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Macroscopic inequivalent entanglement witness in Heisenberg spin chain*

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Motivated by the wise idea of entanglement witness (EW), we present an inequivalent entanglement witness (IEEW) that can analogously classify certain eigenstates entangled in inequivalent ways under stochastic local operations and classical communication (SLOCC) in the Heisenberg spin chain. Since the IEEW is the absolute value of magnetization $|M|$ that is a macroscopically measurable quantity, our conclusions provide a macroscopic method to detect inequivalent entanglement between microscopic spins, on the one hand, and clearly show that inequivalent entanglement can yield different macroscopic effects, on the other hand.

Keywords: Heisenberg spin chain, inequivalent entanglement witness
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1. Introduction

Quantum entanglement plays a vital role in quantum information processing (QIP), such as quantum key distribution, quantum teleportation, dense coding, etc.[1] Other than these novel microscopic phenomena, quantum entanglement can also influence macroscopic world. The quantum heat engine (QHE) is one of the widely explored tools to study macroscopic effects brought about by quantum mechanics.[2−5] A simple entangled QHE[5] was proposed and the second law of thermodynamics was proven valid even when entanglement needs taking into account.

Entanglement witness (EW), which is defined as a Hermitian operator such that its expectation value is positive for every separable state while negative for some entangled states,[6] is another evidence that entanglement can affect macroscopic properties when the witness operator is a macroscopic quantity.[7] As is well known, the main virtue of EW lies in the fact that it is not only an economic indication of entanglement, but also helpful for estimating the entanglement content[8] when the full (tomographic) information of the state is absent. The concept of EW was applied to various problems in statistical systems,[9] quantum cryptography,[10] quantum optics,[11] cluster states[12] and so on.

The EW is a wise way to classify quantum states from separable states to entangled states. However, to distinguish between separable states and entangled states is only the first step of the complete classification of quantum states. The next step is to classify inequivalent entangled states. Concerning the classification of inequivalent entangled states when only a single copy is provided, a common way is to require two states to be equivalent if both of them can be obtained from each other by stochastic local operations and classical communication (SLOCC),[14] that is, through local operations and classical communication (LOCC) without imposing that it has to be accomplished with certainty. The classification of pure states of no more than three qubits has been satisfactorily accomplished. Any two-qubit pure state can be converted to a singlet state by means of the SLOCC. Three-qubit pure state could be fully entangled in two inequivalent ways, represented by either GHZ-state $(|000\rangle + |111\rangle)/\sqrt{2}$ or W-state $(|001\rangle + |010\rangle + |100\rangle)/\sqrt{3}$.[15] So far the classification of four-qubit pure states by the SLOCC has not been fully solved yet. Verstraete *et al*[16] concluded that there existed nine families of states corresponding to nine different ways of entanglement and claimed that by determinant-one SLOCC operations, a pure state

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of four qubits can be transformed into one of the nine families of states. Clearly, this does not say that each family is an SLOCC class. Li *et al*[17] investigated the SLOCC classification for each family.

Motivated by the EW, we present an inequivalent entanglement witness (IEEW) that can sort inequivalent entangled states, therefore, can play an analogous role in the classification of inequivalent entangled states. The witness operators that can detect the different classes of three-qubit mixed states have already been proposed in Ref.[18]. However, the witness operator given in the present paper is a macroscopical observable. So besides the theoretical interest, the IEEW is also expected to be useful for the potential physical realization of future QIP. Similar to the EW, an IEEW is defined as a Hermitian operator whose expectation value is different for states entangled in inequivalent ways under the SLOCC. Clearly, an EW is a coarse graining of any IEEW since separable and entangled states apparently are inequivalent under the SLOCC. Moreover, just as an EW cannot successfully detect some entangled states, an IEEW may fail to completely classify inequivalent entangled states. That is to say, although the entangled states with different values of an IEEW must be inequivalent under the SLOCC, some entangled states with the same value of the IEEW may also be mutually inconvertible by means of SLOCC.

