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Bipartite entanglement of a two-qubit system with anisotropic couplings under nonuniform magnetic fields

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This paper investigates bipartite entanglement of a two-qubit system with anisotropic couplings under an inhomogeneous magnetic field. This work is mainly to investigate the characteristics of a Heisenberg XYZ chain and obtains some meaningful results. By the concept of negativity, it finds that the inhomogeneity of magnetic field may induce entanglement and the critical magnetic field is independent of J_z. The inhomogeneous magnetic field can increase the value of critical magnetic field B_c. It also finds that the magnetic field not only suppresses the entanglement but also can induce it to revival for some time.

Keywords: Heisenberg XYZ chain, bipartite entanglement, negativity
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1. Introduction

Entanglement is one of the most distinctive features of quantum mechanical systems.[1,2] Recently, interesting features of solid-state[3] systems have been revealed. Particularly, in the field of condensed-matter physics, an interesting and natural type of entanglement, the thermal entanglement, is introduced and analysed within various types of quantum spin systems. Most of papers calculate the pairwise entanglement of Heisenberg systems[4−8] by using concurrence.[9]

Vidal *et al* raised negativity[10]as a measure of bipartite entanglement, which has been drawn much attention. Negativity is a measure for the degree of violation of the criterion of positive partial transpose (PPT)[11] in entangled states. As a result of the serviceability the negativity theory to the system $2\otimes2$, we will investigate the bipartite entanglement[12−16] of Heisenberg systems.

This paper investigates the bipartite entanglement in a spin-1/2 Heisenberg XYZ spin chain under nonuniform magnetic fields. In Section 2, we briefly give the Hamiltonian of the model and the definition of the negativity. In Section 3, we investigate the properties of the ground states and the thermal states. Finally, Section 4 contains the concluding remarks.

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2. General formalism

We use the anisotropic Heisenberg XYZ spin chain to study the bipartite entanglement under nonuniform magnetic fields. The Hamiltonian of this model is given by

$$\begin{aligned} H =&\frac{1}{2}\sum_{i=1}^{N}[J(1+\gamma)\sigma_i^x\sigma_{i+1}^x \\ &+ J(1-\gamma)\sigma_i^y\sigma_{i+1}^y + J_z\sigma_i^z\sigma_{i+1}^z \\ &+ (B+b)\sigma_i^z + (B-b)\sigma_{i+1}^z], \end{aligned} \tag{1}$$

where $\sigma^\alpha(\alpha = x, y, z)$ denotes the spin-1/2 operator, $J(1+\gamma)$, $J(1-\gamma)$ and J_z are the couplings which take care of both the ferro- or antiferromagnetic couplings between two spin degrees of freedoms. B and b describe the magnetic fields, applied along the z-direction, so that b controls the degree of inhomogeneity. We consider the Hamiltonian for the N=2 case.

Now, we briefly introduce the definition of the negativity for a state ρ. According to the Peres–Horodecki criterion, a non-entangled state has necessarily a PPT.[12,16] In the case of two spin-halves, and the case of (1/2, 1) mixed spins, a PPT is sufficient. Negativity was firstly introduced by Vidal and Werner and is defined as[11]

$$N(\rho) = \frac{||\rho^{T_2}||_1 - 1}{2}, \tag{2}$$

where the trace norm of ρ^{T_2} is equal to the sum of the absolute values of the eigenvalues of ρ^{T_2}, and T_2 denotes the partial transpose of ρ with respect to the second subsystem. The state at thermal equilibrium[3] is represented by the Gibb's density operator $\rho(T) = Z^{-1}\exp(-H/k_{\rm B}T)$, where $Z = {\rm tr}[\exp(-H/k_{\rm B}T)]$ is the partition function, and $k_{\rm B}$ is the Boltzmann's constant which is set to be 1 hereafter.

As the negativity is a computable measure of entanglement for a bipartite system of any dimension, here we choose it to measure the thermal entanglement. In n-qubit system at finite temperature, it can then be used to detect the entanglement between the components of any bipartition $\{m\}$ and $\{n-m\}$ of the whole system, as well as of any subsystem, beyond the level of pairwise two-qubit entanglement measured by the concurrence. The negativity $N(\rho)$ is equivalent to the absolute value of the sum of the negative eigenvalues of ρ^{T_2}[11,17]

$$N_{1-n}=\sum\nolimits_i |(\mu_{1-n})_i|. \tag{3}$$

3. Results and discussion

For the case of the two-qubit model, if the periodic boundary condition (PBC) is imposed, then by applying the exact diagonalization method, we can analytically obtain the eigenvalues and energy eigenstates of the system

$$\begin{aligned}
&E_1 = J_z + \Gamma, \quad E_2 = J_z - \Gamma,\\
&E_3 = \Upsilon - J_z, \quad E_4 = -\Upsilon - J_z,\\
&|\psi_1\rangle = r_+|11\rangle + \eta_+|00\rangle,\\
&|\psi_2\rangle = r_-|11\rangle + \eta_-|00\rangle,\\
&|\psi_3\rangle = \mu_+|10\rangle + \upsilon_+|01\rangle,\\
&|\psi_4\rangle = \mu_-|10\rangle + \upsilon_-|01\rangle,
\end{aligned} \tag{4}$$

