Vol 18 No 4, April 2009
1674-1056/2009/18(04)/1517-11
Chinese Physics B
© 2009 Chin. Phys. Soc.
and IOP Publishing Ltd

Entropy squeezing for a two-level atom in two-mode Raman coupled model with intrinsic decoherence*

Zhang Jian(张 剑)†, Shao Bin(邵 彬), and Zou Jian(邹 健)

Department of Physics, School of Science, Beijing Institute of Technology, Beijing 100081, *China*

(Received 10 July 2008; revised manuscript received 14 August 2008)

In this paper, we investigate the entropy squeezing for a two-level atom interacting with two quantized fields through Raman coupling. We obtain the dynamical evolution of the total system under the influence of intrinsic decoherence when the two quantized fields are prepared in a two-mode squeezing vacuum state initially. The effects of the field squeezing factor, the two-level atomic transition frequency, the second field frequency and the intrinsic decoherence on the entropy squeezing are discussed. Without intrinsic decoherence, the increase of field squeezing factor can break the entropy squeezing. The two-level atomic transition frequency changes only the period of oscillation but not the strength of entropy squeezing. The influence of the second field frequency is complicated. With the intrinsic decoherence taken into consideration, the results show that the stronger the intrinsic decoherence is, the more quickly the entropy squeezing will disappear. The increase of the atomic transition frequency can hasten the disappearance of entropy squeezing.

Keywords: entropy squeezing, intrinsic decoherence, two-mode Raman coupled model
PACC: 4250

1. Introduction

Much effort has been made to study quantum decoherence theoretically and experimentally, because of its links to the fundamentals of quantum mechanics[1−4] and its applications[5−7] to quantum information processing and quantum computation. There are several candidates[8] for physical implementation of quantum information devices such as cavity quantum electrodynamics (QED), trapped ion, liquid nuclear magnetic resonance (NMR), etc. Recently, the dynamics of composite system composed of a cavity and other quantum systems such as cold atoms,[9] quantum well,[10] etc., has been investigated, and the theoretical improvement on arrays of microcavities in photonic crystal[11] has opened up possibilities for designing quantum information devices. Quantum decoherence is one of main formidable obstructions in using some of these candidates to design quantum information devices. Theoretically, there are at least two approaches to explaining the decoherence phenomena. One approach[12−14] is to ascribe the decoherence of quantum system to the reservoirs with many degrees of freedom. The interaction between quantum system and a reservoir causes the coherence of quantum system to disappear. The other approach[15−18] is to modify the Schrödinger equation so that quantum coherence is damaged as the scale of physical system becomes large. In particular, Milburn[18] has proposed a simple model on the assumption that the system evolves in a stochastic sequence of identical unitary transformations in sufficiently short time steps. This model makes a feature of constants of the motion remaining unchanged. The Milburn model has been widely applied to the investigation of the decoherence problem.

The entropic uncertainty relation[19−22] as an alternative to the Robertson formulation[23] of quantum uncertainty has been widely investigated in quantum optics, quantum information, etc. The Robertson formulation has a lower bound which depends on the state of system. Therefore, it imposes no restriction on the variance of one variable, when the state of system is an eigenstate of the other variable. The new formulation of quantum uncertainty by information entropy can overcome this deficiency of Robertson formulation. The Robertson formulation is expressed in terms of the variances of the system observables, and it contains only second-order statisti-

*Project supported by the National Natural Science Foundation of China (Grant No 10374007).
†Corresponding author.

cal moments. However, the entropic uncertainty relation contains all order moments and directly connects quantum information with the quantum fluctuations of system. The connections[24−26] between entropic uncertainty relations and entanglement have also been discussed, and some separability criteria utilizing the entropic uncertainty relations were obtained. Besides, the entropic uncertainty relation has been used to prove quantum cryptography protocols to be secure.[27] Entropic uncertainty relation has been studied in both discrete and continuous systems and expressed in various forms. Especially, in the two-level atom case, the entropic uncertainty relation used as a general criterion[28] for the squeezing of an atom has been found and discussed in several systems, i.e. JC model and its generalizations,[29−33] Heisenberg model,[34] etc., among which JC model and its generalizations have attracted more attention in discussion. Meanwhile, many effects in field-atom model such as atomic motion,[35,36] time-dependent coupling,[37] nonlinear medium,[38,39] stark shift,[40] etc. have also been considered in some papers dealing with the entropy squeezing. In particular, the effect of intrinsic decoherence on entropy squeezing of superconducting charge qubit interacting with single-mode field has been discussed.[41,42] This, in fact, motivates us to investigate the effect of intrinsic decoherence on entropy squeezing of a two-level atom in Raman coupled model which has a more complicated structure than single-mode case and has been widely applied to several systems such as Josephson junction. Our results are helpful for preparing two-level subsystems and selecting their appropriate parameters. The rest of the present paper is organized as follows. In Section 2, we give a brief review about variance and entropy squeezing. The Milburn equation of the two-mode Raman coupled model in our case is solved in Section 3. The entropy squeezing is discussed in detail in Section 4 and some conclusions are drawn from the present research in Section 5.

2. Variance and entropy squeezing

In this section, to start with, we review some well-known facts about variances and entropy squeezing.

The Robertson formulation of Heisenberg uncertainty relation for a two-level atom characterized by the Pauli operators satisfying the commutation $\left[\hat{S}_x, \hat{S}_y\right] = \mathrm{i}\hat{S}_z$, can be written as

$$\left\langle (\Delta\hat{S}_x)^2 \right\rangle \left\langle (\Delta\hat{S}_y)^2 \right\rangle \geq \frac{1}{4}\left|\left\langle \hat{S}_z \right\rangle\right|^2, \tag{1}$$

where the variances $\left\langle (\Delta\hat{S}_\alpha)^2 \right\rangle = \left\langle \hat{S}_\alpha^2 \right\rangle - \left\langle \hat{S}_\alpha \right\rangle^2$.