In the present paper, we find the absolute value of magnetization $|M|$ can serve as an IEEW for some states. The results provide a macroscopic method to classify certain entangled states. The rest of the present paper is organized as follows. In Section 2, we introduce a Heisenberg spin chain and present essential features of its eigenspectrum. In Section 3, by virtue of the existing classification of quantum states via SLOCC, we show that $|M|$ can act as an IEEW for two-, three- and four-qubit Heisenberg spin chains. We partially generalize the IEEW $|M|$ to an N-party spin chain in Section 4. Finally, the conclusions and further discussions are given in Section 5.

2. Eigenspectrum of Heisenberg spin chain

We first review the Heisenberg spin chain, which describes a system of N linearly arranged spins, each interacting only with its nearest neighbours. The Hamiltonian of the spin chain in a constant external magnetic field B is given by

$$H = \sum_{i=1}^{N} \left(J\boldsymbol{\sigma}^i \cdot \boldsymbol{\sigma}^{i+1} + B\boldsymbol{\sigma}_z^i \right), \tag{1}$$

where $\boldsymbol{\sigma}^i = \left(\sigma_x^i, \sigma_y^i, \sigma_z^i\right)$ are three Pauli matrices for the i-th spin (cyclic boundary condition $N+1 \equiv 1$ is assumed). J is the exchange constant; positive and negative J correspond to antiferromagnet and ferromagnet respectively. As is well known, the exact eigenspectrum of Heisenberg spin chain can be effectively obtained via Bethe ansatz,[19,20] whose application needs two essential symmetries, i.e., the rotational symmetry about the z-axis in spin space and the translational symmetry. The rotational symmetry implies that any eigenstate of H must be a linear combination of product basis vectors which have the same numbers of spin-up. Hence, the eigenstates can be sorted into $N+1$ groups according to the number of spin-up, r. Each eigenstate in the same group always has the same magnetization $M = \sum_{i=1}^{N} \sigma_z^i = 2r - N$ accordingly. We then denote each eigenstate and its eigenvalue as $\left|\psi_k^{(r)}\right\rangle$ and $\lambda_k^{(r)}$, where $k = 1, \ldots, C_N^r$ label the orders of eigenstates in a group separately. Moreover, due to $\left[H, \prod_{i=1}^{N} \otimes \sigma_x^i\right] = 0$, both $\left|\psi_k^{(r)}\right\rangle$ and $\left|\psi_k^{(N-r)}\right\rangle = \prod_{i=1}^{N} \sigma_x^i \left|\psi_k^{(r)}\right\rangle$ are eigenstates of H. $\left|\psi_k^{(r)}\right\rangle$ and $\left|\psi_k^{(N-r)}\right\rangle$ obviously are equivalent under the SLOCC. Consequently when inequivalent entanglement in Heisenberg spin chain is discussed, only the inequivalence of $[N/2]+1$ ($[x]$ denotes the biggest integer no more than x) eigenstate groups needs investigating. We pick eigenstate groups with $r = 0, ..., [N/2]$ as the representatives in the following discussion for simplicity.

In addition, since two parts of H, i.e. $H_1 = J\sum_{i=1}^{N} \boldsymbol{\sigma}^i \cdot \boldsymbol{\sigma}^{i+1}$ and $H_2 = B\sum_{i=1}^{N} \sigma_z^i$ are commutable, the eigenvalue $\lambda_k^{(r)}$ is in the shape of $A(r,k)J - (N-2r)B$, that is, $H_1 \left|\psi_k^{(r)}\right\rangle = A(r,k) \left|\psi_k^{(r)}\right\rangle$ Moreover, $\left[H_1, \prod_{i=1}^{N} \otimes \sigma_x^i\right] = 0$ yields

$$\begin{aligned} & A(N-r,k) \left|\psi_k^{(N-r)}\right\rangle \\ = & H_1 \left|\psi_k^{(N-r)}\right\rangle \\ = & \prod_{i=1}^{N} \otimes \sigma_x^i H_1 \left|\psi_k^{(r)}\right\rangle \\ = & \prod_{i=1}^{N} \otimes \sigma_x^i A(r,k) \left|\psi_k^{(r)}\right\rangle \\ = & A(r,k) \left|\psi_k^{(N-r)}\right\rangle, \end{aligned}$$

i.e., $A(r,k) = A(N-r,k)$. Besides, in an eigenstate group, all eigenstates can be arranged according to the increasing order of its eigenvalue, i.e. $\lambda_1^{(r)}$ is set to be the smallest eigenvalue in eigenstate group with r spin-ups. It is then easily found that the variation of B does not affect the order of each eigenstate in a group. Therefore, the eigenstate set $\mathcal{G} \hat{=} \left\{ \left|\psi_1^{(0)}\right\rangle, \ldots, \left|\psi_1^{([N/2])}\right\rangle \right\}$ covers all possible ground states of the Heisenberg spin chain when B is varied while J is fixed. We call $\mathcal{G}$ the ground state set.

