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Chirality-asymmetry force between α-quartz and copper block*

Hu Yong-Hong(胡永红), Xu Qing(徐 庆), and Liu Zhong-Zhu(刘中柱)†

Department of Physics, Huazhong University of Science and Technology, Wuhan 430074, *China*

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The chirality-asymmetry macroscopic force mediated by light pseudoscalar particles between α-quartz and some achiral matter is studied. If this force between achiral source mass and α-quartz with some chirality is attractive, it will become repulsive when the chirality of the α-quartz crystal is changed. According to the tested limits of the coupling constant $g_s g_p/\hbar c < 1.5\times10^{-24}$ at the Compton wavelength $\lambda = 10^{-3}$ m, the force (F) between a $0.08\times0.08\times0.002\,\mathrm{m}^3$ block of α-quartz and a $0.08\times0.08\times0.01\,\mathrm{m}^3$ copper block with a separation being 0.5×10^{-3} m in between, is estimated from the published data at less than 4.64×10^{-24} N, i.e. $F < 4.64\times10^{-24}$ N.

Keywords: macroscopic force, α-quartz, pseudoscalar, chirality
PACC: 0485

1. Introduction

New interactions are considered to be mediated by most light particles[1] Especially, the axion is hypothesized as the best candidate of dark matter. However, it is still very difficult to detect and so no absolute result on it has been acquired as yet, although one has been exploring it ever since it was proposed.[2,3] Its interaction with matter was discussed and the ratio of the force to the gravity is given in Ref.[4], where the chirality of matter and the chiral approach were mentioned. Motivated by these studies, we study the interactions due to the exchange of axions between chiral matters and the relationship between the macroscopic force induced by such an interaction and the chirality of matters. Left-handed or right-handed materials have distinct geometrical configurations, chiralities of matter.[5] And the chirality is a key feature of materials, which can cause different properties of materials.[6] If a chirality-dependent interaction exists, there may appear different attractions between chiral matter and the same achiral source matter. Focalized spin-dependent interaction mediated by light scalar particles is such a one.[2] It can induce a macroscale force between a chiral material and an achiral body.

This force possesses various features, in which the following scalar–pseudoscalar coupling potential between fermions is of much account:[2]

$$V(r) = \frac{g_s g_p \hbar}{4\pi m_2 c}(\boldsymbol{\sigma}_2\cdot\hat{r})(m_\varphi + \frac{1}{r})\frac{\mathrm{e}^{-rm_\varphi}}{r}, \qquad (1)$$

where g_s and g_p are the scalar and the pseudoscalar coupling constants respectively, $\boldsymbol{\sigma}_2$ is the Pauli matrix, m_2 is the mass of fermion at the pseudoscalar vertex, m_ϕ is the mass of medial boson and r is the distance between two particles. Generally, nucleons are considered to appear at the scalar vertex and electrons reside at the pseudoscalar vertex.[1] Several experiments using spin-polarized test masses have been conducted to determine the magnitude of the coupling $g_s g_p$.[2,3] In this type of experiment, if a chirality-dependent potential exists, the coupling $g_s g_p$ would be eclipsed by strong polarization effects of the matter. The potential (1) would vanish, for the sum of the particle spins in zero-polarizing matters is zero.

Comparing the interaction (1) with the interaction inducing the rotatory power of matter yields a heuristic idea. In fact, the rotatory power of a chiral material is a secondary quantum effect in the nonrelativistic limit.[8] Therefore, we anticipate that the potential (1) may give a finite macroscopic chiral force in considering the practical environment of electrons.

We show that a macroscopic chiral force between a copper block and a block of α-quartz can be determined by the potential (1) with the correction of

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†Corresponding author. E-mail: zzl129@mail.hust.edu.cn

spin–orbit interaction. We estimate the force between a $0.08\times0.08\times0.002\,\mathrm{m}^3$ block of α-quartz and a $0.08\times0.08\times0.02\,\mathrm{m}^3$ copper block with a separation being 0.5 mm in between, for the Compton wave length of the mediatory particle $\lambda = 10^{-3}\,\mathrm{m}$. According to the tested limit of production of the coupling constant $g_\mathrm{s}g_\mathrm{p}/\hbar c < 1.5\times10^{-24}$ and other published data, the upper limit of the force is about $4.64\times10^{-24}\,\mathrm{N}$. The remainder of the present paper is organized as follows. In Section 2 the chiral potential is described. In Section 3 the chirality-asymmetry macroscopic force is estimated. Finally, the conclusions drawn from the present study is given in Section 4.

