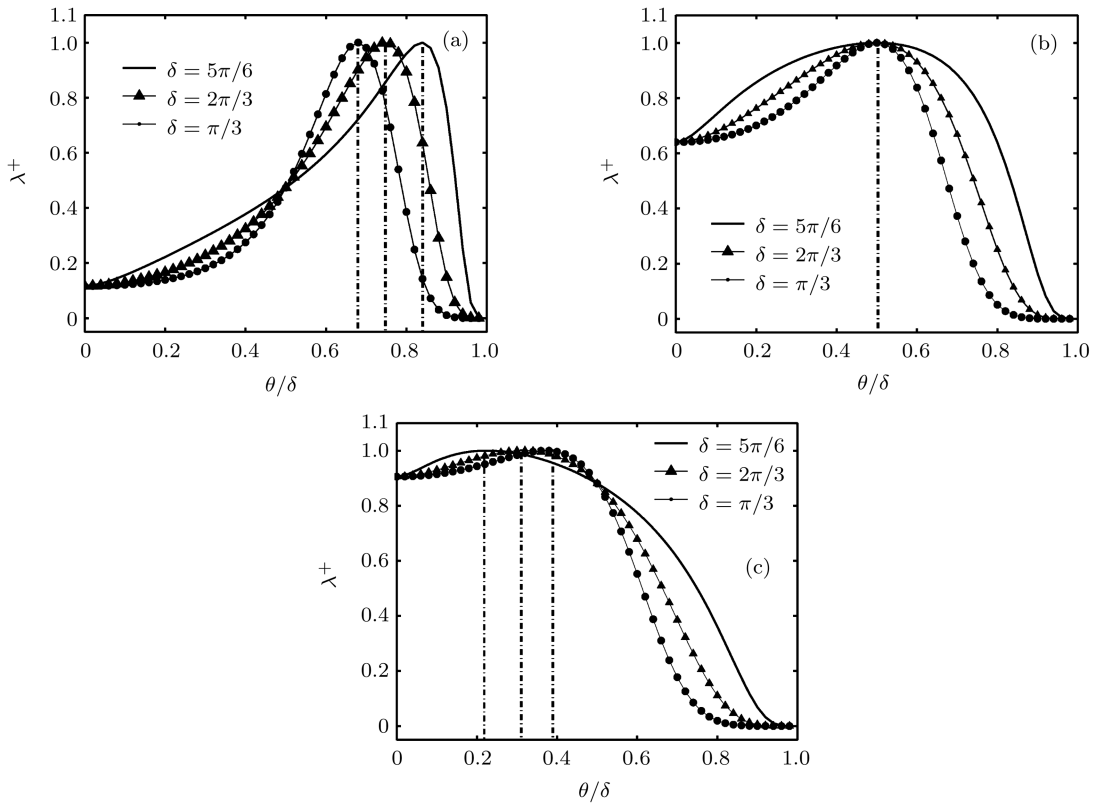


Fig.5. Curves for dimensionless effective thermal conductivity versus ratio θ/δ for different values of δ and $\beta_1 = 0.45$ (a), $\beta_1 = 0.707$ (b), and $\beta_1 = 0.85$ (c).

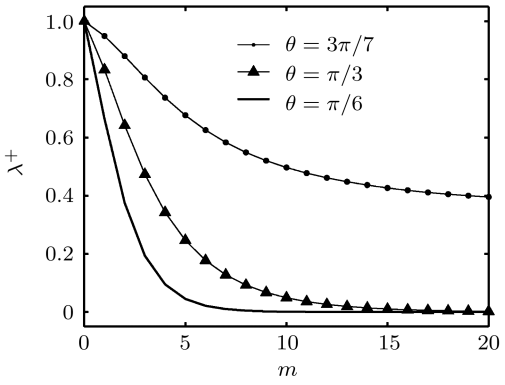


Fig.6. Curves for dimensionless effective thermal conductivity λ^+versus iteration m for different values of θ at $\beta_1 = 0.45$, $\gamma = 0.55$ and $\delta = 2\pi/3$.

5. Conclusions

In this paper, we have investigated the asymmetric tree-like branched network with geometric algorithms. The effective thermal conductivity is studied by using an asymmetric tree-like branched network. It is found that the thermal conductivity of the asymmetric tree-like branched network is closely related to the geometric parameters. Those analyses show that there exists an optical diameter ratio β^* when iteration m, length ratio γ, angles δ and θ are fixed, and the optical diameter ratio β^* does not change with the iteration m and the length ratio γ. The diameter ratio β^* is closely related only to angels δ and θ in two branches (each channel is divided into two). While the longer the branch is, the lower the effective thermal conductivity will be. The bigger the iteration m is, the lower the effective thermal conductivity will be and it approaches to zero when the angle $\theta < \delta/2$. If the diameter ratio $\beta_1 < 0.707$, a bigger angle δ corresponds to a bigger k of the perfect ratio, otherwise a bigger angle δ corresponds to a smaller k of the perfect ratio. We also find that the effective thermal conductivity of the asymmetric tree-like branched network does not obey Murray's law. Effective thermal conductivity is always less than that of single channel material. These properties of the asymmetric tree-like branched network are useful for designing temperature (low) materials, energy transport systems, cooling systems of electronic chips, etc.

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