3. IEEW in two-, three-, four-qubit Heisenberg spin chains

In order to show that $|M|$ is qualified to be an IEEW, we need to explain different eigenstate groups that are inequivalent under the SLOCC. To start with, we study some simple cases, i.e. eigenstates of two-, three-, four-qubit Heisenberg spin chains in this section.

From the Bethe ansatz, the eigenspectra of two-qubit and three-qubit Heisenberg spin chains are respectively presented in Tables 1 and 2. It is shown that for $N = 2$, the eigenstates with $|M| = 2$ and $|M| = 0$ are the separable state and the entangled state respectively. When $N = 3$, the classification via SLOCC can be accomplished by using the method introduced by Dür *et al*,[15] i.e. by using the different values of the local entropy of every spin and the 3-tangle.[21] Therefore the eigenstates with $|M| = 3$ and $|M| = 1$ are found to be separable and W-class entangled states respectively, i.e., $|M|$ can successfully witness inequivalent entangled states under the SLOCC in the 3-qubit Heisenberg spin chain.

Table 1. The eigenspectrum of two-qubit Heisenberg spin chain.

r	$\|M\|$	eigenvalue	eigenstate
0	2	$2J-2B$	$\|00\rangle$
1	0	$-6J$	$(\|01\rangle - \|10\rangle)/\sqrt{2}$
1	0	$2J$	$(\|01\rangle + \|10\rangle)/\sqrt{2}$

Table 2. The eigenspectrum of three-qubit Heisenberg spin chain.

r	$\|M\|$	eigenvalue	eigenstate
0	3	$3J-3B$	$\|000\rangle$
1	1	$3J-B$	$(\|001\rangle + \|010\rangle + \|100\rangle)/\sqrt{3}$
1	1	$J-B$	$\left(\|001\rangle + \mathrm{e}^{\mathrm{i}2\pi/3}\|010\rangle + \mathrm{e}^{-\mathrm{i}2\pi/3}\|100\rangle\right)/\sqrt{3}$
1	1	$J-B$	$\left(\|001\rangle + \mathrm{e}^{-\mathrm{i}2\pi/3}\|010\rangle + \mathrm{e}^{\mathrm{i}2\pi/3}\|100\rangle\right)/\sqrt{3}$

The four-qubit spin chain is a bit complicated. In Ref.[16], the authors claimed that a pure state of four-qubit could (up to permutations) be transformed into one of nine families of states by unit determinant SLOCC operations. Among nine families, three families are relevant to our paper:

$$
\begin{aligned}
G_{abcd} &= \frac{a+d}{2}(|0000\rangle + |1111\rangle) + \frac{a-d}{2}(|0011\rangle + |1100\rangle) + \frac{b+c}{2}(|0101\rangle + |1010\rangle) + \frac{b-c}{2}(|0110\rangle + |1001\rangle),\\
L_{abc_2} &= \frac{a+b}{2}(|0000\rangle + |1111\rangle) + \frac{a-b}{2}(|0011\rangle + |1100\rangle) + c(|0101\rangle + |1010\rangle) + |0110\rangle,\\
L_{ab_3} &= a(|0000\rangle + |1111\rangle) + \frac{a+b}{2}(|0101\rangle + |1010\rangle) + \frac{a-b}{2}(|0110\rangle + |1001\rangle)\\
&\quad + \frac{\mathrm{i}}{\sqrt{2}}(|0001\rangle + |0010\rangle + |0111\rangle + |1011\rangle).
\end{aligned} \tag{2}
$$

The eigenvalues and eigenstates of the four-qubit Heisenberg spin chain are given in Table 3. According to the classification of four-qubit pure states presented above, we find the eigenstate with $|M| = 4$ is a separable state, consequently belongs to the family L_{abc_2} with $a = b = c = 0$; four eigenstates with $|M| = 2$ belong to L_{ab_3} family with $a = b = 0$, six eigenstates with $|M| = 0$ belong to G_{abcd} family with a, b, c, d have some particular values that are also presented in Table 3. Although six eigenstates that belong to G_{abcd} family are not completely equivalent to each other under the SLOCC,[17] they must be inequivalent to any state that belongs to other SLOCC families. Thus the eigenstates of four-qubit spin chain with different values of $|M|$ are sure to be inequivalent under the SLOCC.

