Vol 18 No 4, April 2009
1674-1056/2009/18(04)/1328-05
Chinese Physics B
© 2009 Chin. Phys. Soc.
and IOP Publishing Ltd

# Two-mode excited entangled coherent states and their entanglement properties*

Zhou Dong-Lin(周冬林), and Kuang Le-Man(匡乐满)†

*Laboratory of Low-Dimensional Quantum Structures and Quantum Control of Ministry of Education, and Department of Physics, Hunan Normal University, Changsha* 410081, *China*

(Received 1 April 2008; revised manuscript received 24 August 2008)

This paper introduces two types of two-mode excited entangled coherent states (TMEECSs) $|\Psi_{\pm}(\alpha,m,n)\rangle$, studies their entanglement characteristics, and investigates the influence of photon excitations on quantum entanglement. It shows that for the state $|\Psi_{+}(\alpha,m,m)\rangle$ the two-mode photon excitations affect seriously entanglement character while the state $|\Psi_{-}(\alpha,m,m)\rangle$ is always a maximally entangled state, and shows how such states can be produced by using cavity quantum electrodynamics and quantum measurements. It finds that the entanglement amount of the TMEECSs is larger than that of the single-mode excited entangled coherent states with the same photon excitation number.

**Keywords:** photon excitations, quantum entanglement, entangled coherent states
**PACC:** 0367, 4250, 0530J

## 1. Introduction

As it is well known that quantum entanglement has been viewed as an essential resource for quantum information processing, creation and manipulation of entangled states are essential for quantum information applications. Among these applications are quantum computation,[1] quantum teleportation, and quantum cryptography. In recent years, much attention has been paid to continuous variable quantum information processing[2−22] in which continuous-variable-type entangled pure states play a key role. For instance, two-state entangled coherent states (ECS) are used to realize efficient quantum computation[7] and quantum teleportation.[8] Two-mode squeezed vacuum states have been applied to quantum dense coding.[9] In particular, following the theoretical proposal of Ref. [10], continuous variable teleportation has been experimentally demonstrated for coherent states of a light field[11] by using entangled two-mode squeezed vacuum states. Therefore, it is an interesting topic to create and apply continuous-variable-type entangled pure states. On the other hand, a coherent state is the simplest continuous-variable state. Based on coherent states, two types of continuous-variable states, the photon-added coherent states[23] and ECS,[24] have been introduced and shown to have wide applications in both quantum physics[25] and quantum information processing.[7,8,20,26] In a previous paper,[27] single-mode excited entangled coherent states (SMEECSs) are introduced. It has been shown that the SMEECSs form a type of cyclic representation of the Heisenberg–Weyl algebra and exhibit rich entanglement properties. The purpose of this paper is to propose the concept of two-mode excited entangled coherent states (TMEECSs), study their preparation and entanglement properties. This paper is organized as follows. In Section 2, we present the definition of the TMEECSs and discuss their preparation. In Section 3, we study entanglement character of the TMEECSs and compare them with the SMEECSs. We shall conclude this paper with discussions and remarks in the last section.

## 2. Two-mode excited entangled coherent states and their preparation

In this section we introduce the definition of the TMEECSs and present a possible scheme of producing them from the SMEECSs through atom–field interac-

*Project supported by the National Natural Science Foundation of China (Grant Nos 10775048 and 10325523), the National Fundamental Research Program of China (Grant No 2007CB925204), and the Funds from the Education Department of Hunan Province.
†Corresponding author. E-mail: lmkuang@hunnu.edu.cn

tion. Let us begin with the following two-mode ECSs:

$$|\Psi_{\pm}(\alpha,0)\rangle = \mathcal{A}_{\pm}(\alpha,0)(|\alpha,\alpha\rangle \pm |-\alpha,-\alpha\rangle), \quad (1)$$

where $|\alpha,\alpha\rangle = |\alpha\rangle\otimes|\alpha\rangle$ with $|\alpha\rangle = D(\alpha)|0\rangle$ being the usual Glauber coherent state defined by the action of the displacement operator $D(\alpha) = \exp(\alpha\hat{a}^{\dagger} - \alpha^{*}\hat{a})$ upon the vacuum state $|0\rangle$. The normalization constants are given by

$$\mathcal{A}_{\pm}^{-2}(\alpha,0) = 2\left[1 \pm \exp(-4|\alpha|^2)\right]. \quad (2)$$

