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Theoretical analysis and experimental research on thermal focal length of a YVO_4/Nd:YVO_4 composite crystal

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This paper investigates the temperature field distribution and thermal focal length within a laser diode array (LDA) end-pumped YVO_4/Nd:YVO_4 rectangular composite crystal. A general expression of the temperature field distribution within the Nd:YVO_4 rectangular crystal was obtained by analysing the characteristics of the Nd:YVO_4 crystal and solving the Poisson equation with boundary conditions. The temperature field distributions in the Nd:YVO_4 rectangular crystal for the YVO4/Nd:YVO_4 composite crystal and the Nd:YVO_4 single crystal are researched respectively. Calculating the thermal focal length within the Nd:YVO_4 rectangular crystal was done by an analysis of the additional optical path differences (OPD) caused by heat, which was very identical with experimental results in this paper. Research results show that the maximum relative temperature on the rear face of the Nd:YVO_4 crystal in the composite crystal is 150 K and the thermal focal length is 35.7 mm when the output power of the LDA is 22 W. In the same circumstances, the experimental value of the thermal focal length is 37.4 mm. So the relative error between the theoretical analysis and the experimental result is only 4.5%. With the same conditions, the thermal focal length of the Nd:YVO_4 single crystal is 18.5 mm. So the relative rate of the thermal focal length between the YVO4/Nd:YVO_4 crystal and the Nd:YVO_4 crystal is 93%. So, the thermal stability of the output power and the beam quality of the YVO_4/Nd:YVO_4 laser is more advantageous than the laser with Nd:YVO_4 single crystal.

Keywords: thermal focal length, temperature field distribution, rectangular laser composite crystal
PACC: 4255B, 4255R

1. Introduction

Diode-end-pumped solid-state lasers of multiwatt output power, while maintaining high efficiency and diffraction-limited beam quality, have attracted growing attention due to the wide application in the areas such as industry, communication, military, medicine and scientific research. However, this progress has been hindered by the strong thermal effects which result from the high thermal loading density in end-pumped lasers. When pumping light energy is absorbed by a laser crystal, fluorescent light is produced and some of the pumping light energy will be converted to thermal energy, which will create variation in the refractive index of the material and mechanical deformation. When the laser is in steady operation, a stable field distribution of the temperature gradient will be formed in the laser crystal, which can cause thermal distortion on the end face of the crystal. Such thermal effects make the crystal approximately to be a thermal lens. Until now, much experimental and theoretical research has been done in order to investigate thermal lens effects and power scaling in Nd:YAG, Nd:YLF and Nd:YVO_4 solid-state laser media.[1−7]

In this paper, we establish a thermal analytical physical model which can describe the actual conditions of the YVO_4/Nd:YVO_4 composite crystal. The laser operation characteristics of diode-pumped YVO_4/Nd:YVO_4 composite crystal laser are analysed. On the basis of the analytical thermal analysis theory, a calculated method of thermal lens focal length is adopted to solve the conduction equation with the boundary condition of the rectangular crystal. The results can provide a guide to the design of end-pumped solid-state lasers of the YVO_4/Nd:YVO_4 composite crystals and improve the performances of the YVO_4/Nd:YVO_4 laser.

2. Mathematical model and analytical solution of temperature distribution in the rectangular crystal

The schematic diagram of the diode-pumped YVO_4/Nd:YVO_4 solid-state laser with a linear cavity is shown in Fig.1. The dimensions of the a-cut doped Nd:YVO_4 and the undoped YVO_4 are $4\times4\times7\,mm^3$ and $4\times4\times4\,mm^3$. The end-face of the YVO_4 crystal is anti-reflection (AR) coated at 808 nm and high-reflection (HR) coated at 1064 nm, which is used as the rear mirror of the resonator. The other end-face of the Nd:YVO_4 crystal is AR coated at 1064 nm. The YVO_4/Nd:YVO_4 composite crystal is wrapped with indium foil and mounted in water-cooled coppers to reduce the thermal effects of the crystal. The fibre-coupled laser diode array (LDA) (made by Coherent Inc. USA) with 30 W CW maximum output power is used as the pumping source. The diameter of the fibre is 0.8 mm with N.A=0.22. The LDA is tuned to the Nd:YVO_4 absorption-peak wavelength by means of temperature controller.

