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Analysis of space harmonic selectivity of period-varying folded waveguide*

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A period-varying folded waveguide is formed by varying the period of a folded waveguide. It has the advantages of the space harmonic selectivity and the wide bandwidth. However, the regularities of the variety of these period-varying folded waveguides are unavailable from the published papers. In order to solve this problem, the principle of the space harmonic selectivity of a period-varying folded waveguide is analysed, and the conditions to select the space harmonic for this slow wave system are obtained. In addition, the space harmonic selectivities for a linear period-varying folded waveguide and a hyperbolic sine-varying period folded waveguide are also analysed as examples.

Keywords: varying period, folded waveguide, harmonic selection
PACC: 7870G, 1220, 8760F

1. Introduction

The folded waveguide circuit has a series of advantages over other slow wave structures, such as low-cost fabrication, high strength, simple input and output circuits, and high-power handing capability with a moderate bandwidth in a millimetre wave range.[1−3] Therefore, the folded waveguide travelling wave tube is an appropriate device that may be an answer to the problem of achieving millimetre wave amplification with increased power and bandwidth in combination. However, there is still room for improvement in the coupling impedance and the frequency bandwidth.

A period-varying folded waveguide is formed by varying the period of a folded waveguide. Some problems about the period-varying slow-wave structure have been studied,[4−7] and the obtained results show that the period-varying folded waveguides not only retain their inherent advantages, but also they have the space harmonic selectivity and wide bandwidth additionally. However, the conditions under which the period-varying folded waveguides can surely have the space harmonic selectivity, have not been presented in the published papers.

In this paper, the principle of the space harmonic selectivity of the period-varying folded waveguides is presented, and the relation between the harmonic selectivity and the size of the period-varying folded waveguide is achieved. Finally, the space harmonic selectivities for a linear period-varying folded waveguide and a hyperbolic sine-varying period folded waveguide are analysed as examples. It means that a period-varying folded waveguide can select a space harmonic if its structure satisfies the conditions obtained in this paper.

2. The principle of space harmonic selectivity in period-varying folded waveguides

A schematic diagram of a period-varying folded waveguide is shown in Fig.1, where Q is the period number, L is the total length of the period-varying folded waveguide, and v_e is the electron beam velocity. In this structure, the length of the qth period is $L_q = (3l_q/2)+(l_{q+1}/2)$, $q = 1, 2, \ldots, Q$, where l_q is the length between the gaps of the qth period. Obviously, this structure is a quasi-periodic one.

We make the following assumptions:

a) This period-varying folded waveguide is a lossless system.

b) The TE_{10} mode propagates in this period-varying folded waveguide.

c) The radius of the electron beam is infinitely small. The change of the field in the transverse direction is not considered while the electron beam interacts with the wave.

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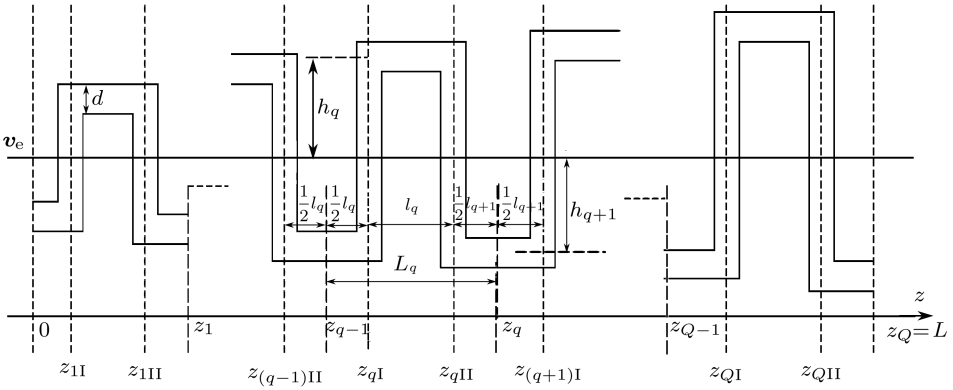


Fig.1. Period-varying folded waveguide.

