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Ultraslow optical solitons in tunnel-coupled double semiconductor quantum well*

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This paper investigates the nonlinear evolution of the pulse probe field in an asymmetric coupled-quantum well driven coherently by a pulse probe field and two controlled fields. This study shows that, by choosing appropriate physical parameters, self-modulation can precisely balance group velocity dispersion in the investigated system, leading to the formation of ultraslow optical solitons of the probe field. The proposed scheme may lead to the development of the controlled technique of optical buffers and optical delay lines.

Keywords: ultraslow optical soliton, quantum well, group velocity
PACC: 4250, 4265

There has been considerable interest in investigating systems of semiconductor quantum wells because such systems are shown to have potentially important applications in optoelectronics and solid-state quantum information processing. For example, some of these phenomena are tunnelling induced transparency,[1] gain without inversion,[2,3] electromagnetically induced transparency,[4,5] pulsed-induced quantum interference,[6] electron intersubband transitions,[7] optical bistability and multistability,[8] etc. In addition, applying quantum optical coherence and interference effects between intersubband transitions in quantum wells, people can devise much useful devices such as ultrafast optical switches,[9] sensitive infrared detectors,[10] and so on. These electronic devices have some inherent advantages, such as large electric dipole moments due to the small effective electron mass, a great flexibility in device design and high nonlinear optical coefficients. Furthermore, the transition energies, dipoles, and symmetries can be engineered as desired. Studying optical nonlinear and coherent effects of quantum wells structure, one can find that many interesting physical phenomena observed in quantum wells are analogous to those seen in the system of atomic coherence and interference. Recently, coherent quantum interferences between the Stark-split of the ground and first excited states of a semiconductor quantum well have been demonstrated by Dynes and his coworkers.[11] At the same time, they report optically mediated coherent population trapping in asymmetric semiconductor quantum wells.[12] The generation of optical solitons in quantum well is introduced in Ref.[13].

Optical soliton describes a class of fascinating shape-preserving propagation phenomena of optical fields in nonlinear media. The key to such a shape-preserving propagation is the interplay between the nonlinear effects and the dispersion properties of the medium under optical excitations. With the development of the technique of electromagnetically induced transparency,[14−16] ultraslow propagation of an optical field has been demonstrated.[17,18] Recently Wu has investigated the possibility of using such an index enhancement technique to facilitate the formation of ultraslow optical solitons in cold atomic media.[19,20] It is well known that semiconductor quantum wells structures are closely analogous to separate atomic levels. Ultraslow optical soliton in a quantum well is investigated without controlled field.[13] In quantum

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coherent media with a strong coherent field, the absorption and dispersion of a weak probe field are interested in.[21−24] One may naturally ask if such quantum coherent technique can also be used to facilitate the formation of an ultraslow optical soliton in semiconductor quantum well driven coherently by controlled laser field. In the present article, we present a systematic study to address this question. We investigate the generation of ultraslow optical solitons in an asymmetric coupled-quantum well driven coherently by a weak probe field and two controlled field. We shall show that the formation of such soliton in tunnel-coupled double quantum well is universal phenomenon, and we provide both analytical and numerical results on our scheme.

We consider the asymmetric semiconductor coupled double quantum well structures (see Fig.1) that consist of 10 pairs of 51-monolayer (145 Å, 1 Å=0.1 nm) thick wide well and a 35-monolayer (100 Å) thick narrow well, separated by a $Al_{0.2}Ga_{0.8}As$ buffer layer.[25,26] Level $|1\rangle$ in the narrow well and $|2\rangle$ in the wide well are localized hole states. Levels $|3\rangle$ and $|4\rangle$, respectively corresponding to bonding state and anti-bonding state, are tunnel coupled new states of the first electron level in the wide well and the narrow well. More details about this system are shown in Refs.[25, 26].