The variance in the component $\hat{S}_\alpha$ of the dipole of a two-level atom is said to be squeezed if and only if $\hat{S}_\alpha$ satisfies the following condition:

$$V\left(\hat{S}_\alpha\right) = \sqrt{\left\langle (\Delta\hat{S}_\alpha)^2 \right\rangle} - \left(\left|\left\langle \hat{S}_z \right\rangle\right|/2\right)^{1/2} < 0, \quad \alpha = 1, 2. \tag{2}$$

For a two-level atom which has three complementary observables with non-degenerate eigenvalues, the corresponding information entropies are defined as[28,43]

$$\begin{aligned} H\left(S_\alpha\right) =& -\sum_{i=1}^{2} P_i\left(S_\alpha\right) \ln P_i\left(S_\alpha\right), P_i\left(S_\alpha\right) \\ =& \left\langle \psi_{\alpha i}\right| \rho \left|\psi_{\alpha i}\right\rangle, \\ \alpha \equiv& x, y, z, \end{aligned} \tag{3}$$

where $\left|\psi_{\alpha i}\right\rangle$ is an eigenvector of the operator $\hat{S}_\alpha$,

$$\hat{S}_\alpha \left|\psi_{\alpha i}\right\rangle = \lambda_{\alpha i} \left|\psi_{\alpha i}\right\rangle, \tag{4}$$

and

$$\begin{aligned} H\left(S_x\right) =& -\left[\frac{1}{2} + \mathrm{Re}\left(\rho_{eg}\right)\right] \ln\left[\frac{1}{2} + \mathrm{Re}\left(\rho_{eg}\right)\right] - \left[\frac{1}{2} - \mathrm{Re}\left(\rho_{eg}\right)\right] \ln\left[\frac{1}{2} - \mathrm{Re}\left(\rho_{eg}\right)\right], \\ H\left(S_y\right) =& -\left[\frac{1}{2} + \mathrm{Im}\left(\rho_{eg}\right)\right] \ln\left[\frac{1}{2} + \mathrm{Im}\left(\rho_{eg}\right)\right] - \left[\frac{1}{2} - \mathrm{Im}\left(\rho_{eg}\right)\right] \ln\left[\frac{1}{2} - \mathrm{Im}\left(\rho_{eg}\right)\right], \\ H\left(S_z\right) =& -\rho_{ee} \ln \rho_{ee} - \rho_{gg} \ln \rho_{gg}. \end{aligned} \tag{5}$$

The entropic uncertainty relation for the two-level atom is derived[28,43] as

$$H(S_x) + H(S_y) + H(S_z) \geq 2\ln 2. \tag{6}$$

The entropic uncertainty relation can be used as a general criterion for the squeezing of a two-level atom. Setting both sides of inequality (6) as the power of the irrational number e, the following inequality which is similar to the Robertson formulation of Heisenberg uncertainty relation can be derived as

$$\exp[H(S_x)]\exp[H(S_y)]\exp[H(S_z)] \geq 4. \tag{7}$$

The information entropy squeezing factors of component S_α can be defined as[28]

$$E(S_\alpha) = \exp[H(S_\alpha)] - 2/[\exp[H(S_z)]]^{1/2}. \tag{8}$$

If the information entropy squeezing factor $E(S_\alpha) < 0$, the fluctuation in the component S_α is said to be 'squeezed in entropy'.

3. Model and solution of Milburn equation

For simplification, we set the Planck constant $\hbar = 1$ throughout our paper. In the following, we solve the Milburn equation for the two-mode Raman coupled model with the Hamiltonian[44]

$$H = \omega_1 a_1^\dagger a_1 + \omega_2 a_2^\dagger a_2 + \frac{\omega}{2}\sigma_z + g\left(a_1 a_2^\dagger \sigma_+ + a_1^\dagger a_2 \sigma_-\right), \tag{9}$$

where σ_z and $\sigma_\pm$ are the spin flip operators characterizing the effective two-level atom with transition frequency ω, and $a_i\left(a_i^\dagger\right)$ $(i = 1, 2)$ are the annihilation (creation) operators of the two field modes of frequencies ω_i, $i = 1, 2$, respectively. The configuration of the two-level atom is illustrated in Fig.1.

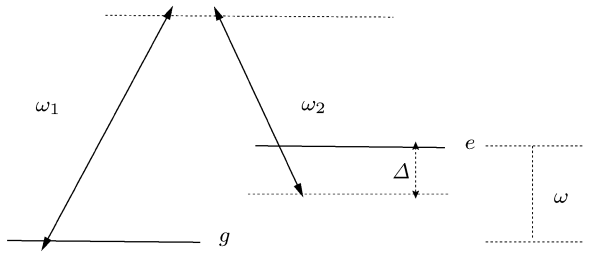


Fig.1. Configuration of atomic levels. The detuning $\Delta = \omega + \omega_2 - \omega_1$.

Here, what we are interested in is the effect of the intrinsic decoherence on the entropy squeezing of the two-level atom. The intrinsic decoherence approach has been studied by Milburn,[18] who has proposed a simple model based on the assumption that on sufficiently short time steps the system does not evolve continuously under unitary evolution, but rather, in a stochastic sequence of identical unitary transformations. This assumption leads to a modification of the Schrödinger equation which contains a term responsible for the decay of quantum coherence in the energy eigenstate bases, without the intervention of a reservoir and the usual energy dissipation associated with normal decay. The stochastic behaviour in the time evolution of the system implies that the density operator satisfies, to the first order, the following equation of motion:[18]

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -\mathrm{i}[H, \rho(t)] - \frac{1}{2\gamma}[H, [H, \rho(t)]], \tag{10}$$

where γ is the intrinsic decoherence parameter. If $\gamma \to \infty$, the Milburn equation reduces to the standard von Neumann equation describing the Schrödinger evolution. The formal solution of Eq.(10) can be written explicitly as

$$\rho(t) = \sum_{k=0}^{\infty}\left(\frac{t}{\gamma}\right)^k \frac{1}{k!} M_k(t)\rho(0)M_k^\dagger(t), \tag{11}$$

where $\rho(0)$ is the initial state of the system and the Kraus operators $M_k(t)$ $(k = 0, 1, ...)$ contain all the information about any quantum-computational process and are given by

$$M_k(t) = H^k \exp\left(-\mathrm{i}Ht - \frac{t}{2\gamma}H^2\right), \qquad k = 0, 1, \ldots. \tag{12}$$

The Milburn equation of Hamiltonian (9) under the intrinsic decoherence has been solved by Xu, *et al.*[45] Utilizing their method, we concentrate on the investigation of entropy squeezing.