Table 3. The eigenspectrum of four-qubit Heisenberg spin chain and their classifications according to Ref.[16]. One paradigmatic group of complex parameters a, b, c, d is given without normalization.

r	$\|M\|$	eigenvalue	eigenstate	SLOCC classification
0	4	$4J-4B$	$\|0000\rangle$	L_{abc_2} $(a=b=c=0)$
1	2	$4J-2B$	$\frac{1}{2}(\|1000\rangle+\|0100\rangle+\|0010\rangle+\|0001\rangle)$	L_{ab_3} $(a=b=0)$
1	2	$-2B$	$\frac{1}{2}(\mathrm{i}\|1000\rangle-\|0100\rangle-\mathrm{i}\|0010\rangle+\|0001\rangle)$	L_{ab_3} $(a=b=0)$
1	2	$-2B$	$\frac{1}{2}(-\mathrm{i}\|1000\rangle-\|0100\rangle+\mathrm{i}\|0010\rangle+\|0001\rangle)$	L_{ab_3} $(a=b=0)$
1	2	$-4J-2B$	$\frac{1}{2}(-\|1000\rangle+\|0100\rangle-\|0010\rangle+\|0001\rangle)$	L_{ab_3} $(a=b=0)$
2	0	$4J$	$\frac{1}{\sqrt{6}}(\|0011\rangle+\|1100\rangle+\|0110\rangle+\|1001\rangle+\|1010\rangle+\|0101\rangle)$	G_{abcd} (a = 1,b = 2,c = 0,d = - 1)
2	0	0	$\frac{1}{2}(-\|0011\rangle+\mathrm{i}\|0110\rangle-\mathrm{i}\|1001\rangle+\|1100\rangle)$	G_{abcd}(a = 1,b = i,c = - i,d = - 1)
2	0	0	$\frac{1}{2}(-\mathrm{i}\|0011\rangle+\|0110\rangle-\|1001\rangle+i\|1100\rangle)$	G_{abcd} (a = i,b = 1,c = - 1,d = - i)
2	0	0	$\frac{1}{\sqrt{6}}(-\mathrm{i}\|0011\rangle+\|1100\rangle+\mathrm{i}\|0110\rangle+i\|1001\rangle-\|1010\rangle-\mathrm{i}\|0101\rangle)$	G_{abcd} (a = - i,b = 1 + i,c = 1 - i,d = i)
2	0	$-4J$	$\frac{1}{\sqrt{2}}(\|1010\rangle-\|0101\rangle)$	G_{abcd} (a = 0,b = 1,c = 1,d = 0)
2	0	$-8J$	$\frac{1}{\sqrt{12}}(\|0011\rangle+\|1100\rangle+\|0110\rangle+\|1001\rangle-2\|1010\rangle-2\|0101\rangle)$	G_{abcd} (a = 1,b = - 1,c = - 3,d = - 1)

So far, we have proven that any two eigenstates in two-, three- and four-qubit Heisenberg spin chains are inequivalent under the SLOCC if they have different values of $|M|$. Hence $|M|$, a macroscopic Hermite operator which can be easily measured, is qualified to serve as an IEEW for three particular Heisenberg spin chains.