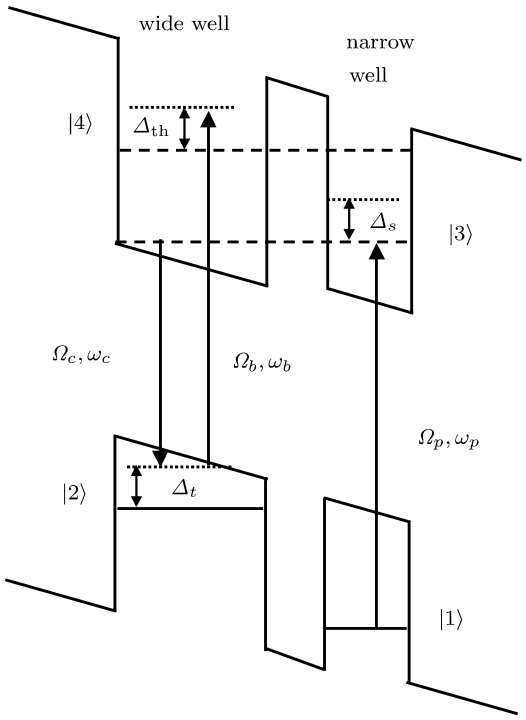


Fig.1. Schematic band diagram of the asymmetric coupled-quantum wells structure consisting of a wide well and a narrow well. Level $|1\rangle$ and level $|2\rangle$ in a valence band are the localized hole states, respectively. Level $|3\rangle$ and level $|4\rangle$ in a conduction band are the bonding and antibonding electron states of the coupled system. This system interacts with two strong lasers (frequency ω_j and Rabi frequency $2\Omega_j$, $j = b, c$) and a weak pulsed probe field (frequency ω_p and Rabi frequency $2\Omega_p$).

Here we assume that there exist a probe field (frequency ω_p and Rabi frequency $2\Omega_p$) and two controlled fields (frequency ω_j and Rabi frequency $2\Omega_j$, $j = b, c$). From the coupled quantum well system, the Hamiltonian has the form as

$$\begin{aligned} H = & \sum_{j=1}^{4} \varepsilon_j |j\rangle \langle j| - \hbar(\Omega_c \mathrm{e}^{-\mathrm{i}\omega_c t + \mathrm{i}\boldsymbol{k}_c \cdot \boldsymbol{r}} |3\rangle \langle 2| \\ & + \Omega_b \mathrm{e}^{-\mathrm{i}\omega_b t + \mathrm{i}\boldsymbol{k}_b \cdot \boldsymbol{r}} |4\rangle \langle 2| \\ & + \Omega_p \mathrm{e}^{-\mathrm{i}\omega_p t + \mathrm{i}\boldsymbol{k}_p \cdot \boldsymbol{r}} |3\rangle \langle 1| + \text{h.c.}), \end{aligned} \tag{1}$$

where $\boldsymbol{k}_{p,\,b,\,c}$ are the wave vectors of the probe and two controlled fields, ε_j is the energy of state $|j\rangle$, and $\Omega_p = D_{31}E_p/(2\hbar)$, $\Omega_c = D_{321}E_c/(2\hbar)$, $\Omega_b = D_{42}E_b/(2\hbar)$ with dipole moment D_{kl} between states $|k\rangle$ and $\langle l|$. Taking $\varepsilon_1 = 0$ and $H_0 = \hbar(\omega_p - \omega_c)|2\rangle\langle 2| + \hbar\omega_p |3\rangle\langle 3| + \hbar(\omega_p - \omega_c + \omega_b)|4\rangle\langle 4|$, we have the Hamiltonian in the interaction picture as follows:

$$\begin{aligned} H_I = & -\Delta_t |2\rangle\langle 2| - \Delta_s |3\rangle\langle 3| - \Delta_{\rm th} |4\rangle\langle 4| \\ & - (\Omega_c \mathrm{e}^{\mathrm{i}\boldsymbol{k}_c \cdot \boldsymbol{r}} |3\rangle\langle 2| + \Omega_b \mathrm{e}^{\mathrm{i}\boldsymbol{k}_b \cdot \boldsymbol{r}} |4\rangle\langle 2| \\ & + \Omega_p \mathrm{e}^{\mathrm{i}\boldsymbol{k}_p \cdot \boldsymbol{r}} |3\rangle\langle 1| + \text{h.c.}), \end{aligned} \tag{2}$$

where $\Delta_s = \omega_p - \varepsilon_3/\hbar$, $\Delta_t = \omega_p - \omega_c - \varepsilon_2/\hbar$, and $\Delta_{\rm th} = \omega_p - \omega_c + \omega_b - \varepsilon_3/\hbar$ are single-, two-, and three-photon detunings, respectively. The wave function of the system can be written in the form