The explicit form of the Kraus operators can be acquired as follows:[45]

$$M_k(t) = \frac{1}{2}\left[\hat{f}_+^k \exp\left(-\mathrm{i}\hat{f}_+ t\right)\exp\left(-\frac{\hat{f}_+^2 t}{2\gamma}\right) + \hat{f}_-^k \exp\left(-\mathrm{i}\hat{f}_- t\right)\exp\left(-\frac{\hat{f}_-^2 t}{2\gamma}\right)\right]$$
$$+ \frac{1}{2}\left[\hat{f}_+^k \exp\left(-\mathrm{i}\hat{f}_+ t\right)\exp\left(-\frac{\hat{f}_+^2 t}{2\gamma}\right) - \hat{f}_-^k \exp\left(-\mathrm{i}\hat{f}_- t\right)\exp\left(-\frac{\hat{f}_-^2 t}{2\gamma}\right)\right]$$
$$\times\left[\frac{\Delta\sigma_z}{2\Omega\left(K_1,K_2\right)} + \frac{g\left(a_1 a_2^\dagger \sigma_+ + a_1^\dagger a_2 \sigma_-\right)}{\Omega\left(K_1,K_2\right)}\right], \tag{13}$$

where

$$\hat{f}_\pm = \omega_1\left(K_1 - \frac{1}{2}\right) + \omega_2\left(K_2 - \frac{1}{2}\right) \pm \Omega\left(K_1,K_2\right),$$
$$\Omega\left(K_1,K_2\right) = \sqrt{\frac{\Delta^2}{4} + g^2 K_1 K_2},$$
$$\Delta = \omega + \omega_2 - \omega_1,$$
$$K_1 = a_1^\dagger a_1 + \frac{1+\sigma_z}{2}, \quad K_2 = a_2^\dagger a_2 + \frac{1-\sigma_z}{2}, \tag{14}$$

which are two constants of motion in Hamiltonian (9).

We assume that the two field modes are initially prepared in a two-mode squeezed vacuum state which is widely studied in quantum information and expressed as

$$|\psi\rangle_f = \sum_{n=0}^{\infty} \frac{\left(\tanh\left(s\right)\right)^n}{\cosh\left(s\right)} |n,n\rangle \tag{15}$$

with field squeezing factor s, and the atom is initially in the coherent state

$$|\varphi\rangle_a = \cos\left(\frac{\theta}{2}\right)|e\rangle + \sin\left(\frac{\theta}{2}\right)\mathrm{e}^{\mathrm{i}\phi}|g\rangle. \tag{16}$$

The two subsystems are separable initially

$$\rho\left(0\right) = \rho_f\left(0\right)\otimes\rho_a\left(0\right) = |\psi\rangle_{f\,f}\langle\psi| \otimes |\varphi\rangle_{a\,a}\langle\varphi|. \tag{17}$$

Substituting expressions (17) and (13) for $\rho\left(0\right)$ and $M_k(t)$ into equation (11), respectively, we obtain the time evolution of the total system

$$\rho\left(t\right) = \frac{1}{2}\sum_{k=0}^{\infty}\left(\frac{t}{\gamma}\right)^k \frac{1}{k!}\Big\{(1+\cos\theta)\sum_{n,m} Q_n Q_m^* M_k |n,n,e\rangle\langle m,m,e| M_k^\dagger$$
$$+ (1-\cos\theta)\sum_{n,m} Q_n Q_m^* M_k |n,n,g\rangle\langle m,m,g| M_k^\dagger$$
$$+ \sin\theta\,\mathrm{e}^{\mathrm{i}\phi}\sum_{n,m} Q_n Q_m^* M_k |n,n,g\rangle\langle m,m,e| M_k^\dagger + \sin\theta\,\mathrm{e}^{-\mathrm{i}\phi}\sum_{n,m} Q_n Q_m^* M_k |n,n,e\rangle\langle m,m,g| M_k^\dagger\Big\}, \tag{18}$$

where

$$M_k|n,n,e\rangle = \frac{1}{2}\left[F_{+,k}(n+1,n)\left(1+\frac{\Delta}{2\Omega(n+1,n)}\right) + F_{-,k}(n+1,n)\left(1-\frac{\Delta}{2\Omega(n+1,n)}\right)\right]|n,n,e\rangle$$
$$+ \frac{1}{2}\left[F_{+,k}(n+1,n) - F_{-,k}(n+1,n)\right]\frac{g\sqrt{(n+1)\,n}}{\Omega(n+1,n)}|n+1,n-1,g\rangle,$$
$$M_k|n,n,g\rangle = \frac{1}{2}\left[F_{+,k}(n,n+1)\left(1-\frac{\Delta}{2\Omega(n,n+1)}\right) + F_{-,k}(n,n+1)\left(1+\frac{\Delta}{2\Omega(n,n+1)}\right)\right]|n,n,g\rangle$$
$$+ \frac{1}{2}\left[F_{+,k}(n,n+1) - F_{-,k}(n,n+1)\right]\frac{g\sqrt{n\,(n+1)}}{\Omega(n,n+1)}|n-1,n+1,e\rangle \tag{19}$$

with

$$F_{\pm,k}(m,n) = f_\pm^k(m,n)\exp[-\mathrm{i}f_\pm(m,n)t]\exp\left(-\frac{f_\pm^2(m,n)t}{2\gamma}\right). \tag{20}$$

To obtain the entropy squeezing factors of the two-level atom, the reduced density operator of two-level atom is obtained by tracing over the two fields' variables as

$$\rho_a(t) = \rho_{ee}|e\rangle\langle e| + \rho_{eg}|e\rangle\langle g| + \rho_{ge}|g\rangle\langle e| + \rho_{gg}|g\rangle\langle g|, \tag{21}$$

where

$$\begin{aligned}
\rho_{ee} &= \frac{1}{8}\sum_{k=0}^{\infty}\left(\frac{t}{\gamma}\right)^k\frac{1}{k!}\Big\{\sum_n |Q_n|^2\{(1+\cos\theta)\\
&\times\left|F_{+,k}(n+1,n)\left(1+\frac{\Delta}{2\Omega(n+1,n)}\right)+F_{-,k}(n+1,n)\left(1-\frac{\Delta}{2\Omega(n+1,n)}\right)\right|^2\\
&+(1+\cos\theta)\left|F_{+,k}(n,n+1)-F_{-,k}(n,n+1)\right|^2\frac{g^2n(n+1)}{\Omega^2(n,n+1)}\Big\},\\
\rho_{eg} &= \frac{1}{8}\sum_{k=0}^{\infty}\left(\frac{t}{\gamma}\right)^k\frac{1}{k!}\Big\{\sin\theta\,\mathrm{e}^{-\mathrm{i}\phi}\sum_n|Q_n|^2\left[F_{+,k}(n+1,n)\left(1+\frac{\Delta}{2\Omega(n+1,n)}\right)\right.\\
&\left.+F_{-,k}(n+1,n)\left(1-\frac{\Delta}{2\Omega(n+1,n)}\right)\right]\left[F^*_{+,k}(n,n+1)\left(1-\frac{\Delta}{2\Omega(n,n+1)}\right)\right.\\
&\left.+F^*_{-,k}(n,n+1)\left(1+\frac{\Delta}{2\Omega(n,n+1)}\right)\right]\Big\},\\
\rho_{eg} &= \rho^*_{ge},\quad \rho_{gg} = 1-\rho_{ee}.
\end{aligned} \tag{22}$$

Using the two-level atom reduced density operator $\rho_a(t)$ and equations (5) and (8), the information entropy squeezing factors of the operators S_x, S_y and S_z can be obtained directly.