2. Chiral potential

α-quartz is a chiral crystal composed of ions and valence electrons. Copper block is comprised of Cu ions and common electrons. Assume that a copper block has N nucleons and a block of α-quartz possesses M primitive cells each of which has m valence electrons. We study the geometrical configuration shown in Fig.1. A coordinate is so arranged that its origin is located at the mass centre of the copper block and its z-axis points to the mass centre of the α-quartz.

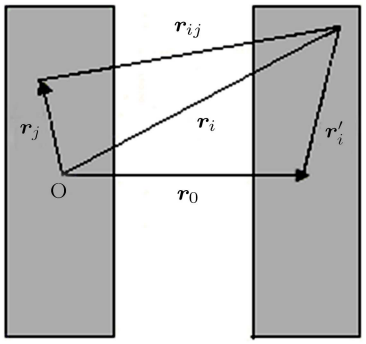


Fig.1. Geometric configuration of the test mass α-quartz (right) and the attractive mass copper block (left). The origin of the coordinate system is situated at the mass centre of the copper block.

In the coordinate system, $\boldsymbol{r}_0$ is the position vector of the mass centre of the α-quartz, $\boldsymbol{r}_i$ is the position vector of the centre of the ith primitive cell in the α-quartz, $\boldsymbol{r}_k$ is the position vector of the kth electron in the ith cell, $\boldsymbol{r}_j$ is the position vector of the jth nucleon in the copper block $\boldsymbol{r}_{kj}$ is the displacement vector from the jth nucleon in the copper block to the kth electron in the quartz and $\hat{r}_{kj}$ is its corresponding unit vector. Moreover, we suppose that the α-quartz crystal is a Z-cut one. In this configuration, the interaction potential between nucleons of the copper block and the electrons of the block of α-quartz is

$$V = g_\mathrm{p}g_\mathrm{s}\sum_{j=1}^{N}\sum_{i=1}^{M}\sum_{k=1}^{m}\frac{\hbar\boldsymbol{\sigma}_k\cdot\hat{r}_{kj}}{4\pi m_2 c}\left(\frac{m_\phi}{r_{kj}}+\frac{1}{r_{kj}^2}\right)\mathrm{e}^{-m_\phi r_{kj}}, \tag{2}$$

and we can write the following vectors: $\boldsymbol{r}_{kj} = \boldsymbol{r}_k - \boldsymbol{r}_j = \boldsymbol{r}_0+\boldsymbol{r}'_i+\boldsymbol{r}_{ik}-\boldsymbol{r}_j$ and $\boldsymbol{r}_k = \boldsymbol{r}_i+\boldsymbol{r}_{ik} = \boldsymbol{r}_0+\boldsymbol{r}_{0i}+\boldsymbol{r}_{ik}$, where $\boldsymbol{r}_{ik}$ is the displacement vector from the centre of the ith primitive cell in the α-quartz to the kth valence electron in the same cell.

Particle systems in crystal are described by wavefunctions satisfying the Schrödinger equation.[7] When we care only the behaviour of valence electrons, the system can be studied in the adiabatic approximation as done in most literature.[8] Let $|\Psi_0\rangle$ be the wavefunction of the valence electron system in α-quartz, the matrix element of potential (2) surely has $\langle\Psi_0|V|\Psi_0\rangle = 0$ due to the zero total magnetic moment of the block of α-quartz.

The Hamiltonian of the spin–orbit interaction of valence electrons is $V' = -\sum_{i=1}^{M}\sum_{k=1}^{m}\boldsymbol{\mu}_k\cdot\boldsymbol{B}(\boldsymbol{r}_k)$, where $\boldsymbol{\mu}_k$ is the spin magnetic moment of the kth valence electron and $\boldsymbol{B}(\boldsymbol{r}_k)$ is the magnetic induction at the position of the electron. The spin–orbit interaction V' corrects the system wavefunction into $|\Psi\rangle$. And the matrix element of the potential (2) is corrected into the following form:

$$\begin{aligned}\langle\Psi|V(r)|\Psi\rangle \approx 2\mathrm{Re}\sum_{n\neq0}\frac{g_\mathrm{s}g_\mathrm{p}}{E_0-E_n}\langle\Psi_0|\sum_{i=1}^{M}\sum_{k=1}^{m}-\boldsymbol{\mu}_k\cdot\boldsymbol{B}(\boldsymbol{r}_k)|\Psi_n\rangle \\ \times\langle\Psi_n|\sum_{j=1}^{N}\sum_{i=1}^{M}\sum_{k=1}^{m}\frac{\boldsymbol{\sigma}_k\cdot\hat{r}_{kj}\hbar}{4\pi m_2 c}\left(\frac{m_\phi}{r_{kj}}+\frac{1}{r_{kj}^2}\right)\mathrm{e}^{-m_\phi r_{kj}}|\Psi_0\rangle + S,\end{aligned} \tag{3}$$

where $|\Psi_n\rangle$ is an eigenfunction corresponding to the energy E_n and it is perpendicular to $|\Psi_0\rangle$. S is the sum of the chirality-independent terms.