$$\begin{aligned} |\Psi(t)\rangle = & A_1(t)|1\rangle + A_2(t)\mathrm{e}^{i(\boldsymbol{k}_p - \boldsymbol{k}_c)\cdot\boldsymbol{r}} |2\rangle \\ & + A_3(t)\mathrm{e}^{\mathrm{i}\boldsymbol{k}_p \cdot \boldsymbol{r}} |3\rangle + A_4(t)\mathrm{e}^{i(\boldsymbol{k}_p - \boldsymbol{k}_c + \boldsymbol{k}_b)\cdot\boldsymbol{r}} |4\rangle, \end{aligned} \tag{3}$$

where $A_j(t)$ is the probability amplitude of finding the electron in state $|j\rangle$. According to the Schrödinger equation in the interaction picture, we obtain the equation of motion for the amplitudes as

$$\begin{aligned} & \frac{\partial A_1}{\partial t} = \mathrm{i}\Omega_p^* A_3, \\ & \frac{\partial A_2}{\partial t} - \mathrm{i}(\Delta_t + \mathrm{i}\gamma_2)A_2 = \mathrm{i}\Omega_c^* A_3 + \mathrm{i}\Omega_b^* A_4, \\ & \frac{\partial A_3}{\partial t} - \mathrm{i}(\Delta_s + \mathrm{i}\gamma_3)A_3 = \mathrm{i}\Omega_c A_2 + \mathrm{i}\Omega_p A_1, \\ & \frac{\partial A_4}{\partial t} - \mathrm{i}(\Delta_{\rm th} + \mathrm{i}\gamma_4)A_4 = \mathrm{i}\Omega_b A_2, \end{aligned} \tag{4}$$

where γ_j ($j = 2, 3, 4$) stands for the total decay rate of level $|j\rangle$. In the quantum wells structure, $\gamma_{j=2,\,3,\,4}$ include the population decay rates γ_{jl} and dephasing

decay rates γ_{jd} and are given by $\gamma_2 = \gamma_{2l} + \gamma_{2d}$, $\gamma_3 = \gamma_{3l} + \gamma_{3d}$, and $\gamma_4 = \gamma_{4l} + \gamma_{4d}$, where γ_{jd} is determined by electron-electron, interface roughness, and phonon scattering processes.[26]

Under slowly varying envelope approximation, the dynamic response of the probe field is governed by Maxwell's equation

$$\frac{\partial \Omega_p}{\partial z} + \frac{1}{c}\frac{\partial \Omega_p}{\partial t} = \mathrm{i}\kappa_{13} A_3 A_1^*, \tag{5}$$

where $\kappa_{13} = 2N\omega_p |D_{13}|^2/(\hbar c)$, N is the electron density in the coupled quantum wells sample.

For the weak probe field, we retain the first order of the probe field Ω_p (i.e. linear term) and assume that the system of quantum well is initially in the state $|1\rangle$ ($A_1 \approx 1$). Under the condition that interaction between pulse laser and electron is weak, then carrying out Fourier transformations $A_j(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\beta_j(\omega)\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}\omega$ with the Fourier-transformation variable ω for Eq.(4) and $\Omega_p(t) = \frac{1}{\sqrt{2}\pi}\int_{-\infty}^{\infty}\Lambda_p(\omega)\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}\omega$ for Eq.(5), we have the results

$$\beta_3 = -\frac{|\Omega_b|^2 - (\Delta_t+\omega+\mathrm{i}\gamma_2)(\Delta_{\mathrm{th}}+\omega+\mathrm{i}\gamma_4)}{|\Omega_c|^2(\Delta_{\mathrm{th}}+\omega+\mathrm{i}\gamma_4) + |\Omega_b|^2(\Delta_s+\omega+\mathrm{i}\gamma_3) - (\Delta_s+\omega+\mathrm{i}\gamma_3)(\Delta_t+\omega+\mathrm{i}\gamma_2)(\Delta_{\mathrm{th}}+\omega+\mathrm{i}\gamma_4)}\Lambda_p, \tag{6a}$$