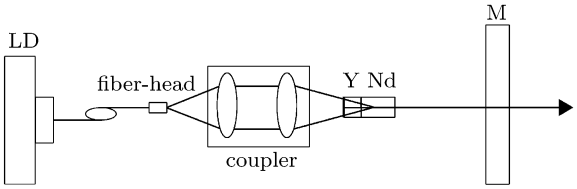


Fig.1. Schematic diagram of a diode-pumped YVO_4/Nd:YVO_4 solid-state laser (Y:YVO_4 crystal, Nd: Nd:YVO_4 crystal).

To analyse the distribution of temperature in the Nd:YVO_4 laser crystal, the following assumption are made in this paper:

(i) Distribution of the pumping laser around the axis of optical oscillator cavity, which is the optical axis of the Nd:YVO_4 laser crystal, is assumed to produce an azimuthally uniform illumination.

(ii) The pump laser is assumed to have a Gaussian emittance profile and travel through the composite crystal only once because of the absorption of pump light energy by the Nd:YVO_4 laser crystal. However, the pump light energy absorbed by the YVO_4 crystal is not considered.

(iii) A single-absorption coefficient is used to describe the absorption process of the Nd:YVO_4 crystal.

(iv) The influence of geometries and materials of coolant can be passed over because of the pump light being described only after it having entered the Nd:YVO_4 crystal. At the same time, reflection and refraction effects are not considered in the boundaries of air/YVO_4 crystal, YVO_4/ Nd:YVO_4 crystal and Nd:YVO_4 crystal/air.

The thermal model of the Nd:YVO_4 laser crystal is shown in Fig.2. The pump light traversed the YVO_4 crystal and arrived to the surface of the Nd:YVO_4 laser crystal, whose three lengths are c, b, $(a-d)$, respectively, and the spatial distribution of the pump laser intensity can be approximately expressed by a Gaussian function.[8] When the pump laser centred around the laser crystal along the axis of the laser crystal ($c/2$, $b/2$, z) arrived the plane $z=0$ in the z direction, the pump light intensity on the plane $z=0$ is given by

$$I(x,y,0)=I_0\exp\left[-2\frac{\left(x-\frac{c}{2}\right)^2+\left(y-\frac{b}{2}\right)^2}{\omega^2}\right], \quad (1)$$

where I_0 is the power density in the centre of the pump laser on the plane $z=0$ and ω is the Gaussian radius of the pump light. When the pump light entered the laser crystal along the z direction, the description of the pump laser distribution on plane $z=z$, according to absorption law, is expressed by

$$I(x,y,z)=I_0\exp\left[-2\frac{\left(x-\frac{c}{2}\right)^2+\left(y-\frac{b}{2}\right)^2}{\omega^2}\right]\times\exp\left[-\beta\left(z-d\right)\right]\quad(d<z\le a)\,. \quad (2)$$

Only the heat generated by fluorescence quantum effect and internal loss in the laser crystal, which played a leading role in the absorption of pump light, was considered. While absorbed energy due to other reasons are neglected. So the thermal density on the plane $z=z$ could be expressed by

$$q_v=\eta I\left(x,y,z\right)=I_0\eta\exp\left[-2\frac{\left(x-\frac{c}{2}\right)^2+\left(y-\frac{b}{2}\right)^2}{\omega^2}\right]\times\exp\left[-\beta\left(z-d\right)\right]\quad(d<z\le a)\,, \quad (3)$$

where η is a thermal conversion fraction determined by the fluorescence quantum effect and internal loss $\eta=1-\lambda_p/\lambda_L$, λ_p is the wavelength of the pump laser

from the LDA (808 nm), and λ_L is the wavelength of the laser from the optical parametric oscillator cavity (1064 nm).

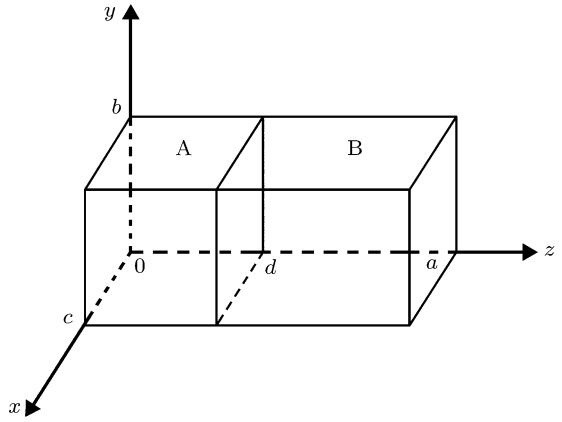


Fig.2. Schematic of the composite crystal thermal model (A and B are YVO_4 and Nd: YVO_4 crystals, respectively).