In a straight waveguide, the TE_{10} mode does not have a longitudinal electric field. But in a folded waveguide, because of the bending of the waveguide, the transverse electric field of the TE_{10} mode is in the same direction as that of the velocity of the electron beam. So we can assume that the electric field that is seen by the electron beam is as follows:

$$E(z) = \begin{cases} E_0 \mathrm{e}^{\mathrm{j}\psi(z)}, & z_{q\mathrm{I}} + d/2 > z > z_{q\mathrm{I}} - d/2, \quad z_{q\mathrm{II}} + d/2 > z > z_{q\mathrm{II}} - d/2; \\ 0, & z_{\mathrm{I}} - d/2 > z > z_{(q-1)\mathrm{II}} + d/2, \ z_{q\mathrm{II}} - d/2 > z > z_{q\mathrm{I}} + d/2, \end{cases} \quad q = 1, 2, ..., Q,$$

$$\psi(z) = \psi_{qk}, \quad k = \mathrm{I}, \mathrm{II}, \tag{1}$$

where $\psi(z)$ is the phase of the qk'th gap, $q = 1, 2, \ldots, Q$, $k = \mathrm{I}$ is the first gap in each period, $k = \mathrm{II}$ is the last gap in each period, and it is determined by the wave propagating in the waveguide.

Above all, it is known that the electron beam only interacts with the longitudinal electric field in the gaps. Therefore, studying period-varying folded waveguides requires the analysis of the longitudinal electric fields of the waves propagating in these systems.

Using the Fourier transform, we define the longitudinal electric field as

$$E(z) = \frac{L}{2\pi} \int_{-\infty}^{+\infty} E(\beta) \mathrm{e}^{\mathrm{j}\beta z} \mathrm{d}\beta,$$
$$E(\beta) = \frac{1}{L} \int_0^L E(z) \mathrm{e}^{-\mathrm{j}\beta z} \mathrm{d}z. \tag{2}$$

It is noticed that the space harmonic with the relation $\beta_n = \beta_0 + 2n\pi/l$ does not exist in this system, because the period is variable. However, there are space harmonics in this system as a consequence of the regular change in period. In expression (2), β is the longitudinal propagation constant.

Substituting expression (1) into expression (2) yields

$$E(\beta) = \frac{MU}{L} \left\{ \sum_{q=1}^{Q} [\mathrm{e}^{\mathrm{j}(\psi_{q\mathrm{I}} - \beta z_{q\mathrm{I}})} + \mathrm{e}^{\mathrm{j}(\psi_{q\mathrm{II}} - \beta z_{q\mathrm{II}})}] \right\},$$
$$M = \sin(\beta d/2) \sin(\beta d/2), \qquad U = E_0 d, \tag{3}$$

where M is the electron–field interaction coefficient in the gap, and U is the gap voltage.

Because the direction of the electric field changes after every half the period in the folded waveguide, one can have

$$\psi_{q\mathrm{II}} - \psi_{q\mathrm{I}} = \beta_{\mathrm{s}}(l_q + 2h_q) + \pi, \quad q = 1, 2, ..., Q, \tag{4}$$

where β_{s} is the propagation constant of the wave propagating along the waveguide.

In order to simplify the expression, we can change it as follows:

$$\psi_{q\mathrm{I}} - \beta z_{q\mathrm{I}} = \sum_{j=1}^{q-1} (s_j \beta_{\mathrm{s}} - L_j \beta) + \beta_{\mathrm{s}}(l_q/2 + h_q) - \beta l_q/2, \quad q = 1, 2, \ldots, Q,$$
$$S_q = 3l_q/2 + l_{q+1}/2 + 3h_q + h_{q+1},$$

$$L_q = 3l_q/2 + l_{q+1}/2, \tag{5}$$

where S_q is the length of the qth period along the waveguide.