$$\beta_2 = \frac{(\Delta_{\mathrm{th}}+\omega+\mathrm{i}\gamma_4)\Omega_c^*}{|\Omega_b|^2 - (\Delta_t+\omega+\mathrm{i}\gamma_2)(\Delta_{\mathrm{th}}+\omega+\mathrm{i}\gamma_4)}\beta_3, \tag{6b}$$

$$\beta_4 = -\frac{\Omega_b}{(\Delta_{\mathrm{th}}+\omega+\mathrm{i}\gamma_4)}\beta_2, \tag{6c}$$

$$\frac{\partial \Lambda_p}{\partial z} - \mathrm{i}\frac{\omega}{c}\Lambda_p = \mathrm{i}\kappa_{13}\beta_3. \tag{7}$$

Substituting Eq.(6a) into Eq.(7), we then obtain

$$\frac{\partial \Lambda_p}{\partial z} = \mathrm{i}K(\omega)\Lambda_p, \tag{8}$$

where the frequency-dependent propagation factor $K(\omega)$ is given as

$$K(\omega) = \frac{\omega}{c} - \frac{\kappa_{13}(|\Omega_b|^2 - (\Delta_t+\omega+\mathrm{i}\gamma_2)(\Delta_{\mathrm{th}}+\omega+\mathrm{i}\gamma_4)}{|\Omega_c|^2(\Delta_{\mathrm{th}}+\omega+\mathrm{i}\gamma_4) + |\Omega_b|^2(\Delta_s+\omega+\mathrm{i}\gamma_3) - (\Delta_s+\omega+\mathrm{i}\gamma_3)(\Delta_t+\omega+\mathrm{i}\gamma_2)(\Delta_{\mathrm{th}}+\omega+\mathrm{i}\gamma_4)}. \tag{9}$$

In order to allow for a partial analytical calculation of the propagation effects of the weak pulsed laser field, the propagation factor is rewritten by using a Taylor expansion as function of ω, and then its Taylor expansion can be expressed as

$$K(\omega) = K(0) + K'(0)\omega + \frac{1}{2}K''(0)\omega^2 + \ldots, \tag{10}$$

where

$$K(0) = -\frac{\kappa_{13}(|\Omega_b|^2 - (\Delta_t+\mathrm{i}\gamma_2)(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)}{|\Omega_c|^2(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4) + |\Omega_b|^2(\Delta_s+\mathrm{i}\gamma_3) - (\Delta_s+\mathrm{i}\gamma_3)(\Delta_t+\mathrm{i}\gamma_2)(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)}, \tag{11}$$

$$\begin{aligned} K'(0) = {} & \frac{1}{c} + \frac{\kappa_{13}(\Delta_t+\mathrm{i}\gamma_2+\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)}{[|\Omega_c|^2(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4) + |\Omega_b|^2(\Delta_s+\mathrm{i}\gamma_3) - (\Delta_s+\mathrm{i}\gamma_3)(\Delta_t+\mathrm{i}\gamma_2)(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)]^2} \\ & + \frac{\kappa_{13}[|\Omega_b|^2 - (\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)(\Delta_t+\mathrm{i}\gamma_2)][|\Omega_c|^2 + |\Omega_b|^2 - (\Delta_s+\mathrm{i}\gamma_3)(\Delta_t+\mathrm{i}\gamma_2) - (\Delta_s+\mathrm{i}\gamma_3+\Delta_t+\mathrm{i}\gamma_2)(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)]}{[|\Omega_c|^2(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4) + |\Omega_b|^2(\Delta_s+\mathrm{i}\gamma_3) - (\Delta_s+\mathrm{i}\gamma_3)(\Delta_t+\mathrm{i}\gamma_2)(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)]^2}, \end{aligned} \tag{12}$$