4. Discussion

On the basis of the analytical solution presented in the previous section, we now compare the entropy squeezing factors with variance squeezing factors and simulate the properties of the entropy squeezing factors of the two-level atom in this section. We consider the two-level atom to be prepared initially in the state $\theta = \pi/2$, $\varphi = 0$ and keep the coupling constant g unchanged, i.e. $g = 0.1$, throughout our paper.

Firstly, we compare the properties of the entropy squeezing factors with those of variance squeezing factors. For instance, we consider the resonance case without intrinsic decoherence. The time evolutions of the entropy squeezing factors and variance squeezing factors are shown in Fig.2. We set the frequencies of two fields $\omega_1 = 2$ and $\omega_2 = 1$, and the field squeezing

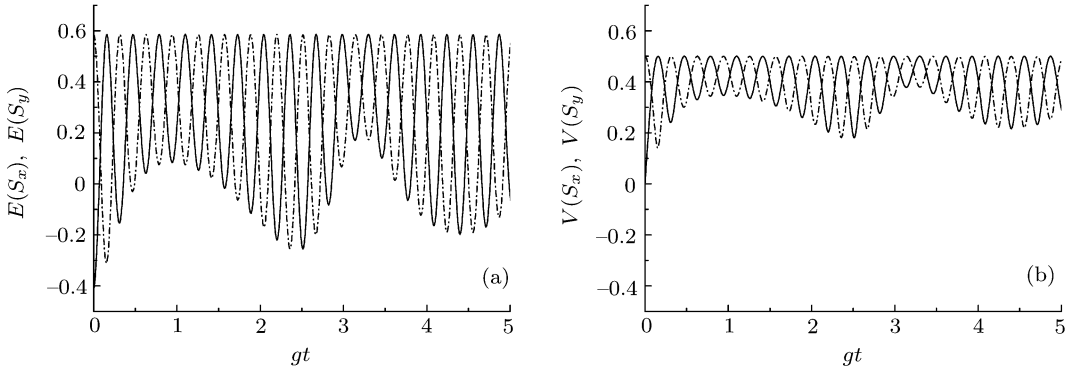


Fig.2. Time evolutions of entropy squeezing factors and variance squeezing factors of a two-level atom in the two-mode Raman-coupled model. The initial two-level atomic state parameters $\theta = \pi/2$ and $\varphi = 0$; the field squeezing factor $s = 0.7$; two field frequencies $\omega_1 = 2$, and $\omega_2 = 1$; the coupling constant $g = 0.1$; detuning $\Delta = 0$; decoherence $\gamma = \infty$. (a) The entropy squeezing factors $E(S_x)$ and $E(S_y)$; (b) the variance squeezing factors $V(S_x)$ and $V(S_y)$.

factor $s = 0.7$. From Fig.2, we find that both $E(S_x)$ and $E(S_y)$ exhibit squeezing in several time intervals, but both $V(S_x)$ and $V(S_y)$ show no variance squeezing always. This phenomenon confirms that the information entropies contain more information than the variances of the two-level atom and shows that entropy squeezing factor is a better measure of atomic squeezing than variance.

Secondly, we focus on the influence of the field squeezing factor denoted by s on the time evolution of the entropy squeezing factors of the two-level atom. We have plotted the time evolution of the entropy squeezing factors in Fig.3 with the same values of the other parameters as those in the above discussion except the value of the field squeezing factor. From Fig.3(a), we can observe that $E(S_x)$and $E(S_y)$ periodically oscillate with a phase difference of $\pi/2$, and their amplitudes all have the maxima 0.586 and minima –0.414 which is the optimal value of entropy squeezing, when the field squeezing factor is small such as $s = 0.05$. As the field squeezing factor increases, the strength of entropy squeezing would become weaker and the total time interval of entropy squeezing would become shorter. When the field squeezing factor is large enough, the entropy squeezing would disappear in the time evolution which is shown in Fig.3(d). The large field squeezing factor breaks the entropy squeezing and the squeezing of information entropies are mainly produced by the small n terms of reduced density operator of the two-level atom. The appearance of large n terms would break the entropy squeezing.