Substituting expressions (4) and (5) into expression (3) yields

$$E(\beta) = \frac{MU}{L}\sum_{q=1}^{Q}\left\{ \mathrm{e}^{\mathrm{j}\left[\sum_{j=1}^{q-1}(s_j\beta_\mathrm{s}-L_j\beta)+\beta_\mathrm{s}(l_q/2+h_q)-\beta l_q/2\right]} + \mathrm{e}^{\mathrm{j}\left[\sum_{j=1}^{q-1}(s_j\beta_\mathrm{s}-L_j\beta)+\pi+\beta_\mathrm{s}(3l_q/2+3h_q)-3\beta l_q/2\right]}\right\}. \tag{6}$$

The electron–field interaction coefficient in the gap M is directly proportional to the amplitude of the wave synchronized with the electron beam.[8] In order to obtain the maximal $E(\beta)$, one needs to synchronize the space harmonic with the electron beam, i.e. $\beta = \beta_\mathrm{max}$ by using

$$\begin{aligned} &\sum_{j=1}^{q-1}(s_j\beta_\mathrm{s} - L_j\beta_\mathrm{max}) + \beta_s(l_q/2 + h_q) - \beta_\mathrm{max}l_q/2 = 2n_{1q}\pi, \\ &\sum_{j=1}^{q-1}(s_j\beta_\mathrm{s} - L_j\beta_\mathrm{max}) + \pi + \beta_\mathrm{s}(3l_q/2 + 3h_q) - 3\beta_\mathrm{max}l_q/2 = 2n_{2q}\pi, \end{aligned} \qquad q = 1, 2, \ldots, Q, \tag{7}$$

where n_{1q} and n_{2q} are arbitrary integers.

According to expression (7), and assuming that β_max has one and only one solution, the rank of the coefficient matrix must equal the rank of the augmented matrix, and it should be equal to 2.

Using the elementary transformation, one can obtain

$$\begin{bmatrix} l_1/2 + h_1 & -l_1/2 & 2n_{11}\pi \\ s_1 & -L_1 & 2[3(n_{12} - n_{11})/2 - (n_{22} - n_{21})/2]\pi \\ s_2 & -L_2 & 2[3(n_{13} - n_{12})/2 - (n_{23} - n_{22})/2]\pi \\ \vdots & \vdots & \\ s_{Q-1} & -L_{Q-1} & 2[3(n_{1Q} - n_{1(Q-1)})/2 - (n_{2Q} - n_{2(Q-1)})/2]\pi \\ l_1 + 2h_1 & -l_1 & 2(n_{21} - n_{11})\pi - \pi \\ (l_2 + 2h_2) - (l_1 + 2h_1) & -(l_2 - l_1) & 2[(n_{22} - n_{21}) - (n_{12} - n_{11})]\pi \\ (l_3 + 2h_3) - (l_2 + 2h_2) & -(l_3 - l_2) & 2[(n_{23} - n_{22}) - (n_{13} - n_{12})]\pi \\ \vdots & \vdots & \\ (l_Q + 2h_Q) - (l_{Q-1} + 2h_{Q-1}) & -(l_Q - l_{Q-1}) & 2[(n_{2Q} - n_{2(Q-1)}) - (n_{1Q} - n_{1(Q-1)})]\pi \end{bmatrix}.$$

To transform this matrix into a matrix in the normalized form, all the lines except the 1st and Q+1th lines need to be zero. So, one can obtain

$$\begin{aligned} \frac{l_1}{l_1 + 2h_1} &= \frac{L_p}{S_p}, \\ \frac{l_1}{l_1 + 2h_1} &= \frac{l_{p+1} - l_p}{(l_{p+1} + 2h_{p+1}) - (l_p + 2h_p)}, \\ p &= 1, 2, \ldots, Q - 1. \end{aligned} \tag{8}$$

Equation (8) can also be expressed as

$$\frac{S_p}{L_p} = 1 + 2\frac{h_{p+1} - h_p}{l_{p+1} - l_p}. \tag{9}$$

Equation (9) can be expressed as

$$\frac{h_{p+1}}{h_p} = \frac{l_{p+1}}{l_p}, \quad p = 1, 2, ..., Q - 1. \tag{10}$$

It can be found that if Eq.(9) or (10) can be satisfied, then only one space harmonic will exist in the period-varying folded waveguide, and it will have maximal amplitude $E(\beta_\mathrm{max}) = 2UMQ/L$.