4. IEEW in N-party Heisenberg spin chain

4.1. Inconvertibility predicted by Schmidt number

We try to generalize the IEEW $|M|$ to an N-qubit Heisenberg spin chain in this section. Before proceeding, we first elucidate the criterion based on which we could tell whether any two multipartite states are inequivalent under the SLOCC. As has been proved,[1] for any bipartite pure state $|\Psi\rangle \in \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$, there exist local orthonormal bases $\{|u_i\rangle\} \in \mathcal{H}_A$ and $\{|v_i\rangle\} \in \mathcal{H}_B$ such that

$$|\Psi\rangle = \sum_i a_i |u_i\rangle \otimes |v_i\rangle, \tag{3}$$

where a_i are non-negative real numbers satisfying $\sum_i a_i^2 = 1$, referred to as Schmidt coefficients. This representation is a Schmidt decomposition of $|\Psi\rangle$ and the number of nonzero a_i is known as the Schmidt number, Sch($|\Psi\rangle$). The Schmidt number proves useful for distinguishing entangled states. In fact, it can be easily deduced that any two mutually convertible bipartite states by means of SLOCC, must have the same Schmidt numbers.[22,23] Therefore two bipartite states with different Schmidt numbers cannot be converted into each other by means of SLOCC. For a multipartite state, a generalized Schmidt decomposition does not exist usually. However, we can always divide any N-party system into two nonempty and disjoint parts, e.g., one part has N_1 $(1 \leq N_1 < N)$ parties and the other possesses $N - N_1$ parties. For every bipartition of a multipartite state, there always exists Schmidt decomposition like expression (2). If the Schmidt numbers of two N-party states $|\Psi\rangle$ and $|\Phi\rangle$ based on the same bipartition are different, then they cannot be converted into each other by any SLOCC. In a word, any two multipartite states $|\Psi\rangle$ and $|\Phi\rangle$ are inequivalent under the SLOCC if we can find a bipartition based on which two states have different Schmidt numbers. This is the foundation of the following discussion.

4.2. IEEW for ground state set in ferromagnet

We now investigate the ground state set $\mathcal{G}$ of ferromagnet. According to Bethe ansatz, each eigenstate in the ferromagnetic ground state set can be given by

$$\left|\psi_1^{(r)}\right\rangle = \frac{1}{\sqrt{C_N^r}} \sum_{1\le n_1<n_2<\ldots<n_r\le N} |n_1 n_2 \ldots n_r\rangle\,, \tag{4}$$

where n_i labels the position of the i-th spin-up. Obviously $\left|\psi_1^{(r)}\right\rangle$ is an equally-weighted supposition of all C_N^r product states with r spin-ups. We use the $\{[N/2]\otimes[(N+1)/2]\}$ bipartition to compute the Schmidt number. It is easy to find $\mathrm{Sch}\left(\left|\psi_1^{(0)}\right\rangle\right)=1$ and $\mathrm{Sch}\left(\left|\psi_1^{(1)}\right\rangle\right)=2$. As concerns other $\left|\psi_1^{(r)}\right\rangle$, the Schmidt decomposition based on $\{[N/2]\otimes[(N+1)/2]\}$ bipartition can be expressed as

$$\begin{aligned}
&|\psi_1^{(r)}\rangle \\
&= \frac{1}{\sqrt{C_N^r}} \sum_{1\le n_1<\ldots<n_r\le N} |n_1\ldots n_r\rangle \\
&= \frac{1}{\sqrt{C_N^r}}[\underbrace{|00\ldots0\rangle}_{[N/2]}\left(\sum_{[N/2]+1\le n_1<\ldots<n_r\le N}|n_1\ldots n_r\rangle\right)+\left(\sum_{1\le n_1\le[N/2]}|n_1\rangle\right)\left(\sum_{[N/2]+1\le n_2<\ldots<n_r\le N}|n_2\ldots n_r\rangle\right) \\
&\quad +\ldots+\left(\sum_{1\le n_1<\ldots<n_r\le[N/2]}|n_1\ldots n_r\rangle\right)\underbrace{|00\ldots0\rangle}_{[(N+1)/2]}.
\end{aligned} \tag{5}$$

It is obvious that $\mathrm{Sch}\left(\left|\psi_1^{(r)}\right\rangle\right) = r+1$. Now we can affirm that $\left|\psi_1^{(r)}\right\rangle$ with different values of $|M| = N-2r$ are all inequivalent under the SLOCC, since we find a bipartition based on which $\mathrm{Sch}\left(\left|\psi_1^{(r)}\right\rangle\right)$ varies with r. Therefore, $|M|$ acts well as an IEEW for the ground state set of ferromagnetic Heisenberg spin chain.