α-quartz crystal has either the left-handed or the right-handed lattices transformed reciprocally under the space reflection. A state vector $|\Psi\rangle_l$ of a left-handed crystal will be transformed into a state vector $|\Psi\rangle_r$ of a right-handed crystal under the reflection P as $|\Psi\rangle_r = P|\Psi\rangle_l$. The reflection induces also transformations of position vectors, spins and magnetic fields into $P\hat{r}_{kj}(r)P^{-1} = -\hat{r}_{kj}$, $P\boldsymbol{\sigma}_k P^{-1} = \boldsymbol{\sigma}_k$ and $P\boldsymbol{B}P^{-1} = \boldsymbol{B}$ respectively. So if potential (3) is nonzero, it will be transformed into

$$\begin{aligned} {}_r\langle\Psi|V(r)|\Psi\rangle_r &= {}_r\langle\Psi|P^{-1}PV(r)P^{-1}P|\Psi\rangle_r \\ &= {}_l\langle\Psi|PV(r)P^{-1}|\Psi\rangle_l \\ &= -{}_l\langle\Psi|V(r)|\Psi\rangle_l . \end{aligned} \tag{4}$$

It means that potential (3) is chiral, which will take the opposite values for left-handed and right-handed crystals.

The distance r_{ik} from the centre of the ith primitive cell to the kth electron of the cell is only on a microscopic scale, while the distance r_0 between the mass centres of the copper block and the α-quartz crystal is on a microscopic scale. Thus the inequality $r_{ik} \ll r_0$ holds true. The wavefunctions $|\Psi_0\rangle$ and $|\Psi_n\rangle$ describing the system of valence electrons in a block of α-quartz crystal are of translation symmetry. Noticing these facts, we can prove the equation

$$\begin{aligned} &\langle\Psi_n|\sum_{k=1}^{N}(\boldsymbol{\sigma}_k\cdot\hat{r}_{jk})\left(m_\varphi+\frac{1}{r_{jk}}\right)\frac{\mathrm{e}^{-r_{jk}m_\varphi}}{r_{jk}}|\Psi_0\rangle \\ &=\langle\Psi_n|\sum_{k=1}^{N}(\boldsymbol{\sigma}_k\cdot\boldsymbol{r}_{ik})\left(m_\varphi+\frac{1}{r_{ji}}\right)\frac{\mathrm{e}^{-r_{ji}m_\varphi}}{r_{ji}^2}|\Psi_0\rangle . \end{aligned} \tag{5}$$

The product of matrix elements in Eq.(3) can be simplified by adopting the single electron approximation.[7] The single-electron wavefunction is the product of its orbital wavefunction $|\Psi\rangle$ and the spin wavefunction $|m\rangle$. Therefore, we can rewrite matrix (3) by using the equality $(\boldsymbol{\sigma}_i\cdot\boldsymbol{A})(\boldsymbol{\sigma}_j\cdot\boldsymbol{B}) = \boldsymbol{A}\cdot\boldsymbol{B} + \mathrm{i}\boldsymbol{\sigma}_i\cdot(\boldsymbol{A}\times\boldsymbol{B})$[9] as follows:

$$\langle\Psi|V(r)|\Psi\rangle = \frac{g_1 g_2\hbar}{2\pi m_2 c}\sum_{j=1}^{K}\sum_{i=1}^{M}\left(m_\varphi+\frac{1}{r_{ij}}\right)\frac{\mathrm{e}^{-r_{ij}m_\varphi}}{r_{ij}^2}\sum_{n\neq 0}\frac{\mathrm{Re}\sum_{k=1}^{m}\langle\psi_{kn}|\boldsymbol{r}_{ik}|\psi_{k0}\rangle\langle\psi_{k0}|\boldsymbol{B}(\boldsymbol{r}_k)|\psi_{kn}\rangle}{E_{k0}-E_{kn}}, \tag{6}$$

where $|\psi_{k0}\rangle$ and $|\psi_{kn}\rangle$ describe the initial and the final states of the kth valence electron in the ith primitive cell respectively.

