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Fatigue life of metal treated by magnetic field

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This paper investigates theoretically the influence of magnetization on fatigue life by using non-equilibrium statistical theory of fatigue fracture for metals. The fatigue microcrack growth rate is obtained from the dynamic equation of microcrack growth, where the influence of magnetization is described by an additional term in the potential energy of microcrack. The statistical value of fatigue life of metal under magnetic field is derived, which is expressed in terms of magnetic field and macrophysical as well as microphysical quantities. The fatigue life of AISI 4140 steel in static magnetic field from this theory is basically consistent with the experimental data.

Keywords: fatigue life, non-equilibrium statistical theory, magnetic field
PACC: 8140N, 6220M

1. Introduction

Decreasing fatigue damage has been a very important research subject. Recently, the method of improving fatigue life of materials by magnetic or electric field has been proposed,[1] and the research in the effects of magnetic treatment on the fatigue life has been conducted.[2−5] Experiments have confirmed that the magnetic field can have an influence on the fatigue damage of metals. Lü *et al*[2] reported that the magnetic field could repair fatigue damage under some conditions. Bhat *et al*[3] found the decrease of fatigue life of mild steel subjected to sustained magnetic field with increasing magnetic field. Fahmy *et al*[4] reported that the magnetic pulse could increase fatigue life of low carbon steel specimens. They suggested that the increase is caused by the influence of magnetic field on domains. Ayhan Çelik *et al*[5] investigated the effect of magnetic field on fatigue life of AISI 4140 steel. They obtained that fatigue life improved when magnetic field was applied especially to the specimens at the stage of fatigue crack initiation. The magnetic field was detrimental on fatigue life if it was applied through the whole process of the test due to overheating.

Although the change in fatigue life of specimen has been observed in experiments, theoretical research in this aspect is very few. It is known that the fatigue life of specimen relates to the initiation and propagation of microcrack and that there is a large scatter in fatigue life data. During fatigue damage process, the internal micro cracks of various sizes in metals nucleate and grow due to the movement and interaction of dislocations in the initial stage, the fatigue fracture happens after a main crack is generated and propagates with a high speed.[6,7] The growth rate of fatigue crack and fatigue life of metal are always structure sensitive, regularly scatter and have size effect because of the inhomogeneity in microscopic compositions, defects, structures and plastic deformation of metals. The fatigue fracture process is an irreversible and non-equilibrium process, stochastic theory of fracture has been used to deal with the fracture damage.[8−10] In this paper, we took a step in the direction of using non-equilibrium statistical theory on fatigue damage to derive an analytical expression of fatigue life of metal under magnetic field.

2. The dynamic equation of transportation growth for micro cracks in magnetic field

The mechanisms of influence of magnetic field on fatigue damage may be different for a metal under different conditions or for different metals, but magnetization of metal due to the presence of magnetic field is one of the most common factors that must be considered in the fatigue damage process. In this paper, we focus on effects caused by magnetization on fatigue damage at the stage of fatigue crack initiation only.

The potential energy of a system with a one di-

mensional fatigue microcrack of unit width has been obtained.[9−11] In the presence of magnetic field, magnetization of metal should be considered. The potential energy U of a system subjected to magnetic field with a one-dimensional microcrack of unit width is written as

$$U = 2\gamma c - \frac{\pi\left(1-\nu^2\right)\sigma_{\rm a}^2 c^2}{2E} - \frac{nbc\sigma_{\rm a}}{2} - \frac{(nb)^2 E}{8\pi\left(1-\nu^2\right)} + 2\lambda c, \tag{1}$$

where c is the length of the micro crack, γ is the surface energy, ν is Poisson's ratio, n is the number of dislocations squeezed in the microcrack wedge, b is the magnitude of Burger's vector, $\sigma_{\rm a}$ is the applied tensile stress normal to the microcrack, E is Young's modulus. The last term in the right side of Eq.(1) is originated from the presence of magnetic field. It equals to the negative work done by the field with the opening of the crack tip.