$$\begin{aligned} K''(0) = {} & \frac{\kappa_{13} - \kappa_{13}[|\Omega_b|^2 - (\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)(\Delta_t+\mathrm{i}\gamma_2)](\Delta_t+\mathrm{i}\gamma_2+\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4+\Delta_s+\mathrm{i}\gamma_3)}{[|\Omega_c|^2(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4) + |\Omega_b|^2(\Delta_s+\mathrm{i}\gamma_3) - (\Delta_s+\mathrm{i}\gamma_3)(\Delta_t+\mathrm{i}\gamma_2)(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)]^2} \\ & + \frac{\kappa_{13}[|\Omega_c|^2 + |\Omega_b|^2 - (\Delta_s+\mathrm{i}\gamma_3)(\Delta_t+\mathrm{i}\gamma_2) - (\Delta_s+\mathrm{i}\gamma_3+\Delta_t+\mathrm{i}\gamma_2)(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)]}{[|\Omega_c|^2(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4) + |\Omega_b|^2(\Delta_s+\mathrm{i}\gamma_3) - (\Delta_s+\mathrm{i}\gamma_3)(\Delta_t+\mathrm{i}\gamma_2)(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)]^3} \\ & \times \frac{\kappa_{13}[|\Omega_c|^2(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)^2 + |\Omega_c|^2|\Omega_b|^2 + |\Omega_b|^4 - 2|\Omega_b|^2(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)(\Delta_t+\mathrm{i}\gamma_2) + (\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)^2(\Delta_t+\mathrm{i}\gamma_2)^2]}{[|\Omega_c|^2(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4) + |\Omega_b|^2(\Delta_s+\mathrm{i}\gamma_3) - (\Delta_s+\mathrm{i}\gamma_3)(\Delta_t+\mathrm{i}\gamma_2)(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)]^3}. \end{aligned} \tag{13}$$

The physical interpretation of Eq.(10) is rather clear. $K(0)=\phi+\mathrm{i}\alpha/2$ describes the phase shift ϕ per unit length and absorption coefficient α of the probe field, $K'(0)$ gives the group velocity $V_g=\mathrm{Re}[1/K'(0)]$, and $K''(0)$ represents the group-velocity dispersion which contributes to the probe pulse's shape change and additional loss of probe field intensity. With the obtained dispersion coefficients, we now investigate the nonlinear evolution of the probe field. Following the method introduced in Refs.[19,20], we take a trial function $\Lambda_p(z,\omega)=\tilde{\Lambda}_p(z,\omega)\exp[\mathrm{i}K(0)z]$ and substitute it into Eq.(8), then we can obtain

$$\exp[\mathrm{i}k(0)z]\frac{\partial\tilde{\Lambda}_p(z,\omega)}{\partial z}=\mathrm{i}\left[K'(0)\omega+\frac{1}{2}K''(0)\omega^2\right]\tilde{\Lambda}_p(z,\omega)\exp[\mathrm{i}k(0)z]. \tag{14}$$

To balance the interplay between the dispersion and nonlinear effects of the quantum well system with three optical fields, we need to reconsider the polarization in the right-hand side of Eq.(5), and to analyse the nonlinear polarization of the pulsed laser field, i.e.

$$\begin{aligned}\mathrm{i}\kappa_{13}A_3A_1^*&=\mathrm{i}\kappa_{13}A_3^{(1)}\left|A_1^{(0)}\right|^2=\mathrm{i}\kappa_{13}A_3^{(1)}\left[1-\left(\left|A_3^{(1)}\right|^2+\left|A_2^{(1)}\right|^2+\left|A_4^{(1)}\right|^2\right)\right]\\&=\mathrm{i}\kappa_{13}A_3^{(1)}-\mathrm{i}\kappa_{13}A_3^{(1)}\left(\left|A_2^{(1)}\right|^2+\left|A_3^{(1)}\right|^2+\left|A_4^{(1)}\right|^2\right)\\&=\mathrm{i}\kappa_{13}A_3^{(1)}+\mathrm{NLT},\end{aligned} \tag{15}$$

where $\mathrm{NLT}=-\mathrm{i}\kappa_{13}A_3^{(1)}\left(\left|A_2^{(1)}\right|^2+\left|A_3^{(1)}\right|^2+\left|A_4^{(1)}\right|^2\right)$ means the nonlinear terms and we have made use of the following relations:

