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Controlling chaos in permanent magnet synchronous motor based on finite-time stability theory*

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This paper reports that the performance of permanent magnet synchronous motor (PMSM) degrades due to chaos when its systemic parameters fall into a certain area. To control the undesirable chaos in PMSM, a nonlinear controller, which is simple and easy to be constructed, is presented to achieve finite-time chaos control based on the finite-time stability theory. Computer simulation results show that the proposed controller is very effective. The obtained results may help to maintain the industrial servo driven system's security operation.

Keywords: chaos control, finite-time stability theory, permanent magnet synchronous motor
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1. Introduction

Permanent magnet synchronous motor (PMSM) is of great interest, particularly for industrial applications in the low-medium power range, since it has superior features such as compact size, high torque/weight ratio, high torque/inertia ratio and absence of rotor losses. Over the past few years, the secure and stable operation of PMSM, which is essential requirement of industrial automation manufacturing, has been received considerable attention. It is found that with certain system parameter values, the PMSM is experiencing chaotic behaviour, see, for instance, the works by Zhang,[1] Hemati,[2] and Jing.[3] Chaotic behaviour in PMSM, which appears mainly intermittent ripples of torque, low-frequency oscillations of rotational speed of motor, can extremely destroy the stability of the motor even induce the collapse of driven system. The existence of chaotic behaviour in PMSM is highly undesirable for its performance. Thus, controlling chaos in PMSM is a significant job. However, to the best of our knowledge, analytical investigation into controlling chaos in PMSM is few and the existing controlled methods are defective and unacceptable to practical use. For example, in our previous work, we suggested that the chaos in PMSM can be controlled by the entrainment and migration control strategy.[4] But, this control strategy does not permit the control objective to be any part of trajectory of the controlled system and the control law cannot put into effect until the states of the system enter into the domain of attractions, which may not be consistent with the requirement of the application. So, this method is not desirable to practical use; Harb has controlled chaos in PMSM by backstepping technique.[5] However, there is a drawback in conventional backstepping method called the problem of 'explosion of terms' caused by the repeated differentiations of virtual input. The main aim of this work is to find a proper and applicable method for controlling chaos in PMSM. On the other hand, recently, most works on chaos control and synchronization have noticed the important role of the stability time; e.g., Wu *et al* applied a terminal sliding mode controller to synchronize a class of chaotic systems with mismatched parametric uncertainties,[6] Wang *et al* and Gao *et al* proposed finite-time chaos control and synchronization for unified chaotic systems with uncertain parameters.[7,8] To control the undesirable chaos in PMSM, we present a finite-time chaos control based on the finite-time stability theory. Simulation results indicate that the proposed control law is very effective.

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2. Chaos in the permanent magnet synchronous motor

The dimensionless mathematical model of PMSM with the smooth air gap is given by[2]

$$\begin{aligned}\frac{\mathrm{d}\omega}{\mathrm{d}t} &= \sigma(i_q - \omega) - \widetilde{T_L},\\ \frac{\mathrm{d}i_q}{\mathrm{d}t} &= -i_q - i_d\omega + \gamma\omega + \widetilde{u_q},\\ \frac{\mathrm{d}i_d}{\mathrm{d}t} &= -i_d + i_q\omega + \widetilde{u_d}, \end{aligned} \tag{1}$$

where, ω, i_q and i_d are state variables, which denote angle speed, quadrature- and direct-axis currents of the motor, respectively, σ and γ are system operating parameters, which are positive, $\widetilde{T_L}$, $\widetilde{u_q}$ and $\widetilde{u_d}$ represent the external load torque, the quadrature- and direct-axis stator voltage components, respectively. In this paper, we only consider the case that the system is unforced. This case can be thought of as that, after an operating period of the system, the external inputs are set to zero, namely, $\widetilde{T_L} = \widetilde{u_q} = \widetilde{u_d} = 0$.[2,3] Then, the system (1) becomes

$$\begin{aligned}\frac{\mathrm{d}\omega}{\mathrm{d}t} &= \sigma(i_q - \omega),\\ \frac{\mathrm{d}i_q}{\mathrm{d}t} &= -i_q - i_d\omega + \gamma\omega,\\ \frac{\mathrm{d}i_d}{\mathrm{d}t} &= -i_d + i_q\omega. \end{aligned} \tag{2}$$

The nonlinear dynamical behaviours of system (2), which depend on parameters σ and γ, have been investigated fully in Refs.[2, 3] by using modern nonlinear theory such as bifurcation and chaos. The results of investigation have shown that with the operating parameters σ and γ falling into a certain area, PMSM is experiencing chaotic behaviour. To make an overall inspection of the dynamical behaviour of the PMSM, the bifurcation diagram of the direct-axis current versus the parameter γ is plotted in Fig.1. The typical chaotic attractors are shown in Fig.2. Chaotic behaviour in PMSM, which appears aperiodic, random, sudden or intermittent morbid oscillations of the motor angle speed, can seriously destroy the stable operation of the motor even induce industrial driven system collapse. Therefore, how to control chaos in PMSM is a significant job.