For convenience, we denote the first and second modes in two-mode ECSs given by Eq.(1) by modes $a$ and $b$, respectively. Then for the ECSs we consider $m$-photon excitations in mode $a$ and $n$-photon excitations in mode $a$ with $m \neq 0$ and $n \neq 0$, respectively, and introduce the TMEECSs defined by

$$\begin{aligned}&|\Psi_{\pm}(\alpha,m,n)\rangle\\ &= \mathcal{N}_{\pm}(\alpha,m,n)\hat{a}^{\dagger m}\hat{b}^{\dagger n}(|\alpha,\alpha\rangle \pm |-\alpha,-\alpha\rangle),\end{aligned} \quad (3)$$

where $\hat{a}^{\dagger}$ and $\hat{b}^{\dagger}$ are the creation operators of the modes $a$ and $b$, respectively. It is straightforward to calculate the normalization constants in Eq.(3) with the following expressions

$$\begin{aligned}\mathcal{N}_{\pm}^{-2}(\alpha,m,n) = 2m!n!\,&\left[L_m(-|\alpha|^2)L_n(-|\alpha|^2)\right.\\ &\left.\pm L_m(|\alpha|^2)L_n(|\alpha|^2)\right],\end{aligned} \quad (4)$$

where $L_m(x)$ is a Laguerre polynomials of order $m$ defined by

$$L_m(x) = \sum_{n=0}^{m}\frac{(-1)^n m! x^n}{(n!)^2(m-n)!}. \quad (5)$$

In the derivation of the Eq.(4) we have used the following expressions:

$$\begin{aligned}\langle\alpha|\hat{a}^m\hat{a}^{\dagger m}|\alpha\rangle &= m!L_m(-|\alpha|^2),\\ \langle\alpha|\hat{a}^m\hat{a}^{\dagger m}|-\alpha\rangle &= m!L_m(|\alpha|^2), \quad (m\neq 0).\end{aligned} \quad (6)$$

If we introduce the following normalized states with respect to modes $a$ and $b$

$$\begin{aligned}|a(\pm\alpha,m)\rangle &= N(\alpha,m)\hat{a}^{\dagger m}|\pm\alpha\rangle,\\ |b(\pm\alpha,m)\rangle &= N(\alpha,m)\hat{b}^{\dagger m}|\pm\alpha\rangle,\end{aligned} \quad (7)$$

where the normalized coefficients are given by

$$N^{-2}(\alpha,m) = m!L_m(-|\alpha|^2), \quad (8)$$

then the TMEECSs $|\Psi_{\pm}(\alpha,m,n)\rangle$ in terms of four normalized states can be rewritten as

$$\begin{aligned}|\Psi_{\pm}(\alpha,m)\rangle = \mathcal{M}_{\pm}(\alpha,m,n)&[|a(\alpha,m)\rangle\otimes|b(\alpha,n)\rangle\\ &\pm|a(-\alpha,m)\rangle\otimes|b(-\alpha,n)\rangle],\end{aligned} \quad (9)$$

where the normalization constants are given by

$$\mathcal{M}_{\pm}^{2}(\alpha,m,n) = \frac{L_m(-|\alpha|^2)L_n(-|\alpha|^2)}{2\left[L_m(-|\alpha|^2)L_n(-|\alpha|^2) \pm L_m(|\alpha|^2)L_n(|\alpha|^2)\right]}. \quad (10)$$

We now present a possible scheme to produce them from the two-mode ECSs through atom–field interaction. Consider an interaction between a two-level atom with a cavity field. The atom makes a transition from the excited state $|e\rangle$ to the ground state $|g\rangle$ by emitting a photon. In the interaction picture, the Hamiltonian of resonant interaction is given by

$$\hat{H} = \hat{\sigma}_{+}\hat{b} + g^{*}\hat{\sigma}_{-}\hat{b}^{\dagger}, \quad (11)$$

where $\hat{\sigma}_{+}$ and $\hat{\sigma}_{-}$ are Pauli operator corresponding to the two-level atom, $\hat{b}^{\dagger}$ and $\hat{b}$ are the creation and annihilation operator of the field mode $b$, $g$ is the coupling constant.

