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# A new hyperchaos system and its circuit simulation by EWB*

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This paper reports a new hyperchaotic system evolved from the three-dimensional Lü chaotic system. The Lyapunov exponents spectrum and the bifurcation diagram of this new hyperchaotic system are obtained. Hyperchaotic attractor, periodic orbit and chaotic attractor are obtained by computer simulation. A circuit is designed to realize this new hyperchaotic system by electronic workbench.

**Keywords:** hyperchaotic system, computer simulation, electronic workbench
**PACC:** 0545

## 1. Introduction

There has been increasing interest in exploiting chaotic dynamics in engineering applications, such as electrical engineering, telecommunications, information processing, material engineering, etc. This is due to the desirable features of chaos in some engineering and technological applications, where chaos can provide certain unique features for special needs. Much attention has been paid to effectively generating chaos via simple devices such as circuitry design.[1−3] Many novel and simple nonlinear circuits exhibiting chaotic dynamics have been proposed in Refs.[4–12].

But, in secure communication, there exists a possibility that the messages, masked by a simple chaotic system with only a single positive Lyapunov exponent, can be easily extracted sometimes when they were intercepted. An example of how to extract messages which are masked by a chaotic signal in a system of two Lorenz systems was shown in Ref.[13]. Because hyperchaotic systems have more than one positive Lyapunov exponent, and have more complex dynamic behaviour. So, hyperchaotic system has the characteristics of high capacity, high security and high efficiency. Typical examples are the four-dimensional (4D) hyperchaotic Rossler system, 4D hyperchaotic Lorenz–Haken system, 4D hyperchaotic Chua circuit, and 4D hyperchaos Chen system.[6]

This paper reports a new 4D hyperchaotic system, which is obtained by adding one state variable into a three-dimensional Lü chaotic system. Reference [6] has reported a hyperchaotic system based on Lü system by using a state feedback controller. But, it has three equilibrium points, and the range of the system parameter generating hyperchaotic attractor is small. But, this new hyperchaotic system has only one equilibrium point, and the range of the system parameter generating hyperchaotic attractor is large. Some basic dynamical behaviours are explored by calculating its Lyapunov exponent spectrum, bifurcation diagram. This new hyperchaotic system also has been physically realized by electronic workbench (EWB).

## 2. A new hyperchaos system

For hyperchaos, some basic properties are described as follows:

(i) The minimal dimension of the phase space of an autonomous system is at least four.

(ii) The number of terms in the equations giving rise to instability is at least two, of which at least one has a nonlinear function.

We can construct a 4D hyperchaotic system by adding one state variable into 3D Lü chaotic system. It is shown in the following system:

$$\begin{aligned} \dot{x} &= a(y-x)+w, \\ \dot{y} &= cy-xz, \\ \dot{z} &= -bz+xy, \\ \dot{w} &= dx+0.5yz, \end{aligned} \tag{1}$$

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where $a = 35$, $b = 3$, $c = 12$, $0 \le d \le 50$.

System (1) is symmetrical about $z$-axis for its invariance under the coordinate transformation ($x$, $y$, $z$, $w$) $\to$ ($-x$, $-y$, $z$, $-w$), and the phase portrait of system (1) in $x-y-w$ 3D space is symmetrical about the origin.

According to system (1), we can obtain

$$\begin{aligned}\nabla V &= (\partial\dot{x}/\partial x + \partial\dot{y}/\partial y + \partial\dot{z}/\partial z + \partial\dot{w}/\partial w)\\ &= -a + c - b < 0.\end{aligned}$$

Therefore, system (1) is dissipative. Hence, a volume element $V_0$ is contracted by the flow into a volume element $V_0 e^{t\nabla V}$ in time $t$. That is, each volume containing the system orbit shrinks to zero as $t \to +\infty$ at an exponential rate $\nabla V$, which is independent of $x$, $y$, $z$, $w$. Consequently, all system orbits will ultimately be confined to a specific subset of zero volume and the asymptotic motion settles onto an attractor. Then, the existence of attractor is proved.