3. Estimation of the force

Now, we estimate the chirality-dependent term in potential (3) according to the published experimental or theoretical data. Let an electron be in a valence band initially and then jump into a conduction band due to a transition induced by the matrix elements of expression (3). The measured energy gap of α-quartz is about 8.3 eV with neglecting a small difference between the X-cut quartz and the Z-cut quartz. The measured lower energy transitions is 10.4 eV as previously reported in Ref.[10]. Therefore, we can take $(E_0 - E_n) \approx 10$ eV for a rough estimation. We can take the energy split due to the spin–orbit interaction as the value of the matrix element $\langle\Psi_0|\boldsymbol{\mu}_k\cdot\boldsymbol{B}(\boldsymbol{r}_k)|\Psi_n\rangle$ approximately[11] whose value can be obtained from published theoretical calculations that $\langle\Psi_0|\boldsymbol{\mu}_k\cdot\boldsymbol{B}(\boldsymbol{r}_k)|\Psi_n\rangle \approx 0.01$ eV.[12] So, we have the estimation

$$\frac{1}{(E_0-E_n)}\langle\Psi_0|\boldsymbol{\mu}_k\cdot\boldsymbol{B}(\boldsymbol{r}_k)|\Psi_n\rangle \approx 10^{-3}. \tag{7}$$

Microscopically, α-quartz is built up of linked tetrahedrons each of which is composed of one Si atom and four O atoms. The Si atom stays at the centre of the tetrahedron and the O atoms all reside on the corners of the tetrahedron. The lattice of α-quartz is a trigonal Bravais lattice with the translational vectors as follows:[13]

$$\boldsymbol{t}_1 = a_0\begin{pmatrix}1\\0\\0\end{pmatrix},\quad \boldsymbol{t}_2 = a_0\begin{pmatrix}-1/2\\\sqrt{3}/2\\0\end{pmatrix},\quad \boldsymbol{t}_3 = c_0\begin{pmatrix}0\\0\\1\end{pmatrix}. \tag{8}$$

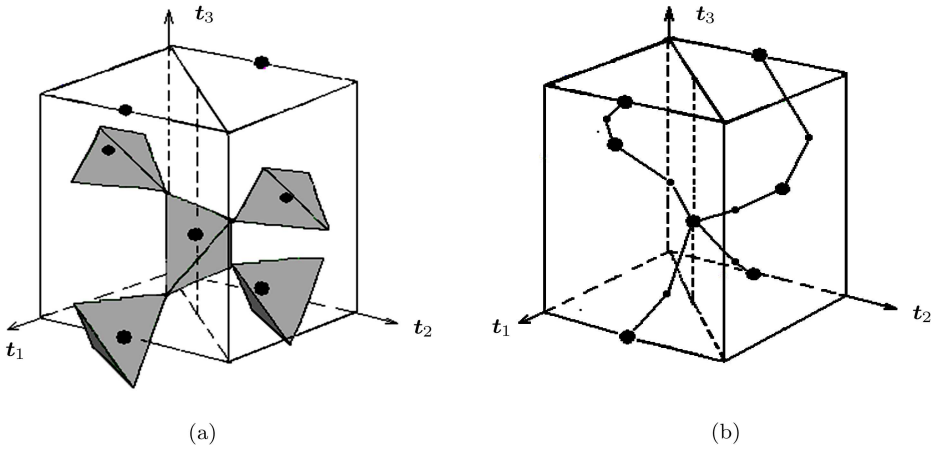
The lattice constants were determined experimentally as $a_0 = 4.89$ Å, and $c_0 = 5.375$ Å (1 Å=0.1 nm).[12]

In potential (6), the detailed expression of the sum of the products of matrix elements for valence

electrons in a primitive cell is

$$\begin{aligned}&\sum_{k=1}^{n}\frac{1}{E_0-E_n}\langle\psi_{kn}|\,\boldsymbol{r}_{ik}\,|\psi_{k0}\rangle\,\langle\psi_{k0}|\,\boldsymbol{B}(\boldsymbol{r}_k)\,|\psi_{kn}\rangle\\ =&\sum_{k=1}^{n}\frac{1}{E_0-E_n}[\langle\psi_{kn}|\,r_{ikx}\,|\psi_{k0}\rangle\,\langle\psi_{k0}|\,B_x(\boldsymbol{r}_k)\,|\psi_{kn}\rangle\\ &+\langle\psi_{kn}|\,r_{iky}\,|\psi_{k0}\rangle\,\langle\psi_{k0}|\,B_y(\boldsymbol{r}_k)\,|\psi_{kn}\rangle\\ &+\langle\Psi_{vn}|\,r_{ikz}\,|\Psi_{k0}\rangle\,\langle\psi_{k0}|\,B_z(\boldsymbol{r}_k)\,|\psi_{kn}\rangle].\end{aligned}\tag{9}$$