Suppose that the atom is initially in the excited state and the two field modes are initially in the SMEECS with $m$ photon excitations ($m \neq 0$) given by

$$|\Psi_{\pm}(\alpha,m)\rangle = N_{\pm}(\alpha,m)\hat{a}^{\dagger m}(|\alpha,\alpha\rangle \pm |-\alpha,-\alpha\rangle), \quad (12)$$

where the normalization constant is given by

$$N_{\pm}^{-2}(\alpha,m) = 2m!\left[L_m(-|\alpha|^2) \pm \mathrm{e}^{-2|\alpha|^2}L_m(|\alpha|^2)\right], \quad (13)$$

where $L_m(x)$ is $m$-th Laguerre polynomial defined in Eq.(5).

Suppose that only the mode $b$ interacts with the atom. Then for the weak coupling case, the state of the atom–field system at time $t$ can be approximated by

$$|\psi(t)\rangle \approx |\Psi_{\pm}(\alpha,m)\rangle\otimes|e\rangle - \mathrm{i}\hat{H}t|\Psi_{\pm}(\alpha,m)\rangle\otimes|e\rangle, \quad (14)$$

which is approximately valid for interaction times such that $gt \ll 1$. Making use of Eq.(11), one can reduce Eq.(14) to

$$|\psi(t)\rangle \approx |\Psi_{\pm}(\alpha,m)\rangle \otimes |e\rangle - \mathrm{i}(g^*t)\hat{b}^{\dagger}|\Psi_{\pm}(\alpha,m)\rangle \otimes |g\rangle, \tag{15}$$

which indicates that if the atom is detected to be in the ground state $|g\rangle$, then after normalization the state of the two optical fields is reduced to the TMEECS $|\Psi_{\pm}(\alpha,m,1)\rangle$ given by Eq.(3). If we consider a succession of $m$ atoms through the cavity and if we detect all the atoms in the ground state $|g\rangle$, then the state of the two optical fields is reduced to the desired state, the TMEECS with $(m+n)$-photon excitations. Hence, we can, in principle, produce the TMEECS $|\Psi_{\pm}(\alpha,m,n)\rangle$.

## 3. The entanglement amount of TMEECS

In this section, we calculate the amount of entanglement of the TMEECSs and investigate the influence of the photon excitations on the entanglement of the TMEECSs. From Eq.(9) we can see that the TMEECSs are two-component entangled states. The degree of quantum entanglement of the two-state entangled states can be measured in terms of the concurrence[12,26,28] which is generally defined for discrete-variable entangled states to be[28]

$$\mathcal{C} = |\langle\Psi|\sigma_y \otimes \sigma_y|\Psi^*\rangle|, \tag{16}$$

where $|\Psi^*\rangle$ is the complex conjugate of $|\Psi\rangle$. The concurrence equals one for a maximally entangled state.

For continuous-variable-type entangled states like Eq.(9), we consider a general bipartite entangled state

$$|\psi\rangle = \mu|\eta\rangle \otimes |\gamma\rangle + \nu|\xi\rangle \otimes |\delta\rangle, \tag{17}$$

where $|\eta\rangle$ and $|\xi\rangle$ are *normalized* states of subsystem 1, $|\gamma\rangle$ and $|\delta\rangle$ *normalized* states of subsystem 2 with complex $\mu$ and $\nu$. Through transforming continuous-variable-type components to discrete orthogonal basis and making use of a Schmidt decomposition,[29] it finds the concurrence of the entangled state Eq.(17) given by the following expression:[12,30]

$$\mathcal{C} = \frac{2|\mu||\nu|\sqrt{(1-|p_1|^2)(1-|p_2|^2)}}{|\mu|^2+|\nu|^2+2\mathtt{Re}(\mu^*\nu\mathtt{p_1}\mathtt{p_2^*})}, \tag{18}$$

where the two overlapping functions are defined by

$$p_1 = \langle\eta|\xi\rangle, \quad p_2 = \langle\delta|\gamma\rangle. \tag{19}$$

For the case of the no-photon excitation ECSs defined in Eq.(1), making use of Eqs.(1) and (2) from Eqs.(18) and (19) we can find that the concurrence is given by

$$\mathcal{C}_-(\alpha,0) = 1, \quad \mathcal{C}_+(\alpha,0) = \frac{1-\mathrm{e}^{-4|\alpha|^2}}{1+\mathrm{e}^{-4|\alpha|^2}}, \tag{20}$$

which implies that the degree of entanglement of the ECS $|\Psi_-(\alpha,0)\rangle$ is independent of the state parameter $\alpha$, and it is a maximally entangled state while the amount of entanglement of the ECS $|\Psi_+(\alpha,0)\rangle$ is less than that of the ECS $|\Psi_-(\alpha,0)\rangle$. The concurrence $\mathcal{C}_+(\alpha,0)$ increases with $|\alpha|^2$, and the state $|\Psi_+(\alpha,0)\rangle$ approaches the maximally entangled coherent state with $\mathcal{C}_+(\alpha,0) \approx 1$ for the strong field case of the large $|\alpha|^2$.