$$2\lambda c = -(nb)\, c \int_M^0 \mu_0 \boldsymbol{H}\cdot \mathrm{d}\boldsymbol{M}, \tag{2}$$

where $\boldsymbol{H}$ is the magnetic field, $\boldsymbol{M}$ is magnetization intensity of metal, μ_0 is permeability of free space. From Eq.(2), we obtain

$$\lambda = \frac{\mu_0 nb}{2}\int_0^M \boldsymbol{H}\cdot \mathrm{d}\boldsymbol{M} = nb\varsigma, \tag{3}$$

$$\varsigma = \frac{\mu_0}{2}\int_0^M \boldsymbol{H}\cdot \mathrm{d}\boldsymbol{M}. \tag{4}$$

The value of λ can be determined via Eq.(3) if the function of the magnetization intensity varying with magnetic field is given for a specific metal. It can be seen from Eq.(3) that λ is a function of magnetic field H. Consider a simple example. Assume that the directions of magnetic field and magnetization intensity are the same and magnetic permeability μ does not vary with magnetic field. Then magnetization intensity can be related to magnetic field by $\boldsymbol{M} = (\mu_{\rm r}-1)\boldsymbol{H}$, and Eq.(3) yields

$$\lambda = \frac{1}{4} nb\,(\mu_{\rm r}-1)\,\mu_0 H^2, \tag{5}$$

where $\mu_{\rm r}$ is relative permeability of metal. In general, $\mu_{\rm r}$ can be a function of temperature and magnetic field, and depends on chemical compositions of metal also.

It is obtained from Eq.(1) that if

$$nb = n_{\rm k} b = \frac{2\pi\left(1-\nu^2\right)\sigma_{\rm a} c}{E}, \tag{6}$$

$$c = c_{\rm k} = \frac{(\gamma+\lambda)E}{\pi\left(1-\nu^2\right)\sigma_{\rm a}^2}, \tag{7}$$

the potential energy of a microcrack has the maximum value. If $c > c_{\rm k}$, then the microcrack would propagate with high speed. $c_{\rm k}$ is the critical length of the fatigue crack in magnetic field.

The kinetic energy of a system with a fatigue microcrack of length c propagating at speed $\dot{c}$ is[12]

$$K = \frac{\kappa\rho c^2 \dot{c}^2 \sigma_{\rm a}^{\frac{4}{1+m}}}{2E\sigma_{\rm A}^{\frac{2}{1+m}}}, \tag{8}$$

where ρ is the density of metal, m is the strain-hardening exponent of the metal, $\sigma_{\rm A}$ denotes the strength coefficient of the metal, $\kappa = 5.45$ and $\dot{c} = \dfrac{\mathrm{d}c}{\mathrm{d}t}$ is the growth rate of microcrack c at time t. Applying conservation of energy, we obtain

$$\frac{\mathrm{d}\,(U+K)}{\mathrm{d}N} = 0. \tag{9}$$

For cyclic stress, the change in stress per cycle is zero. Considering the condition and substituting Eqs.(1) and (8) into Eq.(9), we obtain the dynamic equation of transportation growth for micro cracks in magnetic field

$$\begin{aligned} &c\dot{c}_N\ddot{c}_N + \dot{c}_N^3 + \frac{T_N^2 E\sigma_{\rm A}^{2/(1+m)}}{\kappa\rho\sigma_{\rm a}^{4/(1+m)} c} \\ &\times\left[2(\gamma+\lambda) - \frac{nb\sigma_{\rm a}}{2} - \frac{\pi\left(1-\nu^2\right)\sigma_{\rm a}^2 c}{E}\right]\dot{c}_N \\ &-\frac{T_N^2 E\sigma_{\rm A}^{2/(1+m)}}{\kappa\rho\sigma_{\rm a}^{4/(1+m)} c}\left[\frac{\sigma_{\rm a} c}{2} + \frac{nbE}{4\pi\left(1-\nu^2\right)} - 2\varsigma c\right]\frac{\mathrm{d}\,(nb)}{\mathrm{d}N} \\ &+\frac{2T_N^2 E\sigma_{\rm A}^{2/(1+m)}\dot{\lambda}_H \dot{H}_N}{\kappa\rho\sigma_{\rm a}^{4/(1+m)}} = 0, \end{aligned} \tag{10}$$