3. Analysis of space harmonics in period-varying folded waveguides

Period-varying folded waveguides can be classified into several groups according to the regular variation of the period. Studied in this paper are a linear period-varying folded waveguide ($L_q = L_{q-1} + \Delta L$) and a hyperbolic sine-varying period folded waveguide ($l_q = \mathrm{sh}(aq)$, a is constant) as examples to analyse the space harmonic in period-varying folded waveguides.

The synchronous condition of the electron beam with the field can be expressed as[3]

$$\beta_{\max} z_x = \psi_x + 2m_q\pi, \quad x = 1, 2, \ldots, 2Q,$$
$$m_q = 0, \pm 1, \pm 2, \ldots. \tag{11}$$

Assuming that the electron–field interaction coefficients in gaps are identical with each other, one can obtain

$$\frac{E(\beta)}{E(\beta_{\max})} = \frac{1}{2Q}\sum_{x=1}^{2Q} \mathrm{e}^{\mathrm{j}[(\beta_{\max}-\beta)z_x]}. \tag{12}$$

Figure 2(a) describes the relation between space harmonics and propagation constant in a linear varying-period folded waveguide with period number $Q = 10$, $h_1 = 6.7$ in the first period, and the length between the gaps of the qth period, $l_q = 3.98{+}0.02(q{-}1)$, $q = 1, 2, \ldots, 10$. Figure 2(b) describes that relation in a hyperbolic sine-varying period folded waveguide with $Q = 10$, $h_1 = 5l_q = \mathrm{sh}(1.8{+}0.005(q-1))$, and q = 1,2,...,10. In Figs.2(a) and 2(b), the horizontal ordinate represents $(\beta_{\max} - \beta)$, where $\beta_{\max}$ is the maximal amplitude among the propagation constants of the space harmonics and β is the propagation constant of an arbitrary space harmonic. So the zero point of horizontal ordinate implies $\beta = \beta_{\max}$. The longitudinal ordinate indicates the ratio of the amplitude of an arbitrary space harmonic to the maximal amplitude of the space harmonic, $|E(\beta)/E(\beta_{\max})|$, which can be named the relative amplitude. It is obvious from Fig.2 that if we choose the space harmonic with the maximal amplitude to interact with the electron beam, the coupling impedance will also be maximal.

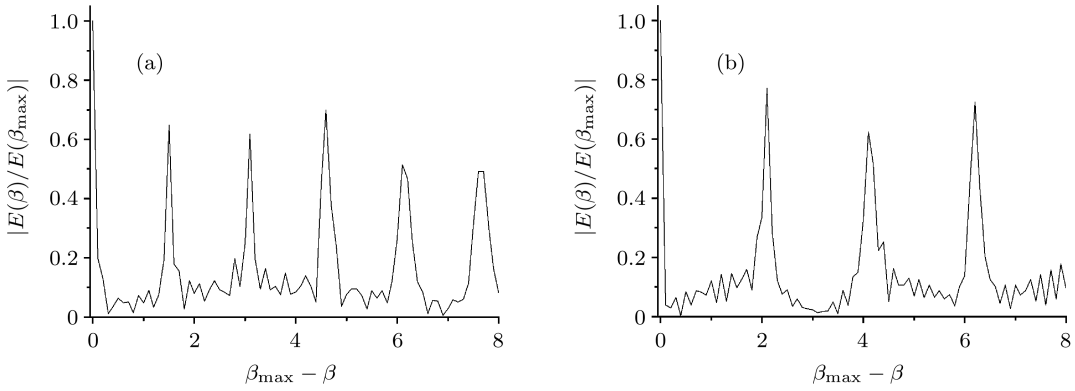


Fig.2. Relations between relative amplitude, $|E(\beta)/E(\beta_{\max})|$, and relative propagation constant, $\beta_{\max} - \beta$, in a linear period-varying folded waveguide (a) and in a hyperbolic sine-varying period folded waveguide (b).