Besides, the composite crystal is edge cooled by the coolant to a relatively constant temperature, which can be assumed T_0. Zero temperature can be intercalated in mathematical treatment of the thermal model, as long as the cooling temperature T_0 is added to the ultimately calculated temperature field. The two end faces of the composite crystal can be considered adiabatic, since the coefficient of heat transfer between the two end faces of the composite crystal and air is very small. So boundary conditions of the thermal model is given by

$$\begin{aligned} &T(0,y,z) = T(c,y,z) = 0,\\ &T(x,0,z) = T(b,0,z) = 0,\\ &\left.\frac{\partial T}{\partial z}\right|_{z=0} = \left.\frac{\partial T}{\partial z}\right|_{z=a} = 0. \end{aligned} \tag{4}$$

However, the process of heat transfer between the forward face of the YVO_4 crystal and the rear face of the Nd:YVO_4 crystal must be considered. So boundary conditions of the Nd:YVO_4 crystal is expressed by

$$\begin{aligned} &T(0,y,z) = T(c,y,z) = 0,\\ &T(x,0,z) = T(x,b,z) = 0,\\ &T(x,y,d) = T_\mu,\\ &\left.\frac{\partial T}{\partial z}\right|_{z=a} = 0, \end{aligned} \tag{5}$$

where T_μ only is a function of (x,y) coordinates.

Since there is a heat source within the Nd:YVO_4 laser crystal, the distribution of the temperature field satisfies the following Poisson equation,

$$k_x\frac{\partial^2 T}{\partial x^2} + k_y\frac{\partial^2 T}{\partial y^2} + k_z\frac{\partial^2 T}{\partial z^2} = -q_v, \tag{6}$$

where k_x, k_y, and k_z are the coefficients of heat conductivity along x, y, and z axes, respectively.

According to the method from Ref.[3], the solution of Eq.(6) can be expressed by

$$\begin{aligned} T(x,y,z) = &\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{l=0}^{\infty} A_{nml}\sin\left(\frac{n\pi}{c}x\right)\\ &\times\sin\left(\frac{m\pi}{b}y\right)\cos\left(\frac{l\pi}{a}z\right), \end{aligned} \tag{7}$$

where A_{nml} is a constant to be determined by the boundary conditions in Eq.(4). Substituting Eq.(7) into Eq.(4) we obtain

$$\begin{aligned} A_{nml} = &\frac{8I_0\eta\left\{\beta a\left[\cos\left(\frac{l\pi d}{a}\right) - \mathrm{e}^{-\beta(a-d)}\cos(l\pi)\right] - l\pi\sin\left(\frac{l\pi d}{a}\right)\right\}}{cb\left[k_x\left(\frac{n\pi}{c}\right)^2 + k_y\left(\frac{m\pi}{b}\right)^2 + k_z\left(\frac{l\pi}{a}\right)^2\right](l^2\pi^2+\beta^2a^2)}\\ &\times\int_0^b\int_0^c \exp\left[\frac{\left(x-\frac{c}{2}\right)^2 + \left(y-\frac{b}{2}\right)^2}{\omega^2}\right]\sin\left(\frac{n\pi}{c}x\right)\sin\left(\frac{m\pi}{b}y\right)\mathrm{d}x\mathrm{d}y. \end{aligned} \tag{8}$$

Noting that the range of integrating z is from d to a, we can obtain the temperature distribution in the Nd:YVO_4 rectangular crystal, which is expressed by

$$T(x,y,z)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{l=0}^{\infty}\frac{8I_0\eta\left\{\beta a\left[\cos\left(\frac{l\pi d}{a}\right)-\mathrm{e}^{-\beta(a-d)}\cos(l\pi)\right]-l\pi\sin\left(\frac{l\pi d}{a}\right)\right\}}{cb\left[k_x\left(\frac{n\pi}{c}\right)^2+k_y\left(\frac{m\pi}{b}\right)^2+k_z\left(\frac{l\pi}{a}\right)^2\right](l^2\pi^2+\beta^2a^2)}$$
$$\times\int_0^b\int_0^c\exp\left[\frac{\left(x-\frac{c}{2}\right)^2+\left(y-\frac{b}{2}\right)^2}{\omega^2}\right]\sin\left(\frac{n\pi}{c}x\right)\sin\left(\frac{m\pi}{b}y\right)\mathrm{d}x\mathrm{d}y$$
$$\times\sin\left(\frac{n\pi}{c}x\right)\sin\left(\frac{m\pi}{b}y\right)\cos\left(\frac{l\pi}{a}z\right). \tag{9}$$