where $\dot{c}_N = \dfrac{\mathrm{d}c}{\mathrm{d}N} = \dot{c}T_N$, $\dot{H}_N = \dfrac{\mathrm{d}H}{\mathrm{d}N} = \dfrac{\mathrm{d}H}{\mathrm{d}t}T_N$, $\ddot{c}_N = \dfrac{\mathrm{d}\dot{c}_N}{\mathrm{d}N}$, T_N is the period of the external stress, T_N is the period of magnetic field.

3. Fatigue microcrack growth rate

For the fatigue microcrack evolution process, the growth rate $\dot{c}_N$ is small, so $\dot{c}_N^3$ and $\ddot{c}_N$ are negligible in Eq.(10). We obtain the fatigue microcrack growth rate from Eq.(10) with the approximation,

$$\dot{c}_N = \frac{\left[\dfrac{\sigma_{\rm a} c}{2} + \dfrac{nbE}{4\pi\left(1-\nu^2\right)} - 2\varsigma c\right]\dfrac{\mathrm{d}\,(nb)}{\mathrm{d}N} - 2\dot{\lambda}_H \dot{H}_N c}{2(\gamma+\lambda) - \dfrac{nb\sigma_{\rm a}}{2} - \dfrac{\pi\left(1-\nu^2\right)\sigma_{\rm a}^2 c}{E}}. \tag{11}$$

Assuming that N_0 is the mean mobile dislocation sources per unit volume and L is the length of the slip plane, σ_0 is the coefficient of applied periodical strength, β is cyclic hardening exponent, then we have[10]

$$\frac{\mathrm{d}\,(nb)}{\mathrm{d}N} = \frac{1}{N_0 L^2}\left(\frac{\sigma_\mathrm{a}}{\sigma_0}\right)^{1/\beta}. \tag{12}$$

At the end of microcrack evolution process, $c \approx c_\mathrm{k}$, the number of dislocations squeezed in the crack is derived from Eq.(6), that yields

$$\frac{n_\mathrm{k} b E}{4\pi\,(1-\nu^2)} = \frac{\sigma_\mathrm{a} c}{2}. \tag{13}$$

Substituting Eqs.(12) and (13) into Eq.(11), and considering $2\,(\gamma+\lambda) \gg \dfrac{2\pi\left(1-\nu^2\right)\sigma_\mathrm{a}^2 c}{E}$, we obtain the fatigue microcrack growth rate in magnetic field H

$$\begin{aligned}\dot{c}_N &= \frac{(\sigma_\mathrm{a}+2\varsigma)\sigma_\mathrm{a}^{1/\beta} c - 2\dot{\lambda}_H \dot{H}_N N_0 L^2 \sigma_0^{1/\beta} c}{2\,(\gamma+\lambda)\,N_0 L^2 \sigma_0^{1/\beta}} \\ &= Ac, \end{aligned} \tag{14}$$

$$A = \frac{(\sigma_\mathrm{a}+2\varsigma)\sigma_\mathrm{a}^{1/\beta} - 2\dot{\lambda}_H \dot{H}_N N_0 L^2 \sigma_0^{1/\beta}}{2\,(\gamma+\lambda)\,N_0 L^2 \sigma_0^{1/\beta}}. \tag{15}$$

Equation (14) is the microcrack growth rate that is proportional to the length of fatigue microcrack in the evolution process. It can be seen from Eq.(14) that the growth rate of microcrack is influenced by magnetic field because it varies with the time rate of the magnetic field, ς and λ.