Obviously, system (1) has only an equilibrium point (0,0,0,0). This is different from the hyperchaotic system in Refs.[5], [6] and [7], they have more than one equilibrium points.

The eigenvalues of equilibrium point (0,0,0,0) are

$$\lambda_1 = c, \lambda_2 = \frac{-a + \sqrt{a^2 + 4d}}{2},$$
$$\lambda_3 = -b, \lambda_4 = \frac{-a - \sqrt{a^2 + 4d}}{2}.$$

Therefore, the equilibrium point is unstable.

Figure 1 shows the Lyapunov exponents spectrum of system (1) with the increasing of parameter $d$. Figure 2 shows the bifurcation diagram of $x$.

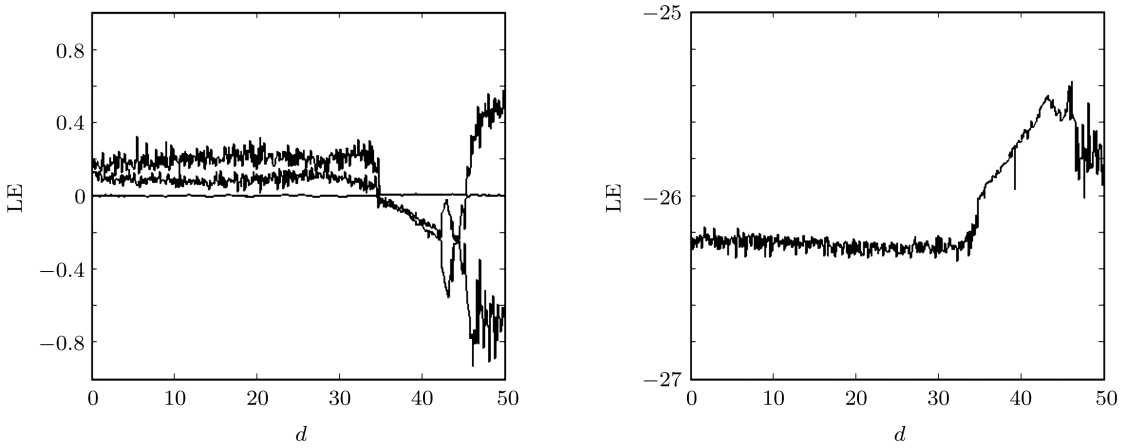



**Fig.1.** Lyapunov exponents spectrum of system (1).

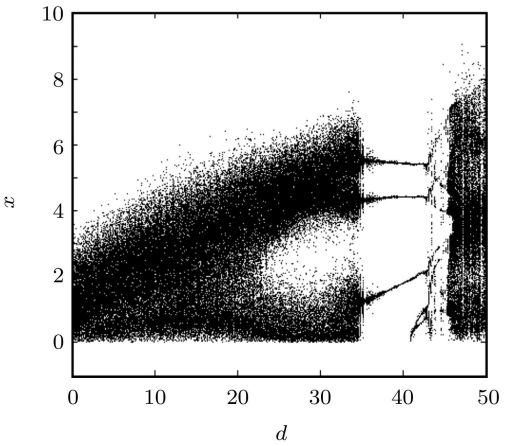



**Fig.2.** Bifurcation diagram of $x$.

According to Fig.1, we have the following three cases:

(i) System (1) is hyperchaotic for $0 \le d \le 34.8$. Where $\lambda_1 > \lambda_2 > 0$, $\lambda_3 = 0$, $\lambda_4 < 0$ and $\lambda_1 + \lambda_2 + \lambda_4 < 0$.

(ii) System (1) is a periodic orbit for $34.8 < d < 45.6$. Where $\lambda_1 = 0$, $\lambda_2 < 0$, $\lambda_3 < 0$, $\lambda_4 < 0$.

(iii) System (1) is chaotic for $45.6 \le d \le 50$. Where $\lambda_1 > 0$, $\lambda_2 = 0$, $\lambda_3 < 0$, $\lambda_4 < 0$ and $\lambda_1 + \lambda_3 + \lambda_4 < 0$.