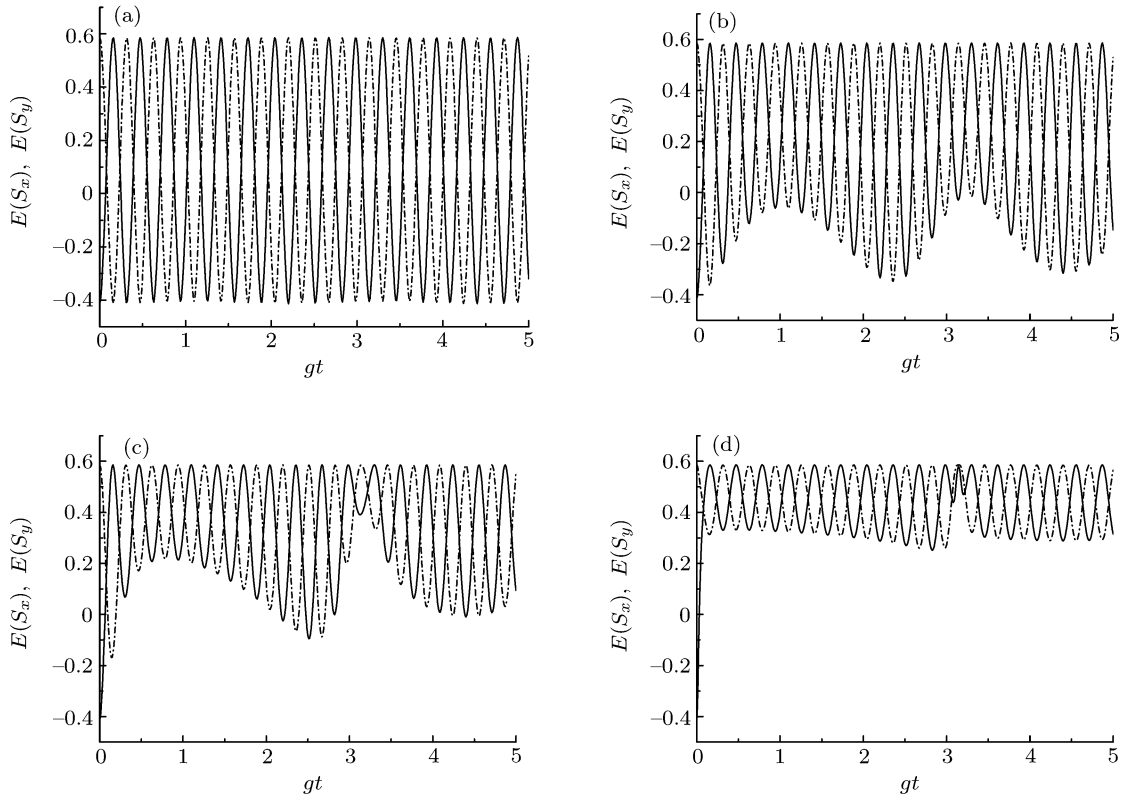


Fig.3. Time evolutions of entropy squeezing factors $E(S_x)$ (solid) and $E(S_y)$ (dash-dot), with initial two-level atomic state parameters, $\theta = \pi/2$ and $\varphi = 0$; two field frequencies $\omega_1 = 2$ and $\omega_2 = 1$; the coupling constant $g = 0.1$; decoherence $\gamma = \infty$; detuning $\Delta = 0$; values of the field squeezing factor (a) $s = 0.05$; (b) $s = 0.5$; (c) $s = 1$; (d) $s = 2$.

The effect of detuning between the fields and the two-level atom, $\Delta = \omega + \omega_2 - \omega_1$, in the case where two field frequencies are fixed, i.e. the effect of the two-level atomic transition frequency, is also of importance when the influence of intrinsic decoherence is excluded. We maintain the two field frequencies unchanged, i.e. $\omega_1 = 2$ and $\omega_2 = 1$ and select the field squeezing factor $s = 0.05$ because of its relatively simple curves. The initial state of the two-level atom, coupling constant and decoherence parameter are main-

tained to be the same as those of Fig.3. From Fig.4(a), we can find that when the detuning between the fields and the two-level atom is equal to -1, i.e. the two levels are degenerate, the entropy squeezing factor $E(S_x)$ oscillates near the optimal value -0.414 at all time and $E(S_y)$ keeps the value at 0.586. When the detuning is larger than -1, the entropy squeezing factors $E(S_x)$ and $E(S_y)$ show the large amplitude oscillations in the time evolution. The frequencies of oscillations of $E(S_x)$ and $E(S_y)$ monotonically increase with the increase of detuning, while the time of squeezing, i.e. half the total time, remains unchanged. The extrema of $E(S_x)$ and $E(S_y)$ have no large change at various values of detuning. When $\gamma = \infty$ and $s = 0.05$, we can find the only oscillation term

$$\left(1 - \frac{\Delta}{|\Delta|}\right)^2 \cos\left[\phi + (\omega_1 - \omega_2 - |\Delta|)\, t\right] + \left(1 + \frac{\Delta}{|\Delta|}\right)^2 \cos\left[\phi + (\omega_1 - \omega_2 + |\Delta|)\, t\right] \tag{23}$$

in the analytic expression of $E(S_x)$ and the similar term in $E(S_y)$, because the terms $Q_0 \approx 1$ and $Q_i \approx 0$ $(i = 1, 2, \ldots)$. When the atomic transition frequency is smaller than the difference between two field frequencies, i.e. $\Delta < 0$, the term becomes $4\cos\left[\phi + (\omega_1 - \omega_2 - |\Delta|)\, t\right]$. When the atomic transition frequency is larger than the difference between two field frequencies, i.e., $\Delta > 0$, the term becomes $4\cos\left[\phi + (\omega_1 - \omega_2 + |\Delta|)\, t\right]$. It is easy to see that the oscillation frequency increases with the increase of Δ. Therefore, the change of the atomic transition frequency only influences the oscillation frequencies of entropy squeezing factors and does not influence the strength of entropy squeezing.