4. Fatigue life

The quantities N_0, L, γ, and σ_0 are structure sensitive, they fluctuates about their averages due to the inhomogeneous fluctuation of microstructure about an average background in metals. N_0, L, γ, and σ_0 used in previous calculation should be regarded as the averages. Assuming that the deviations of N_0, L, γ, and σ_0 are ΔN_0, ΔL, $\Delta\gamma$, and $\Delta\sigma_0$ respectively, replacing N_0, L, γ, and σ_0 in Eq.(14) with $N_0 \pm \Delta N_0$, $L \pm \Delta L$, $\gamma \pm \Delta\gamma$, and $\sigma_0 \pm \Delta\sigma_0$, considering $\dfrac{\Delta N_0}{N_0} \ll 1$, $\dfrac{\Delta\gamma}{\gamma} \ll 1$, $\dfrac{\Delta L}{L} \ll 1$, $\dfrac{\Delta\sigma_0}{\sigma_0} \ll 1$ and neglecting the deviations of higher order, we obtain

$$\dot{c}_N = Ac \pm Ac\eta, \tag{16}$$

where $\eta = \left(1-\dfrac{\alpha}{A}\right)\left(\dfrac{\Delta N_0}{N_0} + \dfrac{2\Delta L}{L} + \dfrac{\Delta\sigma_0}{\beta\sigma_0}\right) + \dfrac{\Delta\,(\gamma+\lambda)}{(\gamma+\lambda)}$ and $\alpha = \dfrac{\dot{\lambda}_H \dot{\mathrm{H}}_N}{(\gamma+\lambda)}$.

Using parameters A and η derived above, we find that the fatigue life in magnetic field can be calculated by the non-equilibrium theory about fatigue damage,[13] it is

$$\begin{aligned}\bar{N}_\mathrm{f} = {} & \frac{N_0 L^2\,(\gamma+\lambda)\,\sigma_0^{1/\beta}}{6.24\eta\left[(\sigma_\mathrm{a}-2\varsigma)\sigma_\mathrm{a}^{1/\beta} - 2\dot{\lambda}_H \dot{H}_N N_0 L^2 \sigma_0^{1/\beta}\right]} \\ & \times \left[\ln\frac{(\gamma+\lambda)\,E}{\pi\,(1-\nu^2)\,c_0\sigma_\mathrm{a}^2}\right]^2 \left[\frac{1}{Y\,(N)}\right]^{\frac{1}{8}}, \end{aligned} \tag{17}$$

where $Y(N)$ is the total number of microcrack in the Nth cycle. The statistical value of fatigue life in magnetic field is described by Eq.(17), it changes with magnetic field besides with the other mechanical quantities, because the parameters λ and ς in Eq.(17) are functions of magnetic field. From the Eq.(17), it can be seen that statistical value of fatigue life is determined also by the amplitude of cyclic stress σ_a and other characteristic quantities of metal, for example N_0, γ, L, σ_0, β and initial microcrack length c_0.

For static magnetic fields, $\dot{H}_N = 0$, the fatigue life of specimen is

$$\begin{aligned}\bar{N}_\mathrm{f} = {} & \frac{N_0 L^2\,(\gamma+\lambda)\,\sigma_0^{1/\beta}}{6.24\eta(\sigma_\mathrm{a}-2\varsigma)\sigma_\mathrm{a}^{1/\beta}} \\ & \times \left[\ln\frac{(\gamma+\lambda)\,E}{\pi\,(1-\nu^2)\,c_0\sigma_\mathrm{a}^2}\right]^2 \left[\frac{1}{Y\,(N)}\right]^{\frac{1}{8}}. \end{aligned} \tag{18}$$

It can be seen from the equation that the fatigue life of specimen can be increased under the static magnetic field if λ is positive. In this case, the fatigue life is larger in higher magnetic field because λ increases with the magnetic field, which can be seen form Eq.(3). The result is proved by some experiments,[4,5] i.e. the data in columns A, B, C of Table 2 in Ref.[4]. The experiment of Ref.[4] also shows that applying continuous magnetic field has a negative effect on fatigue life, but authors point out that the reason may be over heat which is a factor not being considered in the paper.