When there exist photon excitations for both modes, i.e., $m \neq 0$ and $n \neq 0$, the TMEECSs are given by Eqs.(3) and (9). From Eqs.(18) and (19) we find the corresponding concurrence to be

$$\mathcal{C}_{\pm}(\alpha,m,n) = \frac{\sqrt{(1-|p_1(\alpha,m)|^2)(1-|p_2(\alpha,n)|^2)}}{1\pm p_1(\alpha,m)p_2(\alpha,n)}, \tag{21}$$

where the two overlapping functions are given by

$$p_1(\alpha,m) = \frac{L_m(|\alpha|^2)}{L_m(-|\alpha|^2)},$$
$$p_2(\alpha,n) = \frac{L_n(|\alpha|^2)}{L_n(-|\alpha|^2)}. \tag{22}$$

From Eqs.(21) and (22) we can see that the TMEECSs exhibit the exchanging symmetry with respect to the exchange of two-mode photon excitations,

$$\mathcal{C}_{\pm}(\alpha,m,n) = \mathcal{C}_{\pm}(\alpha,n,m),$$
$$\mathcal{C}_{\pm}(\alpha,m+k,n) = \mathcal{C}_{\pm}(\alpha,m,n+k), \tag{23}$$

where $k$ is an arbitrary non-negative integer. Equation (23) indicates that the allotment of photon excitations between two modes does not affect the entanglement amount of the TMEECSs when the total number of photon excitations are fixed.

Equation (21) indicates that the entanglement amount of the TMEECSs is determined by not only the two overlapping functions $p_1(\alpha,m)$ and $p_2(\alpha,n)$ but also the superposition phase between two coherent states $|\alpha,\alpha\rangle$ and $|-\alpha,-\alpha\rangle$, which is denoted by $\pm$ in Eq.(1). This implies that the quantum interference between two coherent states $|\alpha,\alpha\rangle$ and $|-\alpha,-\alpha\rangle$ in the TMEECSs can affect the entanglement amount of the TMEECSs.

In order to observe the influence of the photon excitations on the quantum entanglement of the TMEECS $|\Psi_{\pm}(\alpha,m,n)\rangle$, we consider the case of $m=n$ in which there are the same photon excitations in each field modes of the TMEECS.

Firstly, let us consider the TMEECS $|\Psi_{+}(\alpha,m,m)\rangle$. In this case we find the corresponding concurrence to be

$$\mathcal{C}_{+}(\alpha,m,m)=\frac{1-p_1^2(\alpha,m)}{1+p_1^2(\alpha,m)}, \tag{24}$$

which indicates that the amount of entanglement of the TMEECS $|\Psi_{\pm}(\alpha,m,m)\rangle$ decreases with the increase of the overlapping function $|p_1(\alpha,m)|$. In the weak field regime of $|\alpha|^2\ll 1$, we have $L_m(|\alpha|^2)\approx 1-m|\alpha|^2$. Then from Eqs.(22) and (24) we can get

$$\mathcal{C}_{+}(\alpha,m,m)\approx\frac{1-2m|\alpha|^2}{1+2m|\alpha|^2}, \tag{25}$$

which implies that the concurrence decreases with the increase of the two-mode photon excitations. Hence in the weak field regime the photon excitation can degrade the entanglement amount for the TMEECS $|\Psi_{+}(\alpha,m,m)\rangle$. However, when $m|\alpha|^2\ll 1$, we have $\mathcal{C}_{+}(\alpha,m,m)\approx 1$. This means that the TMEECS $|\Psi_{+}(\alpha,m,m)\rangle$ is approximately a maximally entangled state for enough weak field and low photon excitations.