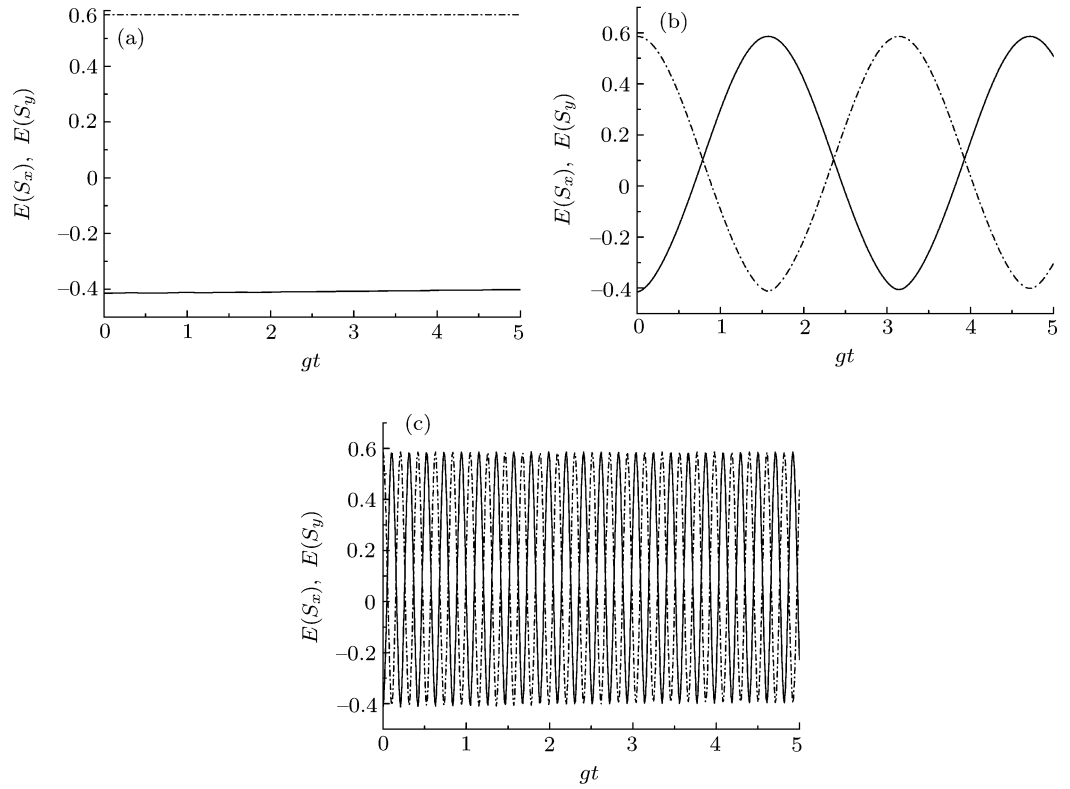


Fig.4. Time evolutions of entropy squeezing factors $E(S_x)$ (solid) and $E(S_y)$ (dash-dot), with initial two-level atomic state parameters, $\theta = \pi/2$ and $\varphi = 0$; two field frequencies $\omega_1 = 2$ and $\omega_2 = 1$; the coupling constant $g = 0.1$; decoherence $\gamma = \infty$; the field squeezing factor $s = 0.05$, detuning (a) $\Delta = -1$; (b) $\Delta = -0.9$; (c) $\Delta = 0.5$.

After discussing the situation where the two-level atomic transition frequency is changed, we consider the situation where the second field frequency ω_2 is changed. The temporal evolutions of $E(S_x)$ when $s = 0.5$ is illustrated in Fig.5 as an example; because in the case of small field squeezing factor, the change

of second field frequency has no obvious effect on the evolution of entropy squeezing factors. We set the first field frequency $\omega_1 = 2$, two-level atomic transition frequency $\omega = 1$ and decoherence parameter $\gamma = \infty$. When the second field frequency is equal to 0, i.e. $\Delta = -1$, the evolution of the entropy squeezing factor $E(S_x)$ displays the squeezing and shows the complex periodicity. It reaches the value near 0.4 when time is between 15 and 20. Accompanied with the second field frequency becoming larger, $E(S_x)$ reaches the value near 0.4 frequently. When the second field frequency reaches 1, i.e. $\Delta = 0$, the time is between 2 and 3. While the second field frequency continues to increase, the time becomes larger again as shown in Fig. 5(c). The behaviour of $E(S_y)$ is similar to that of $E(S_x)$. The influence of the second field frequency change is more complicated than that of two-level atomic transition frequency change.

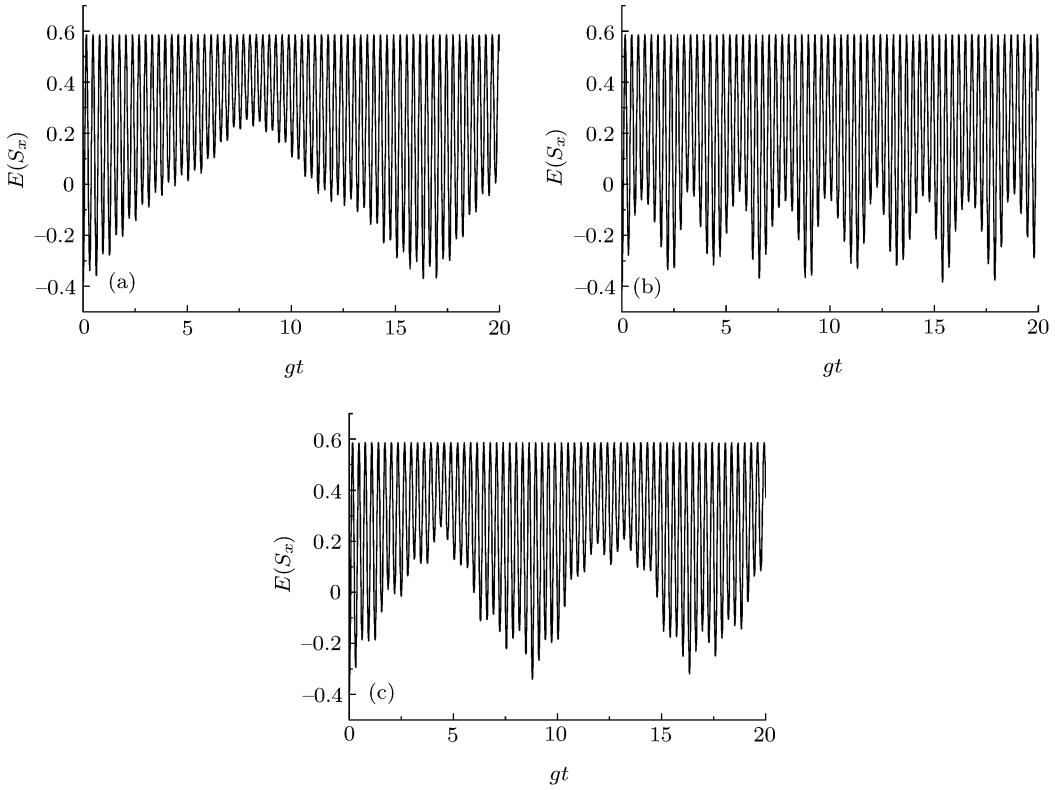


Fig.5. Time evolutions of the entropy squeezing factor $E(S_x)$, with initial two-level atomic state parameters, $\theta = \pi/2$ and $\varphi = 0$; decoherence $\gamma = \infty$; the coupling constant $g = 0.1$; the field squeezing factor $s = 0.5$, the first field frequency $\omega_1 = 2$, the two-level atomic transition frequency $\omega = 1$; (a) $\omega_2 = 0$ and $\Delta = -1$; (b) $\omega_2 = 1$ and $\Delta = 0$; (c) $\omega_2 = 1.5$ and $\Delta = 0.5$.

Finally, we discuss the effect of intrinsic decoherence on the time evolution of the entropy squeezing factors. We set the value of field squeezing factor $s = 0.05$ and two field frequencies $\omega_1 = 2$ and $\omega_2 = 1$. When the difference between two field frequencies is resonant with two-level atomic transition frequency and the intrinsic decoherence parameter $\gamma = 50$, the local minima of entropy squeezing factors $E(S_x)$ and $E(S_y)$ increase with the increase of time as shown in Fig.6(a). Therefore, the squeezing characteristic diminishes gradually. Meanwhile, the peaks of oscillations are not changed at all time. Increasing the strength of intrinsic decoherence such as $\gamma = 10$, the speed of squeezing vanishing would become faster than that in weak decoherence case as shown in Figs.6(a) and 6(b). Fixing the value of intrinsic decoherence parameter and investigating the influence of detuning with two fixed field frequencies, i.e. the influence of the two-level atomic transition frequency, we find that the speeds of squeezing vanishing decrease with $\Delta \to -1$ and increase with the increase of detuning when $\Delta \to \infty$. We find the oscillation terms in the analytic expressions of $E(S_x)$ and $E(S_y)$. For instance, $E(S_x)$ has oscillation terms