Because system (1) is hyperchaotic for $0 \le d \le 34.8$, so this system has a larger hyperchaotic region than the hyperchaotic Lü system reported by Ref.[6] and hyperchaotic Lorenz system reported by Ref.[7],

respectively.

Let $d = 1$, we can obtain the Lyapunov exponents of system (1) as follows:

$$\lambda_1 = 0.1514, \quad \lambda_2 = 0.1226,$$
$$\lambda_3 = 0, \qquad \lambda_4 = -26.2561.$$

The Lyapunov dimensions of system (1) is $D_L = 3 + \dfrac{\lambda_1 + \lambda_2}{|\lambda_4|} = 3.0104$, so the system (1) is fractal. The hyperchaotic attractor of system (1) is shown in Fig.3.

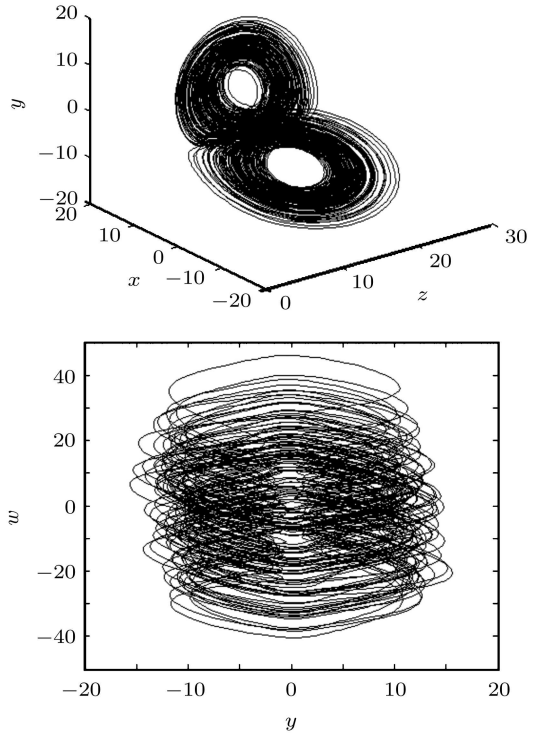



**Fig.3.** The hyperchaotic attractor of system (1) for $d = 1$.

Let $d = 40$, we can obtain the Lyapunov exponents of system (1) as follows:

$$\lambda_1 = 0, \quad \lambda_2 = -0.1398,$$
$$\lambda_3 = -0.1519, \quad \lambda_4 = -25.7128.$$

So, system (1) has a periodic orbit as shown in Fig.4.

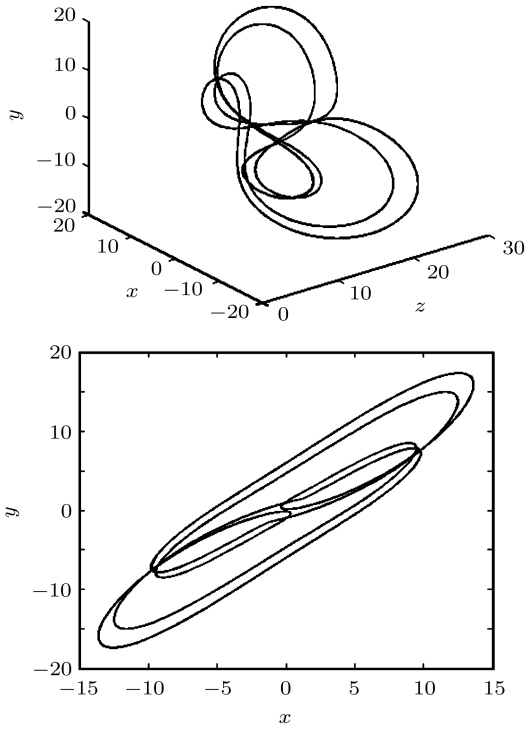



**Fig.4.** The periodic orbit of system (1) for $d = 40$.

Let $d = 48$, we can obtain the Lyapunov exponents of system (1) as follows:

$$\lambda_1 = 0.4678, \quad \lambda_2 = 0,$$
$$\lambda_3 = -0.7045, \quad \lambda_4 = -25.7434.$$

So, system (1) has a chaotic attractor as shown in Fig.5.