We calculated the statistical fatigue life of AISI40 steel in magnetic field of 800 G and 1300 G from Eq.(18) and plot the $S-N$ curve as shown in Fig.1. The parameters are $N_0 = 3\times10^9\,\mathrm{mm}^{-3}$, $L = 0.3\,\mathrm{mm}$, $\sigma_0 = 655\,\mathrm{MPa}$, $\gamma = 10\,\mathrm{J/m}^2$, $Y(N) = 10^4$, $E = 210\,\mathrm{GPa}$, $\nu = 0.27$, $c_0 = 10^{-4}\,\mathrm{mm}$, $\eta = 0.3$, $\mu_\mathrm{r} = 45$, $nb = 0.09\,\mathrm{mm}$, $\beta = 0.1$. The experimental fatigue lives of AISI 4140 steel in the magnetic field of 800 G and 1300 G under the stress 630 MPa, 670 MPa, 710 MPa from Ref.[4] are shown in the Fig.2 represented by asterisks for 800 G and dots for 1300 G. It can be seen

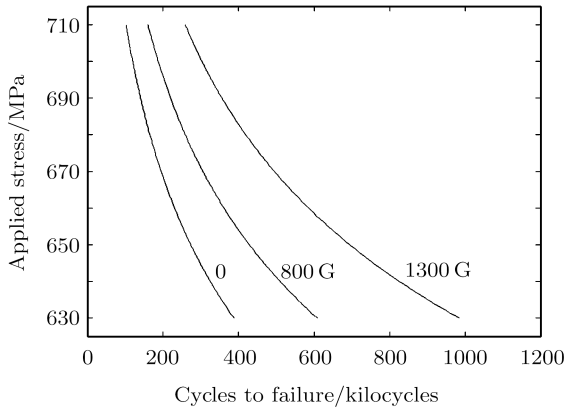


Fig.1. $S-N$ curve for AISI4140.

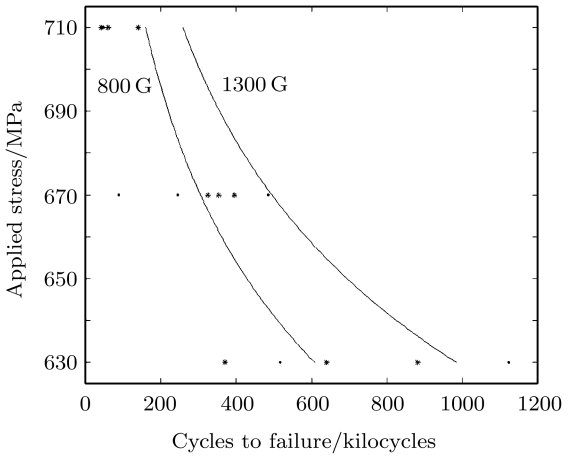


Fig.2. Experimental data from Ref.[4] and $S-N$ curve from the theory for AISI4140 steel under magnetic field of 800 G and 1300 G.

from Fig.2 that the theoretical values from Eq.(18) under the stress 630 MPa, 670 MPa, and 710 MPa are basically consistent with the known experimental data. The experiment on fatigue life influenced by magnetic field is relatively limited till now, so the results in the paper need to be confirmed further in the future.

5. Conclusions

Based on the fact that fatigue fracture is an irreversible process and magnetization mechanism, the fatigue fracture of metals subjected to magnetic field is investigated theoretically in this paper. Using the non-equilibrium statistical theory of fatigue fracture, we derived the analytical expression of microcrack growth rate and the statistical value of fatigue life of metals under magnetic field due to magnetization in terms of magnetic field and macro-physical as well as micro-physical quantities. The theory gives an explanation on the enhancement of fatigue life of metals treated by magnetic field and the fatigue life derived for AISI40 steel is basically consistent with experimental results.

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