where the coefficients are defined as

$$\begin{aligned}
&\Gamma = 2\sqrt{B^2 + (J\gamma)^2},\\
&\Upsilon = 2\sqrt{b^2 + J^2},\\
&\gamma_\pm = \pm\frac{\sqrt{2}}{2}\frac{2B \pm \Gamma}{\sqrt{\Gamma^2 \pm 2B\Gamma}},\\
&\eta_\pm = \pm\frac{\sqrt{2}J\gamma}{\sqrt{\Gamma^2 \pm 2B\Gamma}},\\
&\mu_\pm = \pm\frac{\sqrt{2}}{2}\frac{\Upsilon \pm 2b}{\sqrt{\Upsilon^2 \pm 2b\Upsilon}},\\
&\nu_\pm = \pm\frac{\sqrt{2}J}{\sqrt{\Upsilon^2 \pm 2b\Upsilon}}.
\end{aligned} \tag{5}$$

For the Hamiltonian of the completely anisotropic Heisenberg XYZ chain under the inhomogeneous magnetic field, given in Eq.(1), the partial transpose density matrix of this system has the form

$$\rho_{12}^{T1} = \begin{pmatrix} u_- & 0 & 0 & w\\ 0 & v_- & z & 0\\ 0 & z & v_+ & 0\\ w & 0 & 0 & u_+ \end{pmatrix}, \tag{6}$$

where

$$\begin{aligned}
u_\pm &= \frac{1}{Z\Gamma}{\rm e}^{-\beta J_z}(\Gamma\cosh(\beta\Gamma) \pm 2B\sinh(\beta\Gamma)),\\
v_\pm &= \frac{1}{Z\Upsilon}{\rm e}^{\beta J_z}(\Upsilon\cosh(\beta\Upsilon) \pm 2b\sinh(\beta\Upsilon)),\\
z &= \frac{-2J\gamma}{Z\Gamma}{\rm e}^{-\beta J_z}\sinh(\beta\ \Gamma),\\
w &= \frac{-2J}{Z\Upsilon}{\rm e}^{\beta J_z}\sinh(\beta\ \Upsilon).
\end{aligned} \tag{7}$$

And the partition function is

$$Z = 2{\rm e}^{-\beta J_z}\cosh(\beta\Gamma) + 2e^{\beta J_z}\cosh(\beta\Upsilon). \tag{8}$$

Then using the exact diagonalization method, the eigenvalues of ρ_{12}^{T1} are obtained as

$$\begin{aligned}
\lambda_{1,2} &= \sqrt{u_-u_+} \pm w,\\
\lambda_{3,4} &= \sqrt{v_-v_+} \pm z.
\end{aligned} \tag{9}$$

So we can get the following analytical expression of the negativity

$$N_{1-2} = \max(0, -\lambda_1) + \max(0, -\lambda_3). \tag{10}$$

Case 1 $\gamma = 1$, $J_z = 0$. This model becomes an Ising model. Figure 1 shows the negativity as a function of b for various temperatures. We note that the inhomogeneity of magnetic field will induce entanglement and the value will increase with the increase of b.

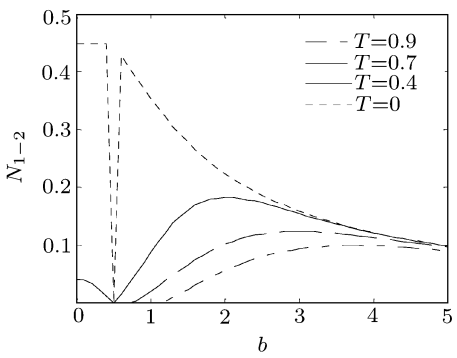


Fig.1. The negativity N_{1-2} versus b for different T ($B = 0.5$, $J = 1$).

There is a revival of the entanglement for magnetic field b a little larger than b_c. In Fig.1 we also plot the conditions of negativity N_{1-2} versus b for ground state ($T = 0$).

Case 2 $J_z = 0$, $\gamma \neq 0$. In this case the XYZ chain reduces to the XY chain. In Fig.2 we depict the negativity N_{1-2} as a function of both the temperature (Fig.1(a), $B = 1$, $T = 0.5$) and the inhomogeneous magnetic field (Fig.2(b), $B = 1$, $T = 0.5$) for different γ. It can be seen from Fig.2(a) that the entanglement vanishes for $T \geq 1.2$. The critical temperature increases with the decrease of γ and comes up to maximum when the model becomes XX chain. In Fig.2(b) we find that for the small nonuniformity, the negativity decreases to the disentangled states with the increase of γ. On the other hand, the negativity shows the oscillatory behaviour for a bigger γ. At the same time it is found that the three curves emerge to one when the inhomogeneity is big enough.