4.3. IEEW for separable states and W-class entangled states

We then elucidate that $|M|$ can serve as a witness for separable states and W-class entangled states in Heisenberg spin chain. In fact, on the one hand, the eigenstate with $|M| = N$ must be a separable state, which is easily found to be $|00\ldots0\rangle$. On the other hand, all eigenstates with $|M| = N-2$ can be converted into $W_N = (1/\sqrt{N})\sum_{n=1}^N |n\rangle$ by local unitary (LU) operations. In fact, the translationally invariant eigenstates of an N-qubit Heisenberg spin chain with $|M| = N-2$ can be written by

$$|\psi^{(1)}\rangle = \frac{1}{\sqrt{N}}\sum_{n=1}^{N} \mathrm{e}^{\mathrm{i}kn}|n\rangle \tag{6}$$

for $k = 2\pi m/N, m = 0,\ldots,N-1$. Apparently a local unitary operator $U = \prod_{n=1}^{N}\otimes\begin{pmatrix}1 & 0\\ 0 & \mathrm{e}^{-\mathrm{i}kn}\end{pmatrix}$ can change $\left|\psi^{(1)}\right\rangle$ into W_N, i.e. $U\left|\psi^{(1)}\right\rangle = W_N$. Notice in addition that the LU is a particular case of SLOCC, so all eigenstates with $|M| = N-2$ have equivalence under the SLOCC. Therefore, $|M| = N$ and $|M| = N-2$ can respectively witness separable state and W-class entangled states in an N-qubit Heisenberg spin chain.

So far, we have proved the validity of IEEW $|M|$ for certain eigenstates in the N-qubit Heisenberg spin chain. For other eigenstates, now we cannot judge whether $|M|$ can also act as an IEEW, due to the complex form of these eigenstates. The further study is still under way.

5. Conclusions

We find that the absolute value of magnetization $|M|$ can be an IEEW in the widely-studied Heisenberg spin chain. The IEEW is defined as a Hermitian operator whose expectation value is different from those states that are entangled in inequivalent ways under the SLOCC. We first sort the eigenstates of Heisenberg spin chain into $[N/2]+1$ different groups by the abso-

lute value of magnetization $|M|$. Based on the classification of pure states of no more than four qubits, we then examine the whole eigenspectra of two-, three- and four-qubit Heisenberg spin chains and find that $|M|$ acts well as an IEEW. Furthermore, we can partially generalize the IEEW $|M|$ to an N-qubit Heisenberg spin chain. Each eigenstate in the ferromagnetic ground state set $\mathcal{G}$ is proved to be inequivalent under the SLOCC, and $|M|$ can successfully witness the inequivalence. The separable eigenstate and W-class entangled eigenstates could also be witnessed by $|M| = N$ and $|M| = N-2$ respectively. Since $|M|$ is a macroscopically measurable quantity, then on certain occasions the existence and the type of entanglement in Heisenberg spin chain can be detected conveniently by solely measuring a macroscopic observable. Considering that the entanglement is a concrete exhibition of quantum nonlocality, our results show different types of nonlocalities can be manifested by the different values of macroscopic quantities. Therefore, to explore other macroscopic behaviours induced by various nonlocalities is an intriguing subject in the research of nonlocality.

Moreover, the two cases of $|M|$ being a successful IEEW in an N-qubit Heisenberg spin chain have extra significance. In the first case, $\mathcal{G}$ covers all the possible states of the system at absolute zero temperature when the system is frozen to its ground state. Every time when $|M|$ undergoes an abrupt change, the entangled way of the state of the system also changes suddenly. The synchronous occurrence of quantum phase transition (QPT)[24] (where the QPT means the abrupt change of $|M|$ and is obviously of first-order) and the variation of entanglement type has not yet be mentioned before, to the best of our knowledge. The second case provides a macroscopic quantitative standard to judge the generation of W-class entangled states in Heisenberg spin chain, i.e. the system is sure to be in W-class entangled states when $|M| = N - 2$ is measured. This makes it convenient to accomplish some QIP tasks by virtue of Heisenberg spin chain.

However, our results are incomplete. Firstly, the straightforward generalization that any two eigenstates with different $|M|$-values are inequivalent under the SLOCC is still unproved and needs further researching. Secondly, our conclusions may be appropriately used at zero temperature when the state of the system is in one of its eigenstates, since what the classification of entangled states deals with are pure states.

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