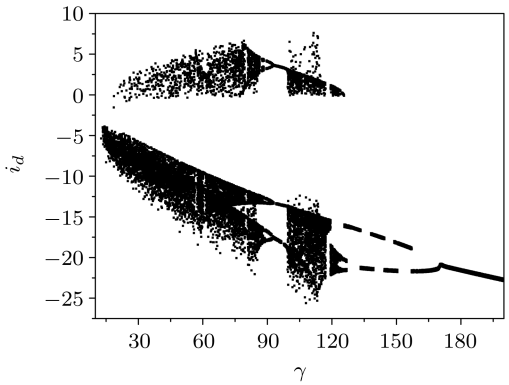


Fig.1. The bifurcation structure of i_d versus the parameter γ with $\sigma = 5.45$.

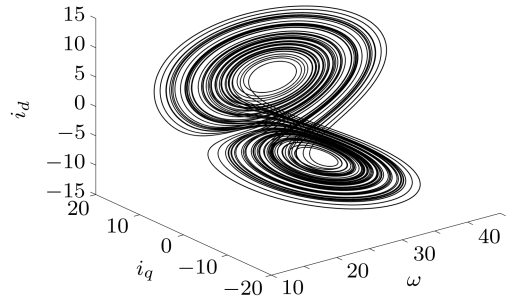


Fig.2. Typical chaotic attractor in PMSM with system parameters $\sigma = 5.45$, $\gamma = 25$.

3. Basic conception of finite-time stability

The meaning of finite-time stability is that the state of the dynamic system converges to a desired target within a finite time. First, some necessary definitions and lemmas are introduced.

Definition 1[9] Consider the following dynamic system:

$$\dot{X} = f(X), \tag{3}$$

where the state variable $X \in \Re^n$, if there exists a constant $T > 0$ (T may depend on the initial state $X(0)$), such that

$$\lim_{t \to T} \|X(t)\| = 0,$$

and $\|X(t)\| \equiv 0$, if $t \geq T$, then the system (3) is finite time stable.

Lemma 1[8] Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \leq -\varepsilon V^{\rho}(t), \quad \forall t \geq t_0, \quad V(t_0) \geq 0, \tag{4}$$

where $\varepsilon > 0$ and $0 < \rho < 1$ are constants. Then for any initial time t_0, $V(t)$ satisfies:

$$V^{1-\rho}(t) \leq V^{1-\rho}(t_0) - \varepsilon(1-\rho)(t-t_0), \quad t_0 \leq t \leq 1 \tag{5}$$

and

$$V(t) \equiv 0, \quad \forall t \geq t_1, \tag{6}$$

where t_1 is given by

$$t_1 = t_0 + \frac{V^{1-\rho}(t_0)}{\varepsilon(1-\rho)}. \tag{7}$$

Proof[11] Consider the following differential equation:

$$\dot{x}(t) = -\varepsilon x^{\rho}(t), \quad x(t_0) = V(t_0). \tag{8}$$

Although this differential equation does not satisfy the global Lipschitz condition, the unique solution to this equation can be found as

$$x^{1-\rho}(t) = x^{1-\rho}(t_0) - \varepsilon(1-\rho)(t-t_0), \quad t_0 \leq t \leq t_1, \tag{9}$$

and

$$x(t) \equiv 0, \quad \forall t \geq t_1, \tag{10}$$

It directly prove that $x(t)$ is differential for $t > t_0$. From the comparison lemma,[10] one obtains

$$V^{1-\rho}(t) \leq V^{1-\rho}(t_0) - \varepsilon(1-\rho)(t-t_0), \quad t_0 \leq t \leq 1 \tag{11}$$

and

$$V(t) \equiv 0, \quad \forall t \geq t_1 \tag{12}$$

with t_1 given by Eq.(7).