In further studies on the space harmonic in period-varying folded waveguides, we derive the relations of β, β_{s} and $|E(\beta)/E(\beta_{\max})|$ from Eq.(6). The calculated results are shown in Figs.3, 4 and 5.

The structure parameters used in Figs.3 and 4 are the same as those in Figs.2(a) and 2(b). Figure 5 depicts an equal periodic folded waveguide with period number $Q = 10$ and the same size as that of the first period in the linear varying-period folded waveguide, e.g. $h = 6.7$, and $l = 4$. In Figs.3(a), 4(a) and 5(a), the horizontal coordinates represent the propagation constant along the waveguide, β_{s}, while the longitudinal coordinates refer to the relative amplitude $|E(\beta)/E(\beta_{\max})|$. In Figs.3(b) 4(b) and 5(b), the horizontal coordinates represent the longitudinal propagation constant of the space harmonic, β, while the longitudinal coordinates indicate the relative amplitude $|E(\beta)/E(\beta_{\max})|$.

From Figs.3(a) 4(a) and 5(a) it follows that when β is determined, if the system is equally periodic, there will exist innumerable β_{s}'s that maximize the amplitude of this space harmonic and these β_{s}'s vary periodically. If the period is variable in the system, there will exist only one β_{s} that maximizes the amplitude of the space harmonic and its coupling impedance is also maximal.

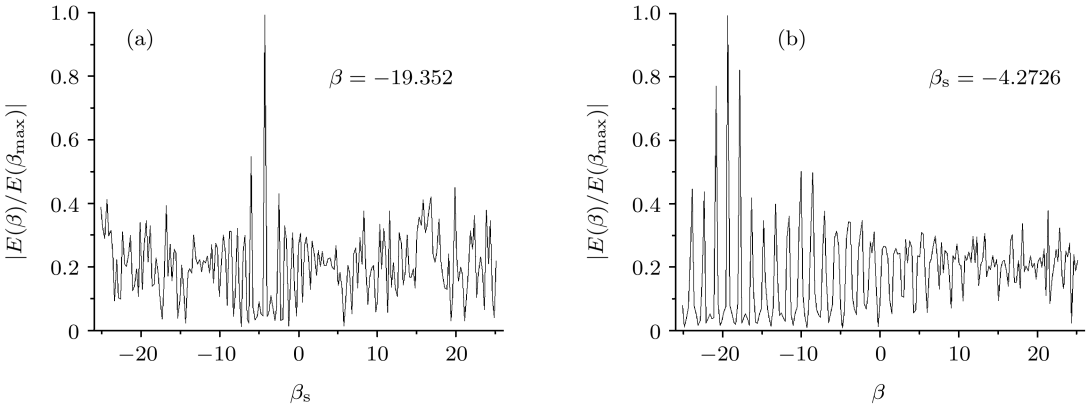


Fig.3. Relations between relative amplitude, $|E(\beta)/E(\beta_{\max})|$, and propagation constant along the waveguide, β_s, (a) and between relative amplitude, $|E(\beta)/E(\beta_{\max})|$, and longitudinal propagation constant, β (b), in a linear period-varying folded waveguide.