It would be interesting to compare entanglement character of the TMEECS $|\Psi_{+}(\alpha,m,m)\rangle$ which contains $2m$ photon excitations of the SMEECS $|\Psi_{+}(\alpha,2m)\rangle$ which involves $2m$ photon excitations as well. The SMEECSs $|\Psi_{\pm}(\alpha,2m)\rangle$ with $m\neq 0$[27] are defined by

$$|\Psi_{\pm}(\alpha,2m)\rangle=\mathcal{N}_{\pm}(\alpha,2m)\hat{a}^{\dagger 2m}(|\alpha,\alpha\rangle\pm|-\alpha,-\alpha\rangle), \tag{26}$$

where the normalization constants Eq.(3) are given by

$$\mathcal{N}_{\pm}^{-2}(\alpha,2m)=4m!\left[L_{2m}(-|\alpha|^2)\pm \mathrm{e}^{-2|\alpha|^2}L_{2m}(|\alpha|^2)\right]. \tag{27}$$

The concurrence of the SMEECS $|\Psi_{+}(\alpha,2m)\rangle$ is given by the following expression:

$$\mathcal{C}_{+}(\alpha,2m)=\frac{\left[\left(1-\mathrm{e}^{-4|\alpha|^2}\right)\left(L_{2m}^2(-|\alpha|^2)-L_{2m}^2(|\alpha|^2)\right)\right]^{1/2}}{L_{2m}^2(-|\alpha|^2)+\mathrm{e}^{-2|\alpha|^2}L_{2m}^2(|\alpha|^2)}, \tag{28}$$

which implies that in the weak field regime of $|\alpha|^2\ll 1$, we can get

$$\mathcal{C}_{+}(\alpha,2m)=2\sqrt{2m}|\alpha|^2, \tag{29}$$

which indicates that the photon excitation can enhance the entanglement amount for the SMEECS $|\Psi_{+}(\alpha,m)\rangle$. In particular, when $m|\alpha|^2\ll 1$, we have $\mathcal{C}_{+}(\alpha,2m)\ll 1$. Therefore, in the weak field regime the TMEECS $|\Psi_{\pm}(\alpha,m,m)\rangle$ exhibits quite different entanglement character from that of the SMEECS $|\Psi_{+}(\alpha,m)\rangle$.

Secondly, we consider the TMEECS $|\Psi_{-}(\alpha,m,m)\rangle$. In this case the corresponding concurrence is found to be

$$\mathcal{C}_{-}(\alpha,m,m)=1, \tag{30}$$

which implies that the TMEECS $|\Psi_{-}(\alpha,m,m)\rangle$ is always a maximally entangled state and $2m$ photon excitations do not affect the entanglement amount of the state. This property is very different from that of the SMEECS $|\Psi_{-}(\alpha,2m)\rangle$ defined in Eq.(26). The concurrence of the SMEECS $|\Psi_{-}(\alpha,2m)\rangle$ is given by

$$\mathcal{C}_{-}(\alpha,2m)=\frac{\left[\left(1-\mathrm{e}^{-4|\alpha|^2}\right)\left(L_{2m}^2(-|\alpha|^2)-L_{2m}^2(|\alpha|^2)\right)\right]^{1/2}}{L_{2m}^2(-|\alpha|^2)-\mathrm{e}^{-2|\alpha|^2}L_{2m}^2(|\alpha|^2)}. \tag{31}$$

In the weak field regime of $|\alpha|^2 \ll 1$, we find that

$$\mathcal{C}_-(\alpha, 2m) \approx \frac{\sqrt{2m}}{4m + (1 - 2m|\alpha|^2)}, \qquad (32)$$

which indicates that when $m|\alpha|^2 \ll 1$, we have $\mathcal{C}_-(\alpha, 2m) \approx \sqrt{2m}/(1 + 4m)$. This means that the concurrence $\mathcal{C}_-(\alpha, 2m)$ decreases with the increase of the photon excitation number $2m$. In particular, when $m \gg 1$ we have $\mathcal{C}_-(\alpha, 2m) \ll 1$. Hence, the photon excitation suppresses the amount of entanglement for the SMEECS $|\Psi_-(\alpha, 2m)\rangle$ in the weak field regime.