3. Calculation method of thermal focal length for the Nd:YVO_4 laser crystal

When there is an asymmetric temperature rise in a rectangular crystal, it will produce a thermal lens effect, which is often expressed by thermal focal length. The thermal lens effect mainly results from the thermal expansion induced variation of the length of the laser crystal, and variation of the refractive index from thermal gradient and birefringence.[9] However, to Nd:YVO_4 crystal, the variation of the refractive index from birefringence can be neglected since the Nd:YVO4 crystal is a uniaxial birefringent crystal. So, the thermal lens effect mainly results from the previous mentioned two reasons. These factors will produce an additional optical path difference (OPD). Hence the total additional OPD is approximately expressed by[2]

$$\mathrm{OPD}(x,y,z)=\left[(n_0-1)\alpha+\frac{\partial n}{\partial T}\right]\times\int_d^a\int_0^b\int_0^c T(x,y,z)\mathrm{d}x\mathrm{d}y\mathrm{d}z, \tag{10}$$

where α is the thermal expansion coefficient along the z direction, and n_0 is the refractive index of o light. While the thermal focal length is expressed by[9]

$$f=\frac{r_\mathrm{e}^2}{2[\mathrm{OPD}(0)-\mathrm{OPD}(r_\mathrm{e})]}, \tag{11}$$

where r_e is the effective radius of pump light, which is 2ω (ω is the Gaussian radius of pump laser). OPD(0) is the OPD in the centre and OPD(r_e) is the OPD at the effective radius of pump light. Substituting Eqs.(9)and (10) into Eq.(11) we get

$$f=\frac{\omega^2}{\frac{2abc\left[(n_0-1)\alpha+\frac{\partial n}{\partial T}\right]}{\pi^3}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{l=0}^{\infty}\frac{A_{nml}}{nml}\left[1-\cos(n\pi)\right]\left[1-\cos(m\pi)\right]\sin\left(\frac{l\pi d}{a}\right)}. \tag{12}$$

4. Data analysis and discussion

The pump power of the LDA is 22 W and the radius of the pump laser is 0.32 mm. For the a-cut Nd:YVO_4 crystal doped with 1 at% Nd^{3+}, the absorption coefficient of the crystal about pump laser (808 nm) is $\beta=5.32\,\mathrm{cm}^{-1}$, the coefficient of heat conductivity is $5.10\,\mathrm{W\cdot m^{-1}\cdot K^{-1}}$, the crystal refractive index gradient with temperature is $\partial n/\partial T=7.73\times10^{-6}\,\mathrm{K}^{-1}$ and the index of the crystal is $n_0=1.82$.[10] The distribution of the calculated temperature field in the crystal is shown in Fig.3. When the YVO_4 crystal contacts the Nd:YVO_4 crystal, the inside temperature field distribution of the Nd:YVO_4 crystal will evidently be affected by the YVO_4 crystal. Furthermore, the temperature field will smoothly rise at the

interface between the forward face of the YVO_4 crystal and the rear face of the Nd:YVO_4 crystal. And the relative temperature of the Nd:YVO_4 crystal in the composite crystal maximally arose to 200 K, while in the single Nd:YVO_4 crystal it arose to 270 K. Comparing Fig.3(a) with Fig.3(c), we find that the gradient of temperature in the Nd:YVO_4 for the composite crystal is smaller than that for the single Nd:YVO_4 crystal along direction of the z axis. Furthermore, the maximum relative temperature in the Nd:YVO_4 for the single crystal presents to the rear end plane ($z = 0$), while it does not present to the rear end plane ($z = d$) for the composite crystal. At the same time, comparing Fig.3(b) with Fig.3(d), we find that the relative temperature at the rear end plane of the Nd:YVO_4 crystal is 150 K for the composite crystal, while it is 270 K for the Nd:YVO_4 single crystal; the temperature of the composite crystal decreases 44.4% compared with the single Nd:YVO_4 crystal for the existence of YVO_4 crystal. In a word, it is because the YVO_4 crystal absorbs the heat from the Nd:YVO_4 crystal that the thermal effect of the composite crystal evidently decreases.