$$\begin{aligned}&A_1=A_1^{(0)},\quad A_2=A_2^{(1)}A_1^{(0)},\quad A_3=A_3^{(1)}A_1^{(0)},\quad A_4=A_4^{(1)}A_1^{(0)},\\&\left|A_1^{(0)}\right|^2+\left|A_2^{(1)}\right|^2+\left|A_3^{(1)}\right|^2+\left|A_4^{(1)}\right|^2=1,\\&A_3^{(1)}=\frac{|\Omega_b|^2-(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)(\Delta_t+\mathrm{i}\gamma_2)}{|\Omega_c|^2(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)-|\Omega_b|^2(\Delta_s+\mathrm{i}\gamma_3)+(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)(\Delta_s+\mathrm{i}\gamma_3)(\Delta_t+\mathrm{i}\gamma_2)},\\&A_2^{(1)}=\frac{(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)\Omega_c^*}{|\Omega_b|^2-(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)(\Delta_t+\mathrm{i}\gamma_2)}A_3^{(1)},\\&A_4^{(1)}=-\frac{\Omega_b}{\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4}A_2^{(1)}.\end{aligned}$$

In order to obtain the nonlinear evolution equation for the weak probe field Ω_p, we perform the inverse Fourier transformation $\tilde{\Omega}_p(z,t)=\dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{-\infty}^{\infty}\tilde{\Lambda}_p(z,\omega)\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}\omega$ for the Eq.(14), we get

$$-\mathrm{i}\frac{\partial\tilde{\Omega}_p(z,t)}{\partial z}-\mathrm{i}K'(0)\frac{\partial\tilde{\Omega}_p(z,t)}{\partial t}+\frac{1}{2}K''(0)\frac{\partial^2\tilde{\Omega}_p(z,t)}{\partial t^2}=W\exp(-\alpha z)\left|\tilde{\Omega}_p(z,t)\right|^2\tilde{\Omega}_p(z,t), \tag{16}$$

where the $\tilde{\Omega}_p(z,\ t)$ is the slowly varying envelope, and the nonlinear coefficient W is given by

$$\begin{aligned}W=&\mathrm{i}\kappa_{13}\frac{|\Omega_b|^2-(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)(\Delta_t+\mathrm{i}\gamma_2)}{|\Omega_c|^2(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)-|\Omega_b|^2(\Delta_s+\mathrm{i}\gamma_3)+(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)(\Delta_s+\mathrm{i}\gamma_3)(\Delta_t+\mathrm{i}\gamma_2)}\\&\times\left|\frac{|\Omega_b|^2-(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)(\Delta_t+\mathrm{i}\gamma_2)}{|\Omega_c|^2(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)-|\Omega_b|^2(\Delta_s+\mathrm{i}\gamma_3)+(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)(\Delta_s+\mathrm{i}\gamma_3)(\Delta_t+\mathrm{i}\gamma_2)}\right|^2\\&\times\left(\left|-\frac{\Omega_b}{\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4}\right|^2+\left|-\frac{\Omega_b}{\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4}\right|^2\left|\frac{(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)\Omega_c^*}{|\Omega_b|^2-(\Delta_{\mathrm{th}}+\mathrm{i}\gamma_4)(\Delta_t+\mathrm{i}\gamma_2)}\right|^2+1\right).\end{aligned}$$

Defining a moving frame represented by $\xi = z$, and $\eta = t - K'(0)z$, we can get the following relations:

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial \xi} - K'(0)\frac{\partial}{\partial \eta},$$
$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \eta}.$$

Basing on the above relations, we rewrite Eq.(16) as

$$\mathrm{i}\frac{\partial \tilde{\Omega}_p(z,t)}{\partial \xi} - \frac{1}{2}K''(0)\frac{\partial^2 \tilde{\Omega}_p(z,t)}{\partial \eta^2} = -W\exp(-\alpha\xi)\left|\tilde{\Omega}_p(z,t)\right|^2 \tilde{\Omega}_p(z,t). \quad (17)$$

According to mathematical expressions of $K(0)$, $K''(0)$, and W, if a reasonable and realistic set of physical parameters can be chosen to achieve the absorption coefficient $\alpha \approx 0$, $K''(0) = K_r''(0) + \mathrm{i}K_i''(0) \approx K_r''(0)$, and $W = W_r + \mathrm{i}W_i = W_r$, we can derive the standard nonlinear Schrödinger equation governing the pulse evolution

$$\mathrm{i}\frac{\partial \tilde{\Omega}_p(z,t)}{\partial \xi} - \frac{1}{2}K_r''(0)\frac{\partial^2 \tilde{\Omega}_p(z,t)}{\partial \eta^2} = -W_r\left|\tilde{\Omega}_p(z,t)\right|^2 \tilde{\Omega}_p(z,t). \quad (18)$$