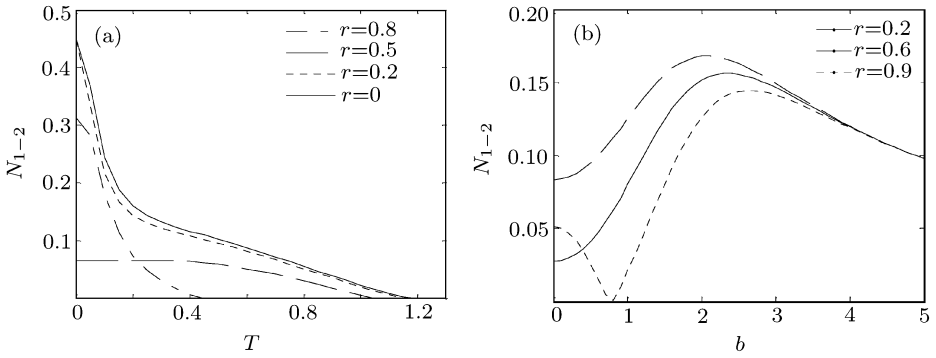


Fig.2. The negativity N_{1-2} versus T (left) and b (right) for different temperatures ($J = 1$).

Case 3 $\gamma = 0$, $J_z \neq 0$. In this case the XYZ chain reduces to the XXZ chain. In Fig.3 we give a plot of the negativity as a function of the temperature T and magnetic fields B for different anisotropic parameters J_z ($B = b = 1$). We can notice that the critical temperature increases with the increase of J_z. When $J_z = 1$, this model becomes an XXX spin chain. It is clear in Fig.3(b) that the critical magnetic field B_c is independent of J_z and we may get strong entanglement by enhancing J_z.

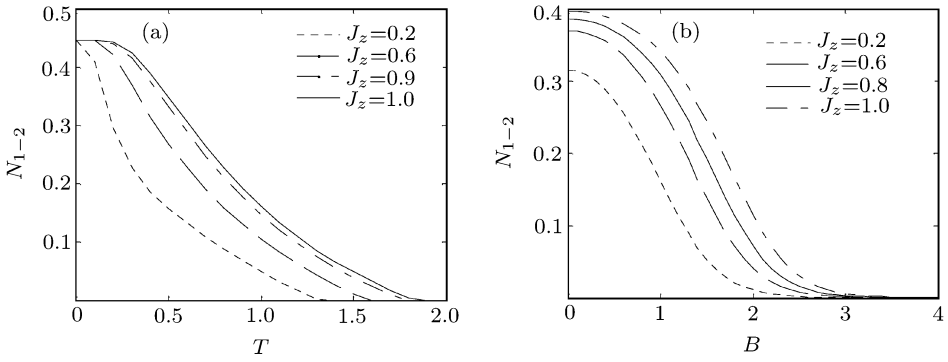


Fig.3. The negativity N_{1-2} versus T (left, $B = b = 1$)) and B (right, $b = 0.6$, $T = 0.5$) ($J = 1$).

Case 4 $\gamma \neq 0$, $J_z \neq 0$, for the Heisenberg XYZ chain. Figure 4 shows the negativity as a function of B and T for different inhomogeneity b. We give the condition $b = 0$ (left) and $b = 2$ (right). Comparing the two figures we find that the magnetic field not only suppresses the entanglement but also can make it revival. The inhomogeneity b can increase the value of the critical magnetic field B_c. We also note that the critical temperature T_c here does not behave as a monotonic function of B.

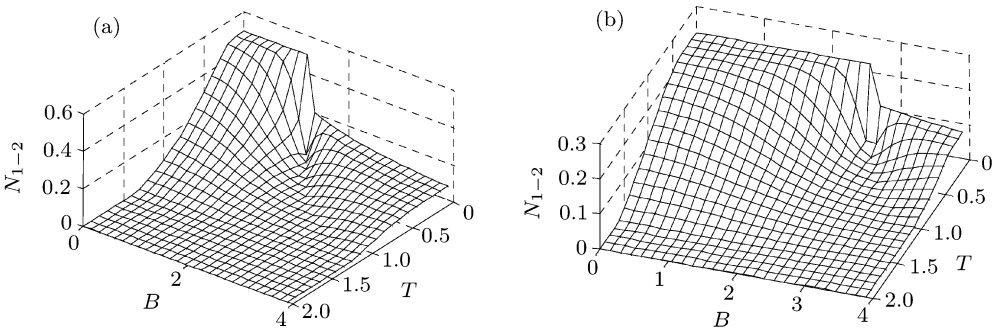


Fig.4. The negativity N_{1-2} versus B and T for different inhomogeneity b ($J = 1$, $\gamma = 0.6$, $J_z = 0.4$).

4. Conclusions

In this paper, the thermal bipartite entanglement in the Heisenberg XYZ chain is investigated in the presence of an external inhomogeneous magnetic field. We obtain analytical expressions of the negativity N_{1-2} and analyse it for four cases in detail. Our results show that the inhomogeneity of magnetic field will induce entanglement and the bipartite entanglement shows the oscillatory behaviour for a bigger anisotropy γ. The inhomogeneity b can increase the value of the critical magnetic field B_c. We also note that the critical temperature T_c does not behave as a monotonic function of B.

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