Magnetic induction in the above expression has the symmetry of D_3^4 group. A primitive cell of the α-quartz crystal is shown in Fig.2(a), where a trigonal coordinate ($\boldsymbol{t}_1$, $\boldsymbol{t}_2$, $\boldsymbol{t}_3$) combining translational vectors $\boldsymbol{t}_1$, $\boldsymbol{t}_2$, $\boldsymbol{t}_3$ is adapted. In the coordinate, the centre of a primitive cell has coordinates (1/2, 1/2, 1/2), which is also the starting point of the displacement vector $\boldsymbol{r}_{ik}$ of a valence electron in the primitive cell. For each primitive cell, there are four tetrahedrons whose centres lie on the boundary of the primitive cell. Their coordinates of centre are (1, 1/2, 0), (0, 1/2, 0), (1, 1/2, 1) and (0, 1/2, 1) separately. They are superposable on each other by shifting some translation vectors. Each tetrahedron has four bonds distributing separately in four neighbouring primitive cells. In other words, each of these tetrahedrons contributes one bond to a primitive cell. Otherwise, the centres of one pair of tetrahedrons, (1/2, 0, 2/3) and (1/2, 1, 2/3), are in the interfaces of primitive cells. Each tetrahedron contributes one pair of bonds to the primitive cell. There is an individual tetrahedron that contributes four bonds to the primitive cell, whose centre is at (1/2, 1/2, 1/3).



(a) (b)

Fig.2. (a) Primitive cell of α-quartz crystal in the trigonal coordinates and each grey tetrahedron is composed of one Si atom in the centre and four O atoms on the corners and (b) the efficient bonds or valence electrons contained in a primitive cell, where the bigger solid circle represents Si atom, the smaller one denotes O atom, and the line connecting them refers to a bond.

Altogether, each primitive cell contains twelve bonds or twenty-four valence electrons as shown in the Fig.2(b). These bonds are not symmetrical with respect to the centre of primitive cell. So, matrices of the displacement vector $\boldsymbol{r}_{ik}$ of valence electron will not counteract with each other. Otherwise, the centres of the four tetrahedrons on the boundary of a primitive cell are symmetrical with respect to the centre of primitive cell. And the centres of the two tetrahedrons in the interfaces are symmetrical with respect to the point (1/2, 1/2, 2/3). These symmetries will depress their contribution to potential (6). We chronically describe the depression by introducing the asymmetry factor χ less than 1 in the sum (11).[14] With the asymmetry factor, we can calculate only the homogeneous part of the magnetic field in a primitive cell approximately as

$$|\langle\psi_{k0}|\, B_x(\boldsymbol{r}_k)\,|\psi_{kn}\rangle| = |\langle\psi_{k0}|\, B_y(\boldsymbol{r}_k)\,|\psi_{kn}\rangle| = |\langle\psi_{k0}|\, B_z(\boldsymbol{r}_k)\,|\psi_{kn}\rangle| = \frac{1}{3}\,|\langle\psi_{k0}|\,\boldsymbol{B}\,|\psi_{kn}\rangle|\,.$$

And expression (9) can be written as

$$\frac{1}{3}\sum_{k=1}^{N}\frac{\chi}{E_0-E_n}\,|\langle\psi_{k0}|\,\boldsymbol{B}\,|\psi_{kn}\rangle|\,(|\langle\psi_{kn}|\,r_{ikx}\,|\psi_{k0}\rangle| + |\langle\psi_{kn}|\,r_{iky}\,|\psi_{k0}\rangle| + |\langle\psi_{kn}|\,r_{ikz}\,|\psi_{k0}\rangle|). \tag{10}$$