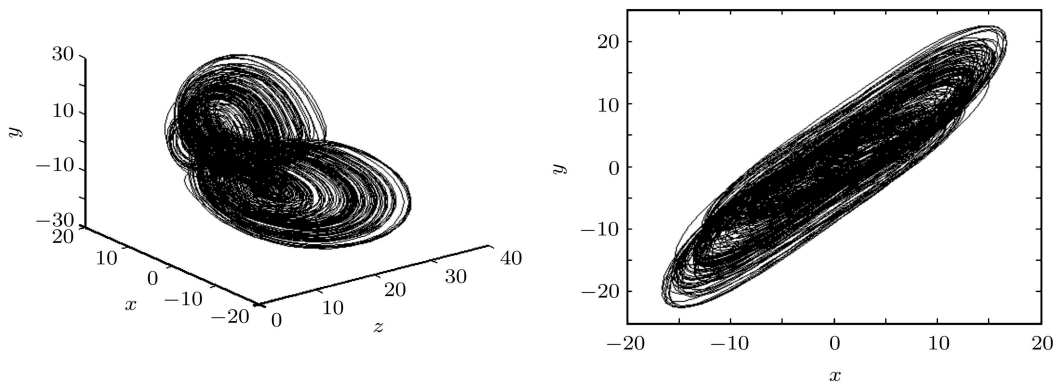



**Fig.5.** The chaotic orbit of system (1) for $d = 48$.

## 3. The electronic workbench (EWB) for hyperchaotic system (1)

A circuit is designed to realize the hyperchaotic system (1). Figure 6 shows the circuit diagram. Because of the restriction in the acceptable voltage of operational amplifier and multiplier, the amplitude of every variable in system (1) is decreased.

Let $x_1 = 10x$, $x_2 = 10y$, $x_3 = 10z$, $x_4 = 10w$. So, system (1) can be changed as the following system (2),

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_4, \\ \dot{x}_2 &= cx_2 - 10x_1x_3, \\ \dot{x}_3 &= -bx_3 + 10x_1x_2, \\ \dot{x}_4 &= dx_1 + 5x_2x_3. \end{aligned} \tag{2}$$

Where $\mathrm{R}_1 = 10\,\mathrm{k\Omega}$, $\mathrm{R}_2 = 3\,\mathrm{k\Omega}$, $\mathrm{R}_3 = 100\,\mathrm{k\Omega}$, $\mathrm{R}_4 = 8\,\mathrm{k\Omega}$, $\mathrm{R}_5 = 33\,\mathrm{k\Omega}$, $\mathrm{R}_6 = 20\,\mathrm{k\Omega}$, $\mathrm{C} = 10\,\mathrm{nF}$, $\mathrm{R}_7$ is a variable resistors. If $\mathrm{R}_7 = 100\,\mathrm{k\Omega}$, then $d = 1$. If $\mathrm{R}_7 = 2.5\,\mathrm{k\Omega}$, then $d = 40$. If $\mathrm{R}_7 = 2.0833\,\mathrm{k\Omega}$, then $d = 48$. The multiplier is AD633, and operational amplifier is LF353N, the power supply is $\pm 15\,\mathrm{V}$.

Utilizing EWB to simulate system (2), we obtain some phase portraits as shown in the following.