Analysing Eq.(18), we shall demonstrate the formation of ultraslow bright and dark optical solitons. As is well known, equation (18) admits of solutions describing the bright solitons or the dark solitons depending on the sign of product $K_r''(0)\cdot W_r$ ($K_r''(0)\cdot W_r < 0$ for the bright solitons and $K_r''(0)\cdot W_r > 0$ for the dark solitons). The fundamental bright soliton is given by

$$\Omega_P = \Omega_{p0}\,\mathrm{sech}\,(\eta/\tau)\exp(-\mathrm{i}\xi W_r\left|\Omega_{p0}\right|^2/2) \times \exp(-\mathrm{i}K(0)\xi), \quad (19)$$

where $\mathrm{sech}(\eta/\tau)$ is the hyperbolic secant function. Amplitude Ω_{p0} and width τ are arbitrary constants, subjecting only to the constraint $\left|\Omega_{p0}\right|^2 = 2K_r''(0)/W_r$. By numerical method, we now choose a set of appropriate physical parameters to demonstrate the existence of ultraslow bright optical soliton in our coupled double quantum well. We consider a system where the decay rates are $\gamma_4 = 2.5\,\mathrm{meV}$, $\gamma_3 = 1.2\,\mathrm{meV}$, $\gamma_2 = 0.02\,\mathrm{meV}$, $\Delta_t = 0\,\mathrm{meV}$, $\Delta_{\mathrm{th}} = 12\,\mathrm{meV}$, $\Delta_t = 10\,\mathrm{meV}$, $\Omega_c = 0.5\,\mathrm{meV}$, $\Omega_b = 0.11\,\mathrm{meV}$, and $\kappa_{13} = 10\,\mathrm{meV}$. Basing on this set of parameters, we obtain the absorption coefficient $\alpha \approx 0.015\,\mathrm{cm}^{-1}$, $V_g/c \approx 0.0084$, and $K_r''(0)\cdot W_r < 0$, which show that the ultraslow optical soliton can be generated in this system . In Fig.2, we have given numerical simulations of Eq.(17) for the fundamental bright soliton and the result shows a good agreement with Eq.(19).

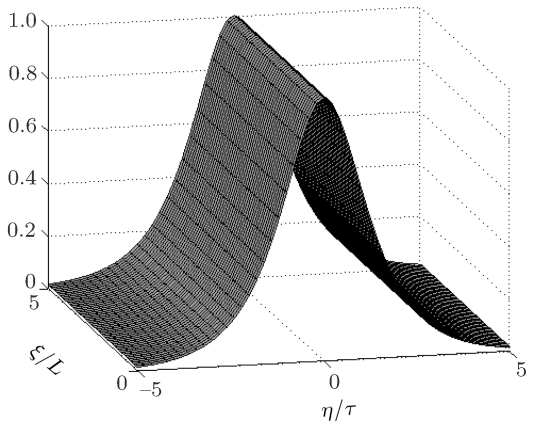


Fig.2. Plot of the amplitude of the generated fundamental bright soliton $|\Omega_p/\Omega_{p0}|^2\exp(-\alpha\xi)$ versus dimensionless time η/τ and distance ξ/L under the boundary condition $|\Omega_p(\xi=0,\eta)/\Omega_{p0}|^2 = \mathrm{sech}^2(\mu/\tau)$ by the numerical simulations. The fitting parameters $L = 1.0\,\mathrm{cm}$ and $\tau = 1.0\times10^{-6}\,\mathrm{s}$.

To sum up, we investigate the nonlinear evolution of the pulse probe field in an asymmetric coupled-quantum well driven coherently by a pulse probe field and two controlled fields. Applying the traditional research method, we establish the standard nonlinear Schrödinger equation governing the pulse evolution. Our studies show that, by choosing appropriate physical parameters, self-modulation can precisely balance group velocity dispersion in our system, leading to the formation of ultraslow optical solitons of the probe field. Such ultraslow optical solitons may provide a new possibility for the development of the controlled technique of optical buffers and optical delay lines.

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