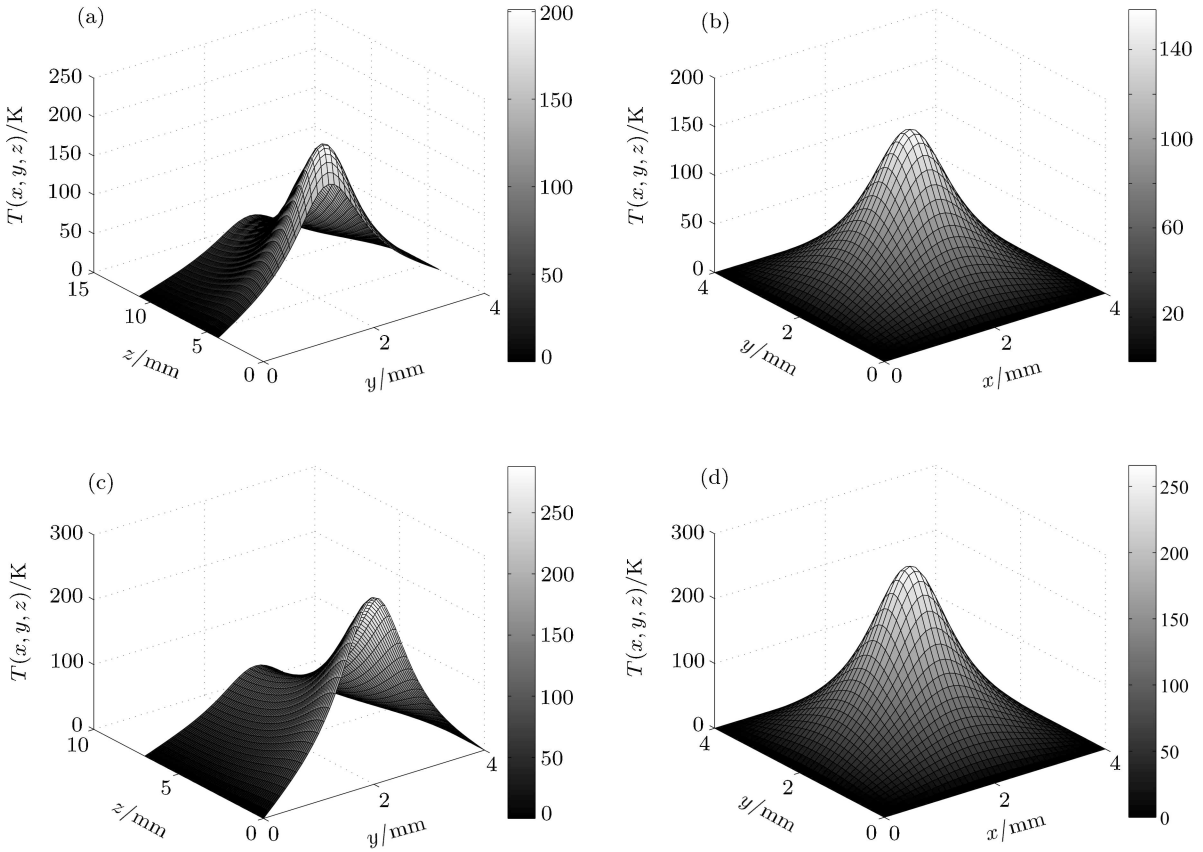


Fig.3. (a) Temperature field distribution diagram of the Nd:YVO_4 crystal (the YVO_4/Nd:YVO_4 composite crystal) ($x = c/2$). (b) Temperature field distribution diagram of the Nd:YVO_4 crystal (the YVO_4/Nd:YVO_4 composite crystal) ($z = d$). (c) Temperature field distribution diagram of the Nd:YVO_4 crystal (the Nd:YVO_4 single crystal) ($x = c/2$). (d) Temperature field distribution diagram of the Nd:YVO_4 crystal (the Nd:YVO_4 single crystal) ($z = 0$).