$$\exp\left[-\frac{(\omega_1-\omega_2-|\Delta|)^2 t}{4\gamma}\right]\left(1-\frac{\Delta}{|\Delta|}\right)^2\left(\frac{\omega_1-\omega_2-|\Delta|}{2}\right)^{2k}$$
$$+\exp\left[-\frac{(\omega_1-\omega_2+|\Delta|)^2 t}{4\gamma}\right]\left(1+\frac{\Delta}{|\Delta|}\right)^2\left(\frac{\omega_1-\omega_2+|\Delta|}{2}\right)^{2k} \tag{24}$$

and

$$\exp\left[-\frac{(\omega_1-\omega_2-|\Delta|)^2}{4\gamma}t\right]\left(1-\frac{\Delta}{|\Delta|}\right)^2\cos\left[\phi+(\omega_1-\omega_2-|\Delta|)\,t\right]\left(\frac{\omega_1-\omega_2-|\Delta|}{2}\right)^k\left(\frac{-\omega_1+\omega_2+|\Delta|}{2}\right)^k$$
$$+\exp\left[-\frac{(\omega_1-\omega_2+|\Delta|)^2}{4\gamma}t\right]\left(1+\frac{\Delta}{|\Delta|}\right)^2\cos\left[\phi+(\omega_1-\omega_2+|\Delta|)\,t\right]\left(\frac{\omega_1-\omega_2+|\Delta|}{2}\right)^k\left(\frac{-\omega_1+\omega_2-|\Delta|}{2}\right)^k,$$
$$k=0,1,\ldots. \tag{25}$$

From the above expressions (24) and (25), when decoherence strength increases, i.e. γ becomes smaller,

$$\exp\left[-\frac{(\omega_1-\omega_2\pm|\Delta|)^2}{4\gamma}t\right]$$

will decrease more quickly, which causes the entropy squeezing to disappear quickly. Similarly, when the atomic transition frequency is smaller than the difference between two field frequencies, i.e. $\Delta<0$, expressions (24) and (25) have the following forms, respectively:

$$4\exp\left[-\frac{(\omega_1-\omega_2-|\Delta|)^2 t}{4\gamma}\right]\left(\frac{\omega_1-\omega_2-|\Delta|}{2}\right)^{2k} \tag{26}$$

and

$$4\exp\left[-\frac{(\omega_1-\omega_2-|\Delta|)^2}{4\gamma}t\right]\cos\left[\phi+(\omega_1-\omega_2-|\Delta|)\,t\right]\left(\frac{\omega_1-\omega_2-|\Delta|}{2}\right)^k\left(\frac{-\omega_1+\omega_2+|\Delta|}{2}\right)^k. \tag{27}$$

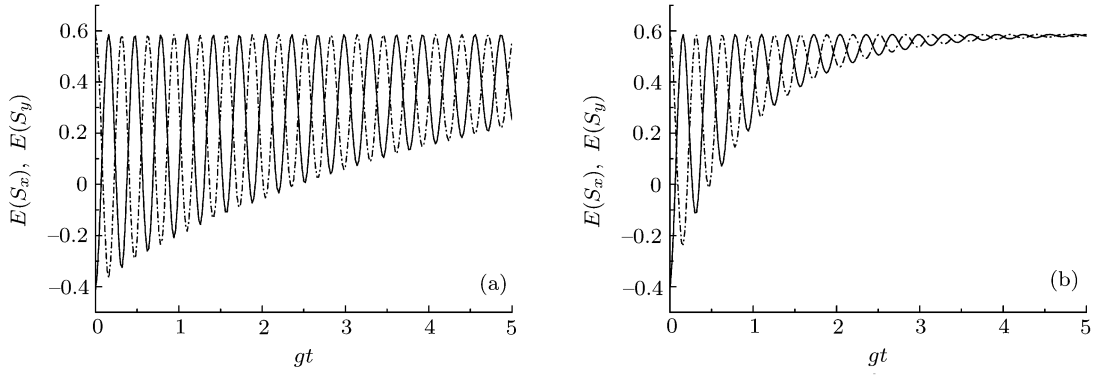
When the atomic transition frequency is larger than the difference between two field frequencies, i.e. $\Delta>0$, expressions (24) and (25) have the following forms:

$$4\exp\left[-\frac{(\omega_1-\omega_2+|\Delta|)^2 t}{4\gamma}\right]\left(\frac{\omega_1-\omega_2+|\Delta|}{2}\right)^{2k} \tag{28}$$

and

$$4\exp\left[-\frac{(\omega_1-\omega_2+|\Delta|)^2}{4\gamma}t\right]\cos\left[\phi+(\omega_1-\omega_2+|\Delta|)\,t\right]\left(\frac{\omega_1-\omega_2+|\Delta|}{2}\right)^k\left(\frac{-\omega_1+\omega_2-|\Delta|}{2}\right)^k. \tag{29}$$

Therefore, the entropy squeezing properties turn to vanish quickly with the increase of the detuning, Δ. The vanishing speeds of entropy squeezing properties are determined by not only decoherence strength but also the two-level atomic transition frequency. Through selecting the atomic transition frequency, we can reduce the effect of decoherence.



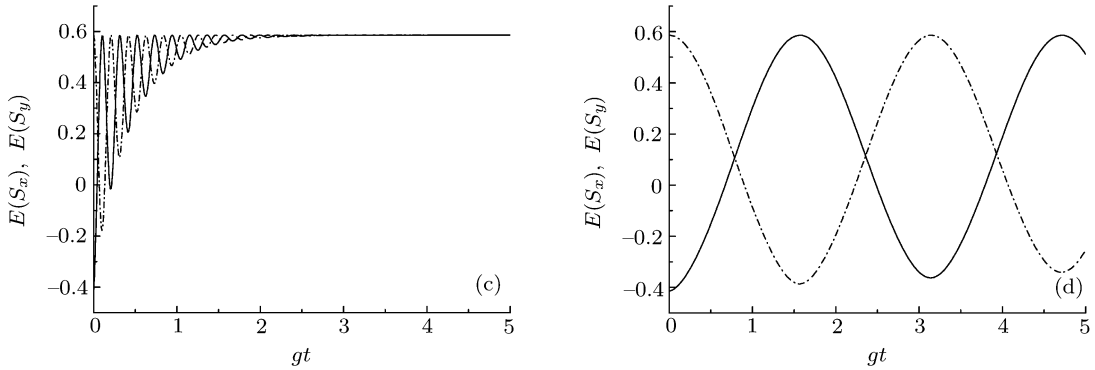


Fig.6. Time evolutions of entropy squeezing factors $E(S_x)$ (solid) and $E(S_y)$ (dash-dot), with initial two-level atomic state parameters, $\theta = \pi/2$ and $\varphi = 0$; the field squeezing factor $s = 0.05$; the coupling constant $g = 0.1$; two field frequencies $\omega_1 = 2$ and $\omega_2 = 1$; (a) $\Delta = 0$ and $\gamma = 50$; (b) $\Delta = 0$ and $\gamma = 10$; (c) $\Delta = 0.5$ and $\gamma = 10$; (d) $\Delta = -0.9$ and $\gamma = 10$.