Lemma 2 Let $0 < \beta < 1$. Then for positive real numbers a and b, the following inequality holds:

$$(a+b)^{\beta} \leq a^{\beta} + b^{\beta}. \tag{13}$$

4. Chaos control in PMSM based on the finite-time stability theory

In order to control chaotic oscillation in PMSM, we add the controls u_1, u_2 and u_3 to system (2) and then the controlled system can be expressed by

$$\begin{aligned} \frac{\mathrm{d}\omega}{\mathrm{d}t} &= \sigma(i_q - \omega) + u_1, \\ \frac{\mathrm{d}i_q}{\mathrm{d}t} &= -i_q - i_d\omega + \gamma\omega + u_2, \\ \frac{\mathrm{d}i_d}{\mathrm{d}t} &= -i_d + i_q\omega + u_3. \end{aligned} \tag{14}$$

Now, we use the finite-time stability theory to design a controller to globally stabilize the unstable equilibrium $X_0 = (0,0,0)$[2] in a finite time. As to other equilibria, we can also adopt the technique presented below to realize the chaotic control. With the above knowledge, we propose our main results in the following.

Theorem 1 If the controller is designed as

$$\begin{aligned} u_1 &= -\sigma i_q - \omega^{\lambda}, \\ u_2 &= H i_q - i_q^{\lambda}, \\ u_3 &= -i_d^{\lambda}, \end{aligned} \tag{15}$$

where $H \leq 1$, $\lambda = i/j$ and $j > i$ are positive odd integers, the chaos in PMSM will be controlled, namely, the PMSM will be asymptotically stabilized at the desired fixed state.

Proof By the control u_1, the first equation of Eq.(14) is rewritten as follows:

$$\dot{\omega} = -\sigma\omega - \omega^{\lambda}. \tag{16}$$

Consider the candidate Lyapunov function $V_1 = \omega^2/2$. The time derivative of V_1 along the trajectory of Eq.(16) is

$$\begin{aligned} \dot{V}_1 &= \omega(-\sigma\omega - \omega^{\lambda}) \\ &= -\sigma\omega^2 - \omega^{\lambda+1} \\ &\leq -\omega^{\lambda+1} \\ &= -\left(\frac{1}{2}\right)^{-\frac{\lambda+1}{2}} \left(\frac{1}{2}\omega^2\right)^{\frac{\lambda+1}{2}} \\ &= -\left(\frac{1}{2}\right)^{-\frac{\lambda+1}{2}} \left(\frac{1}{2}V_1\right)^{\frac{\lambda+1}{2}}. \end{aligned} \tag{17}$$

Since $0 < \lambda < 1$, then $0 < \lambda/2 < 1$. From Lemma 1, the state ω will reach $\omega = 0$ at a finite time $T_1 = \omega(0)/(1-\lambda)$.

Substituting control u_2, u_3 and $\omega = 0$ into the second and the third equations of system (14), we obtain

$$\begin{aligned} \frac{\mathrm{d}i_q}{\mathrm{d}t} &= -i_q + H i_q - i_q^{\lambda}, \\ \frac{\mathrm{d}i_d}{\mathrm{d}t} &= -i_d - i_d^{\lambda}. \end{aligned} \tag{18}$$

Choose Lyapunov function for Eq.(18) as follows: $V_2 = i_q^2/2 + i_d^2/2$. The derivative of V_2 along the trajectories of Eq.(18) is

$$\begin{aligned}\dot{V}_2 &= i_q(-i_q + Hi_q - i_q^{\lambda}) - i_d(i_d + i_d^{\lambda}) \\ &= -(1-H)i_q^2 - i_q^{\lambda+1} - i_d^2 - i_d^{\lambda+1} \\ &\leq -i_q^{\lambda+1} - i_d^{\lambda+1} \\ &= -\left(\frac{1}{2}\right)^{-\frac{\lambda+1}{2}} \left(\frac{1}{2}i_q^2\right)^{\frac{\lambda+1}{2}} - \left(\frac{1}{2}\right)^{-\frac{\lambda+1}{2}} \left(\frac{1}{2}i_d^2\right)^{\frac{\lambda+1}{2}} \\ &= -\left(\frac{1}{2}\right)^{-\frac{\lambda+1}{2}} \left(\left(\frac{1}{2}i_q^2\right)^{\frac{\lambda+1}{2}} + \left(\frac{1}{2}i_d^2\right)^{\frac{\lambda+1}{2}}\right) \\ &\leq -\left(\frac{1}{2}\right)^{-\frac{\lambda+1}{2}} \left(\frac{1}{2}i_q^2 + \frac{1}{2}i_d^2\right)^{\frac{\lambda+1}{2}} \\ &= -\left(\frac{1}{2}\right)^{-\frac{\lambda+1}{2}} \left(\frac{1}{2}v_2\right)^{\frac{\lambda+1}{2}}. \end{aligned} \tag{19}$$

From Lemma 1, the states i_q and i_d will converge to $i_q = 0$, $i_d = 0$ at a finite time T_2. Then after T_2, the state of the system (14) will stay at $\omega = 0$, $i_q = 0$ and $i_d = 0$. This means that the controlled chaotic systems (14) are finite-timely stabilized by the proposed controller.