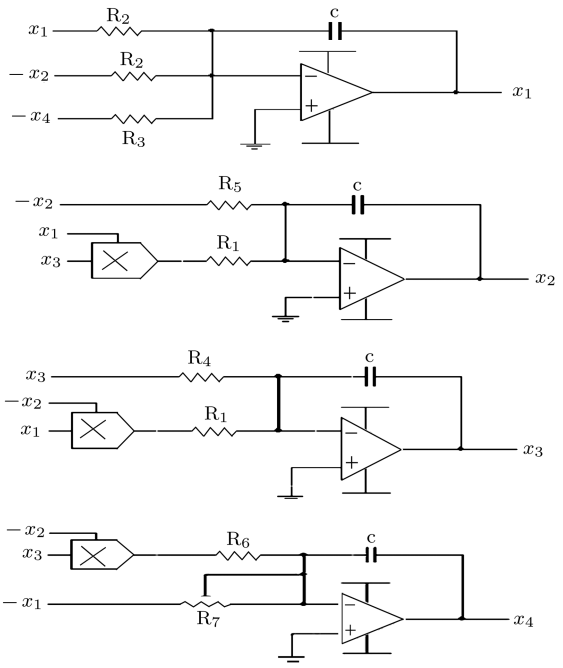



**Fig.6.** Circuit diagram for realizing hyperchaotic system (1).

Figure 7 shows the hyperchaotic phase portraits for $d = 1$. Figure 8 shows the periodic orbit phase portraits for $d = 40$. Figure 9 shows the chaotic phase portraits for $d = 48$.

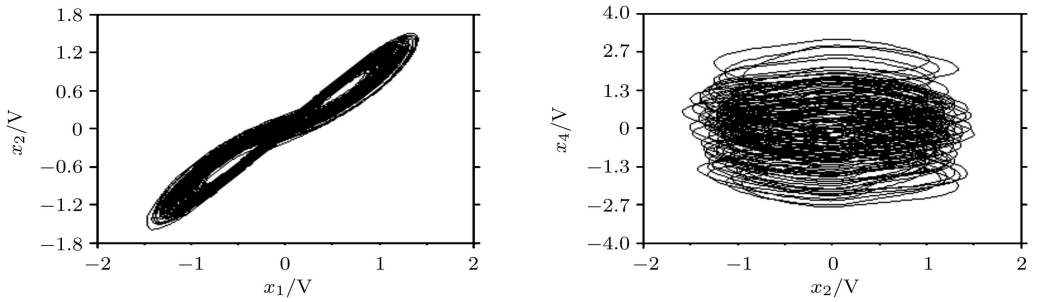



**Fig.7.** The hyperchaotic phase portraits of system (2) for $d = 1$ simulated by EWB.

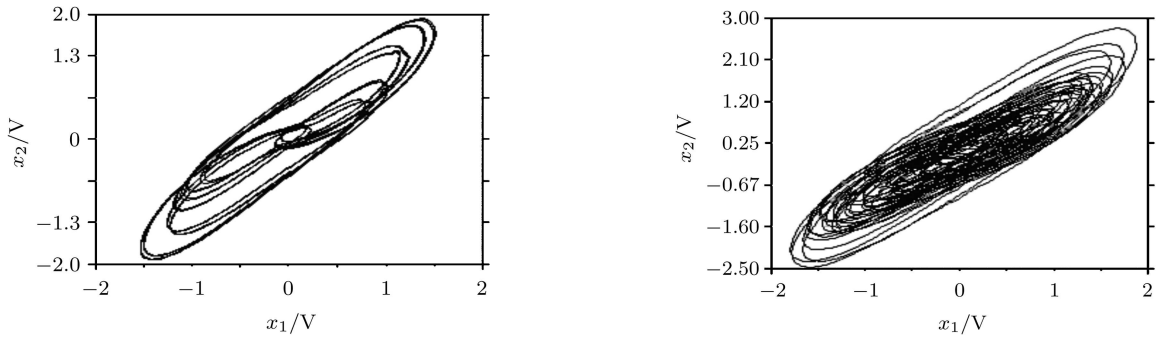



**Fig.8.** The periodic orbit phase portraits of system (2) for $d = 40$ simulated by EWB.

**Fig.9.** The chaotic phase portraits of system (2) for $d = 48$ simulated by EWB.

# 4. Conclusions

We construct one 4D new hyperchaotic system. This new hyperchaotic system has only one equilibrium point, and the range of the system parameter generating hyperchaotic attractor is large. Some basic dynamical behaviours are explored by calculating its Lyapunov exponent spectrum, bifurcation diagram. Simulation results support and verify brief theoretical derivations. A circuit is designed to realize the new hyperchaotic system by EWB. The EWB results show agreement with numerical simulations.

Further work on this new system (2) is an ongoing research topic in our group.

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