The change of the thermal focal length of the crystal with LDA pump power is shown in Fig.4. For the YVO_4/Nd:YVO_4 composite crystal, the thermal focal length is 35.7 mm when the output power of the LDA is 22 W, while the experimental value of the thermal focal length is 37.4 mm in the same conditions with Fig.4(a). The relative error between the theoretical analysis and the experimental result is only 4.5%. Hence the theoretical calculated values are mainly identical with the experimental values obtained by using the method of Ref.[7]. The deviation between theoretical and experimental results mainly depend on the birefringent effect from the thermal temperature field of the Nd:YVO_4 crystal, which can be neglected,

and the other factors are discussed in Ref.[2]. The deviation will decrease when the pump power of the LDA rises. It is because that the birefringent effects from the thermal temperature field and from the Nd:YVO_4 oneself will counteract each other. Comparing Fig.4(a) with Fig.4(b), we find that the thermal focal length of the YVO4/Nd:YVO_4 composite crystal is 35.7 mm when the output power of the LDA is 22 W, while the thermal focal length of the single crystal Nd:YVO_4 is 18.5 mm in the same conditions. Hence the thermal focal length will evidently increase when the YVO_4 crystal contacts the Nd:YVO_4 crystal. So the relative rate of the thermal focal length between the YVO4/Nd:YVO_4 crystal and the Nd:YVO_4 crystal is 93%. In a word, it is because the YVO_4 crystal absorbs the heat from the Nd:YVO_4 crystal that not only the temperature evidently decreases but also the thermal focal length evidently increases. So the thermal lens effect of the Nd:YVO_4 crystal evidently decreases for the YVO_4/Nd:YVO_4 composite crystal, which is highly advantageous to the thermal stability of output power and the beam quality of the output laser.

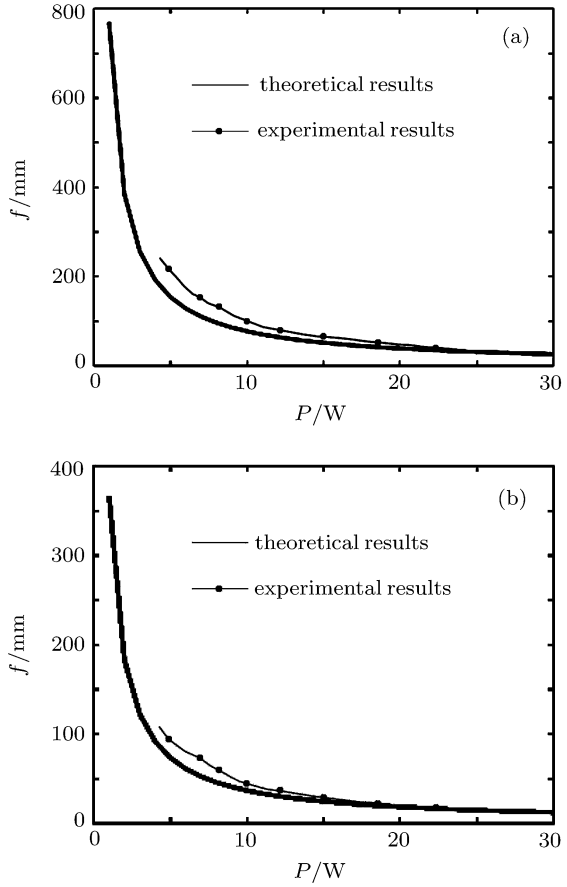


Fig.4. Thermal focal length of the Nd:YVO_4 crystal as a function of LDA pump power: (a) the YVO_4/Nd:YVO_4 composite crystal; (b) the Nd:YVO_4 single crystal.

5. Conclusion

In conclusion, a theoretical model was presented to solve realistic questions based on the characteristics of LDA end-pumped YVO_4/Nd:YVO_4 rectangle composite crystals with Gaussian distribution. The solutions of the temperature field distribution and the thermal focal length are achieved by the Poisson equation and the OPD formula, respectively. The theoretical analysis is verified by the comparison between the theoretical results and the experimental results in this paper. The main reason that makes the deviations between the theoretical results and the experimental results was discussed. At last, we compare the thermal focal length of the YVO_4/Nd:YVO_4 composite crystal with that of the Nd:YVO_4 single crystal. So, the thermal stability of the output power and the beam quality of the YVO_4/Nd:YVO_4 laser is more advantageous than the laser with Nd:YVO_4 single crystal.

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