## 4. Concluding remarks

We have proposed two types of TMEECSs $|\Psi_\pm(\alpha, m, n)\rangle$, studied their entanglement characteristics, and investigated the influence of photon excitations on quantum entanglement. We have indicated that it is possible to produce such states by using cavity quantum electrodynamics and quantum measurements. It is found that the two TMEECSs $|\Psi_\pm(\alpha, m, n)\rangle$ exhibit quite different entanglement properties. In particular, for the state $|\Psi_+(\alpha, m, m)\rangle$ the two-mode photon excitations affect seriously entanglement character, and the entanglement amount decreases with the two-mode photon excitations in the weak field regime. However, the state $|\Psi_-(\alpha, m, m)\rangle$ is always a maximally entangled state, the two-mode photon excitations do not change the entanglement amount of the state. Physically, this can be understood as a quantum interference effect between two coherent states $|\alpha, \alpha\rangle$ and $|-\alpha, -\alpha\rangle$ in the TMEECSs. We have also made comparisons between the TMEECSs $|\Psi_\pm(\alpha, m, m)\rangle$ and the SMEECSs $|\Psi_\pm(\alpha, 2m)\rangle$. It has been shown that two-mode photon excitations have more advantages than single-mode photon excitations such as the entanglement amount of the TMEECSs $|\Psi_\pm(\alpha, m, m)\rangle$ is larger than that of the SMEECSs $|\Psi_\pm(\alpha, 2m)\rangle$ for the same photon excitation number $2m$. This approach of enhancing entanglement by using two-mode excitations of continuous-variable quantum sates opens a new way to create new entanglement resources with continuous variables.

## References


[1] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[2] Cerf N J, Leuchs G and Polzik E S 2007 *Quantum Information with Continuous Variables of Atoms and Light* (London: Imperial College Press)
[3] Su X, Tan A, Jia X, Zhang J, Xie C and Peng K 2007 *Phys. Rev. Lett.* **98** 070502
[4] Chen H and Zhang J 2007 *Phys. Rev.* A **75** 022306
[5] Zhang J, Xie C and Peng K 2005 *Phys. Rev. Lett.* **95** 170501
[6] Braunstein S L and Loock P 2005 *Rev. Mod. Phys.* **77** 513
[7] Jeong H and Kim M S 2002 *Phys. Rev.* A **65** 042305
[8] van Enk S L and Hirota O 2001 *Phys. Rev.* A **64** 022313
[9] Ban M 1999 *J. Opt. B* **1** L9
[10] Braunstein S L and Kimble H J 1998 *Phys. Rev. Lett.* **80** 869
[11] Furusawa A, Sørensen J L, Braunstein S L, Fuchs C A, Kimble H J and Polzik E S 1998 *Science* **282** 706
[12] Kuang L M and Zhou L 2003 *Phys. Rev.* A **68** 043606
[13] Zhou L and Kuang L M 2004 *Chin. Phys. Lett.* **21** 2101
[14] Zhou L and Kuang L M 2003 *Phys. Lett.* A **315** 426
[15] Zhou L and Kuang L M 2004 *Phys. Lett.* A **330** 48
[16] Cai X H and Kuang L M 2002 *Phys. Lett.* A **302** 273
[17] Guo Y and Kuang L M 2005 *Chin. Phys. Lett.* **22** 595
[18] Zeng A H and Kuang L M 2005 *Phys. Lett.* A **338** 323
[19] Kuang L M, Zeng A H and Kuang Z H 2003 *Phys. Lett.* A **319** 24
[20] Munro W J, Milburn G J and Sanders B C 2000 *Phys. Rev.* A **62** 052108
[21] Liao J Q and Kuang L M 2006 *Chin. Phys.* **15** 2246
[22] Cai X H and Kuang L M 2002 *Chin. Phys.* **11** 876
[23] Agarwal G S and Tara K 1991 *Phys. Rev.* A **43** 492
[24] Sanders B C 1992 *Phys. Rev.* A **45** 6811
[25] Zavatta A, Viciani S and Bellini M 2004 *Science* **306** 660
Zavatta A, Viciani S and Bellini M 2005 *Phys. Rev.* A **72** 023820
[26] Wang X G 2002 *J. Phys.* A: *Math. Gen.* **35** 165
Wang X G 2001 *Phys. Rev.* A **64** 022302
[27] Xu L and Kuang L M 2006 *J. Phys.* A: *Math. Gen.* **39** L191
[28] Hill S and Wootters W K 1997 *Phys. Rev. Lett.* **78** 5022
Wootters W K 1998 *Phys. Rev. Lett.* **80** 2245
[29] Mann A, Sanders B C and Munro W J 1995 *Phys. Rev.* A **50** 989
[30] Rungta P, Buzěk V, Caves C M, Hillery M and Milburn G J 2001 *Phys. Rev.* A **64** 042315