5. Conclusion

In this paper, we investigate the time evolution of the system composed of a two-level atom interacting with two quantized fields through Raman-coupling. Through solving the Milburn equation in which the effect of intrinsic decoherence is included, we obtain the temporal evolution of density operator to discuss the behaviour of the information entropy squeezing. It is shown that the information entropies squeeze to more extent than the variances in our case without intrinsic decoherence, which means that the entropy squeezing is a better measure of the two-level atomic squeezing than variance squeezing. We discuss the effects of field squeezing factor, the two-level atomic transition frequency, the second field frequency and intrinsic decoherence on the entropy squeezing. We find that the entropy squeezing strength decreases with the increase of field squeezing factor. The two-level atomic transition frequency changes only the frequency of oscillation but not the strength of entropy squeezing, whereas the effect of the second field frequency change is complex. The intrinsic decoherence breaks the entropy squeezing and the squeezing properties vanishing becomes fast with the increase of decoherence strength or the two-level atomic transition frequency. The two-mode Raman coupled model is applied to describing several systems which are used as candidates for qubit. Therefore, our investigation and analytical expressions should be useful for preparing two-level subsystems and selecting their appropriate parameters.

References

[1] Bohr N 1928 *Nature* **121** 580
[2] Zurek W H 1991 *Phys. Today* **44** 36
[3] Zurek W H 2003 *Rev. Mod. Phys.* **75** 715
[4] Eberly J H and Yu T 2007 *Science* **316** 555
[5] Plenio M B and Huelga S F 2002 *Phys. Rev. Lett.* **88** 197901
[6] Yi X X, Yu C S, Zhou L and Song H S 2003 *Phys. Rev.* A **68** 052304
[7] Xu J B and Li S B 2005 *New J. Phys.* **7** 72
[8] For a review, see Brandt H E 1998 *Prog. Quantum Electron.* **22** 257
[9] Zhang J M, Liu W M and Zhou D L 2008 *Phys. Rev.* A **77** 033620
[10] Jin G R and Liu W M 2004 *Phys. Rev.* A **70** 013803
[11] Ji A C, Xie X C and Liu W M 2007 *Phys. Rev. Lett.* **99** 183602
[12] Leggett A J, Chakravarty S, Dorsey A T, Fisher M P A, Garg A and Zweger W 1987 *Rev. Mod. Phys.* **59** 1
[13] Zurek W H 1981 *Phys. Rev.* D **24** 1516
[14] Zurek W H 1982 *Phys. Rev.* D **26** 1862
[15] Diosi L 1989 *Phys. Rev.* A **40** 1165
[16] Ghirardi G C, Pearle P and Rimini A 1990 *Phys. Rev.* A **42** 78
[17] Ellis J, Mohanty S and Nanapoulos D 1989 *Phys. Lett.* B **221** 113
[18] Milburn G J 1991 *Phys. Rev.* A **44** 5401
[19] Hirschman I I 1957 *Am. J. Math.* **79** 152
[20] Deutsch D 1983 *Phys. Rev. Lett.* **50** 631
[21] Maassen H and Uffink J B M 1988 *Phys. Rev. Lett.* **60** 1103
[22] de Vicente J I and Sánchez-Ruiz J 2008 *Phys. Rev.* A **77** 042110
[23] Robertson H P 1929 *Phys. Rev.* **34** 163
[24] Giovannetti V 2004 *Phys. Rev.* A **70** 012102

[25] Gühne O and Lewenstein M 2004 *Phys. Rev.* A **70** 022316
[26] Vollbrecht K G H, Werner R F and Wolf M M 2004 *Phys. Rev.* A **69** 062304
[27] Ballester M A and Wehner S 2007 *Phys. Rev.* A **75** 022319
[28] Fang M F, Zhou P and Swain S 2000 *J. Mod. Opt.* **47** 1043
[29] Abdel-Aty M 2002 *Commun. Theor. Phys.* **37** 723
[30] Liu X J and Fang M F 2004 *Commun. Theor. Phys.* **42** 103
[31] El-Orany F A A, Wahiddin M R B and Obada A S F 2008 *Opt. Commun.* **281** 2854
[32] Li C X and Fang M F 2003 *Chin. Phys.* **12** 0294
[33] Liu X J, Zhou B J, Fang M F and Zhou Q P 2006 *Acta Phys. Sin.* **55** 704 (in Chinese)
[34] Abdalla M S, Lashin E and Sadiek G 2008 *J. Phys.* B **41** 015502
[35] Liao X P and Fang M F 2004 *Physica* A **332** 176
[36] Zhou B J, Liu X J, Zhou Q P and Liu M W 2007 *Chin. Phys.* **16** 420
[37] Abdel-Aty M, Abdalla M S and Obada A S F 2002 *J. Phys.* B **35** 4773
[38] Abdel-Aty M, Abdalla M S and Obada A S F 2002 *J. Opt.* B **4** 134
[39] El-Shahat T M, Abdel-Khalek S, Abdel-Aty M and Obada A S F 2003 *J. Mod. Opt.* **50** 2013
[40] Obada A S F, Khalil E M and Ahmed M M A 2006 *J. Mod. Opt.* **53** 1149
[41] Yan X Q, Shao B and Zou J 2007 *Commun. Theor. Phys.* **48** 63
[42] Yan X Q, Shao B and Zou J *Chaos, Solitons & Fractals* In Press, Corrected Proof
[43] Sánchez-Ruiz J 1995 *Phys. Lett.* A **201** 125
[44] Gerry C C and Eberly J H 1990 *Phys. Rev.* A **42** 6805
[45] Xu J B, Zou X B and Yu J H 2000 *Eur. Phys. J.* D **10** 295