5. Numerical simulation and analysis

In this section, simulation on controlling chaos in PMSM based on the finite-time stability theory will be conducted. In the numerical simulations, the fourth-order Runge–Kutta method is used to solve the systems with time step size $h = 0.005$, the initial condition of the chaotic system is set as $(\omega(0), i_q(0), i_d(0)) = (12, 13, 10)$ and the systematic parameters take the values as those in Section 2. The PMSM is experiencing chaotic oscillations before the controller was carried out. To show how the presented controller is so effective in controlling the chaos in PMSM, the control signal is put into effect at 150 s. Figures 3 and 4 show the results of control when the control parameters are set as $\lambda = 0.1$, $H = 0.3$ and $\lambda = 0.7$, $H = 0.9$, respectively, from which one can see clearly that the state of PMSM reaches the desired goal in a finite time. The number simulation results also show that with the increase of control parameter, the systematic overshoot is decreasing and transition time is reducing such that it takes very short time for the system to be fast stabilized at the fixed point.

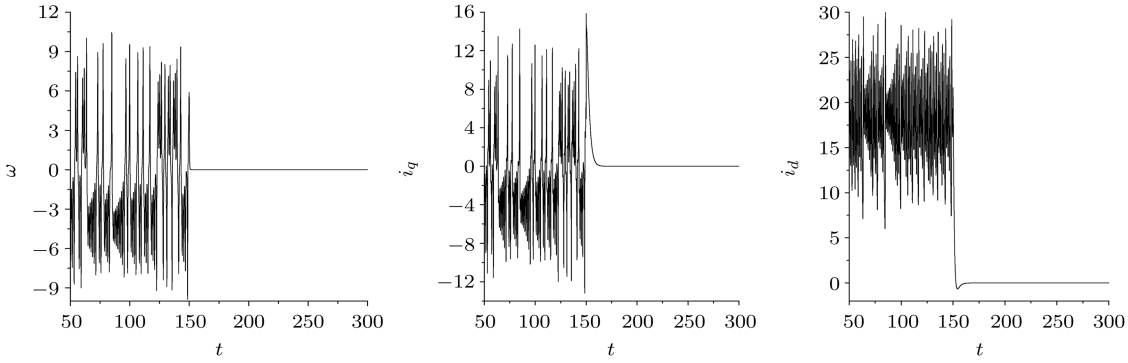


Fig.3. Control the chaotic system to the desired goal with control parameters $\lambda = 0.1, H = 0.3$.

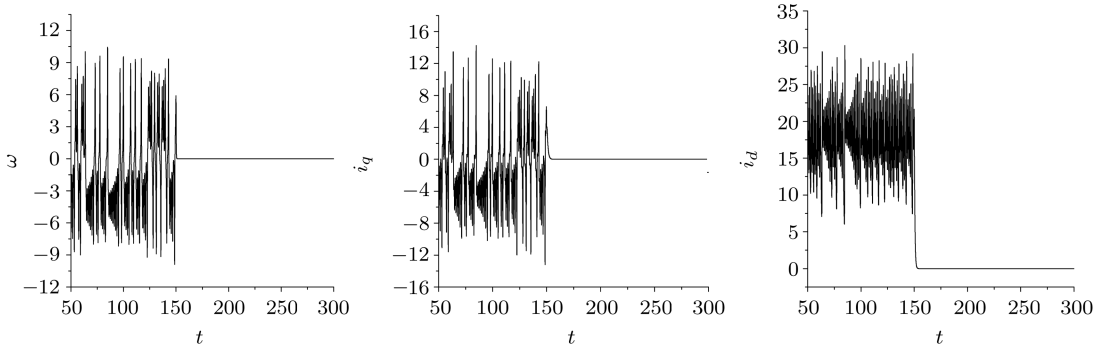


Fig.4. Control the chaotic system to the desired goal with control parameters $\lambda = 0.7, H = 0.9$.

6. Conclusion

In this paper, a new method is developed to eliminate chaos in PMSM based on the finite-time stable theory. The basic conception of finite-time stability is introduced first, and then the controller is presented. Last, we prove that the proposed control law is very effective by the theoretical method and simulation results. It can be seen that the controller is simple and easy to be implemented. This work is significant because it is helpful to maintain the industrial servo driven system's security operation and it is a beneficial try which applies chaos control method to special plant.

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