The module $|\langle\psi_{kn}|\,r_{ika}\,|\psi_{k0}\rangle|$ of the kth valence electron can be taken approximately as the distance between the centres of the tetrahedron and the ith primitive cell, where $a = x,\ y,\ z$. For valence electrons in the tetrahedron whose centre is located on the boundary of primitive cell, we can take $|\langle\psi_{kn}|\,r_{ikx}\,|\psi_{k0}\rangle| = a_0/2$, $|\langle\psi_{kn}|\,r_{iky}\,|\psi_{k0}\rangle| = 0$ and $|\langle\psi_{kn}|\,r_{ikz}\,|\psi_{k0}\rangle| = c_0/2$.[12] Modules of matrices of valence electrons in the tetrahedron centred at interface can be taken as $|\langle\psi_{kn}|\,r_{ikx}\,|\psi_{k0}\rangle| = 0$, $|\langle\psi_{kn}|\,r_{iky}\,|\psi_{k0}\rangle| = a_0/2$ and $|\langle\psi_{kn}|\,r_{ikz}\,|\psi_{k0}\rangle| = c_0/6$. A valence electron in the tetrahedron within a primitive cell has the nonzero module $|\langle\psi_{kn}|\,r_{ikz}\,|\psi_{k0}\rangle| = c_0/6$.

Finally, the sum of matrix products in expression (10) for valence electrons in a primitive cell becomes $2.5\times10^{-12}\chi$. Some theoretical and experimental results concerning the chirality phenomena give the magnitude range of the asymmetry factor as $10^{-5} < \chi < 10^{-2}$.[13] Without loss of generality, we take $\chi = 10^{-3}$ in estimation. After comprehensive analyses of all these results, from potential (6) we can obtain

$$V_c = -\frac{g_1 g_2\hbar}{2\pi m_2 c}\sum_{j=1}^{K}\sum_{i=1}^{N}\left(m_\varphi+\frac{1}{r_{ji}}\right)\frac{\mathrm{e}^{-r_{ji}m_\varphi}}{r_{ji}^2}\times 2.5\times10^{-15}. \tag{11}$$

The problem about the axion mass and the coupling $g_{\mathrm{s}}g_{\mathrm{p}}$ is still an open one. We carry out a measurement of potential (6) practically to determine the distance between test mass and source mass, comparable with the axion Compton wavelength.[4,5] Let the dimensions of the copper block and the block of α-quartz be $0.08\times0.08\times0.01\,\mathrm{m}^3$ and $0.08\times0.08\times0.002\,\mathrm{m}^3$ respectively. The distance between the copper block and the α-quartz is $0.5\times10^{-3}\,\mathrm{m}$ and the interaction range (axion Compton wavelength) is $10^{-3}\,\mathrm{m}$. The interaction force between the copper block and the α-quartz crystal, F_x, can be calculated to be $9.78\times10^{25}g_1g_2$, i.e. $F_x = 9.78\times10^{25}g_1g_2$ according to potential (11). The limit of the tested production of the coupling constants is $g_1g_2 < 1.5\times10^{-24}\hbar c$ for $\lambda = 10^{-3}\,\mathrm{m}$.[5] So, the magnitude of the force is less than $4.64\times10^{-24}\,\mathrm{N}$. And the ratio of the force to the gravity for the copper block and the vessel of homochiral molecules is less than 1.2×10^{-15}, which is consistent with the result given by Pospelov in Ref.[4].

4. Conclusions

The scalar-pseudoscalar coupling mediated by light bosons can induce a long-range chiral force between a block of α-quartz and a copper block. The force F between a $0.08\times0.08\times0.002\,\mathrm{m}^3$ block of α-quartz and a $0.08\times0.08\times0.02\,\mathrm{m}^3$ copper block with a separation being 0.5 mm in between, for the Compton wavelength of the mediatory particle $\lambda = 10^{-3}\,\mathrm{m}$ is estimated with the published data as less than $4.64\times10^{-24}\,\mathrm{N}$, i.e. $F < 4.64\times10^{-24}\,\mathrm{N}$. Best equipment to detect the force with this magnitude is based on the torsion balance. At present, the resolution is achieved to be about $10^{-16}\,\mathrm{N\cdot m}$.[14] The measurable magnitude of the force by a torsion balance with an arm length of $10^{-1}\,\mathrm{m}$ is $10^{-15}\,\mathrm{N}$. The magnitude of the force between a block of α-quartz and a copper block is far lower than the measurable one at present. There must exist many influencing factors which will disturb its detection. But we can confirm that such a force caused by axion is macroscopically chirality-dependent. At present no other force is found to be macroscopically chiral. So some day when the measurement accuracy of torsion pendulum reaches the required one, we can discriminate such macroscopic chirality-dependent force from other influencing factors.

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