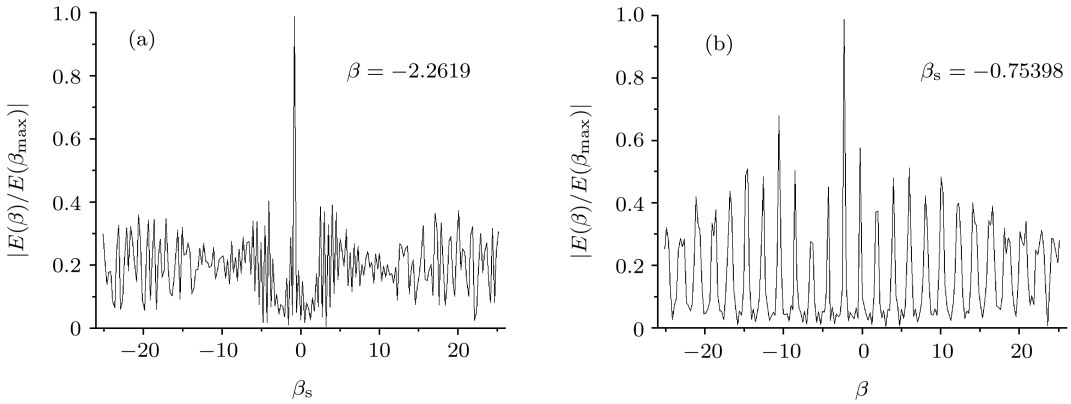


Fig.4. Relations between relative amplitude, $|E(\beta)/E(\beta_{\max})|$, and propagation constant along the waveguide, β_s, (a) and between relative amplitude, $|E(\beta)/E(\beta_{\max})|$, and longitudinal propagation constant, β (b), in a hyperbolic sine-varying period folded waveguide.

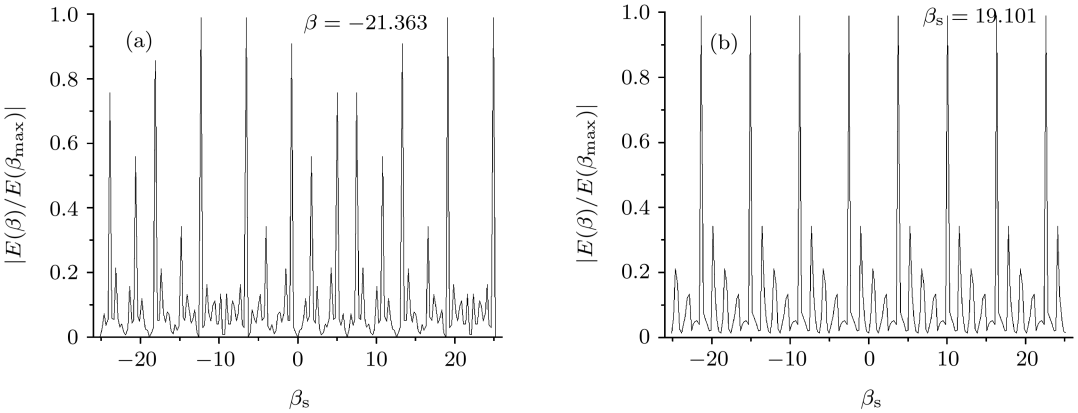


Fig.5. Relations between relative amplitude, $|E(\beta)/E(\beta_{\max})|$, and propagation constant along the waveguide, β_s, (a) and between relative amplitude, $|E(\beta)/E(\beta_{\max})|$, and longitudinal propagation constant, β (b), in an equal period folded waveguide.

It can also be found from Figs.3(b), 4(b) and 5(b) that when β_s is fixed, a similar result can be obtained. If the system is equally periodic, innumerable amplitudes of the space harmonic are maximal, and the relative β varies periodically. But this period is not the same as the period of β_s analysed above. If the period is variable in the system, depending on whether the size is selected appropriately, only one space harmonic has the maximal amplitude.

It is noticed that only $\beta = \beta_{max}$ has the maximal amplitude in the period-varying folded waveguide, and the relative β and β_s determined according to Figs.3 and 4 are not arbitrary.

4. Conclusion

Period-varying folded waveguides have space harmonic selectivity, and the space harmonic that synchronizes with the electron beam has the maximal amplitude. These waveguides are useful for increasing the coupling impedance. Using a linear period-varying folded waveguide and a hyperbolic sine-varying period folded waveguide as examples, the principle of space harmonic selectivity in period-varying folded waveguides is analysed, and the performances for space harmonic selectivity are derived in this paper. Further study on period-varying folded waveguides would be significant.

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