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Collision induced splitting of bright soliton in Bose–Einstein condensate*

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This paper studies the collision dynamics of bright soliton in Bose–Einstein condensate with trapezoid potential. It is found that besides the total reflection and total transmission, one bright soliton can be divided into two bright solitons with different amplitudes in a controllable manner.

Keywords: soliton, Bose–Einstein condensate, collision
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The experimental realization of Bose–Einstein condensate (BEC) in dilute alkali–metal atomic vapours has stimulated intensive studies on the nonlinear excitations of atomic matter waves,[1] such as dark[2,3] and bright solitons,[4−6] vortices[7] and vortex lattices.[8] An important issue of these investigations is the dynamics of BEC solitons, for their potential applications in atomic interferometry and different kinds of the quantum phase transitions, as well as in the context of nonlinear physics.

It is well known that BEC admits bright and dark solitons with negative and positive s-wave scattering length, respectively. In the past few years, the interaction of bright and dark solitons with harmonic and periodic potential, as well as the interaction of dark soliton with a localized impurity (potential), is well established.[9−13] For example, Frantzeskakis *et al* [10] found that a dark soliton can be reflected from or transmitted through a repulsive impurity in a controllable manner, while near the critical point the soliton can be quasi-trapped by the impurity, and an immobile soliton may be captured and dragged by an adiabatically moving attractive impurity. Parker *et al* [11] showed that a dark soliton can pass over, be reflected with sound emission or even collides elastically when incident on a step-like potential. Note that the emission of sound corresponds to the splitting of a dark soliton.

As for the bright soliton, although some exact soliton solutions were obtained,[14−16] the head-on collision of BEC bright soliton with hardwalled obstacle of finite size has not yet been analysed in detail. Recently, Cheng *et al* [17] found that a moving bright soliton can be totally reflected from or transmitted through a step-like potential in a controllable fashion, and they obtained the critical velocity which depends on the soliton and the step-like potential parameters. In fact, this critical velocity can be well described by the theory based on the particle-like behaviour of bright soliton.[18] Because of the similarity between bright and dark solitons, the splitting of the BEC bright soliton should take place when it incident on a localized potential. In the present work, we shall investigate the collision dynamics of BEC bright soliton with a localized potential.

We begin with the cigar-shaped BEC,[1] which is effectively one-dimensional and the governing equation is given in the following dimensionless form:

$$\mathrm{i}\frac{\partial u}{\partial t}+\frac{1}{2}\frac{\partial^2 u}{\partial x^2}+|u|^2u=V(x)u, \tag{1}$$

where $u(x,t)$ is the normalized macroscopic wave function measured in the unit $\sqrt{a_\perp}$, t and x are the temporal and spatial coordinates measured in units $\omega_\perp^{-1}$ and $a_\perp=\sqrt{\hbar/m\omega_\perp}$, respectively. $V(x)$ is the external trapping potential in x direction, which is measured in the unit $\hbar\omega_\perp$, and $2a_{\rm s}/a_\perp=-1$ with $a_{\rm s}$ being the

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s-wave scattering length controlled by Feshbach resonance. Note that in deriving the above equation, we have assumed that the transversal (y and z directions) dynamics of the condensate is frozen by the harmonic potential with confinement frequency $\omega_\perp$.

When the external potential $V(x)$ is absent, Eq.(1) becomes the well-known nonlinear Schrödinger equation, which possesses an exact bright soliton solution

$$u = A\,\mathrm{sech}[A(x - x_0 - v_0 t)]\mathrm{e}^{\mathrm{i}[v_0 x + (A^2 - v_0^2)t/2 + \phi_0]}, \quad (2)$$

where A, v_0, x_0 and ϕ_0 are the amplitude, velocity, initial centre and initial phase of the bright soliton, respectively.

In the presence of the external potential, however, it is hard to find the exact soliton solution of Eq.(1), except for very little forms of $V(x)$ such as the linear ramp potential $V(x) = kx$, where the solution can be given by the transformation[19] $x \to x + kt^2/2, u \to \exp[\mathrm{i}(k^2t^3/6 - kxt)]u$ to Eq.(2). Thus, one finds that the such a potential does not change the shape of the BEC bright soliton, but only change its motion. The centre of the bright soliton is now $\xi = x_0 + v_0 t - kt^2/2$, which implies that the bright soliton experiences a constant acceleration $-k$. Therefore, when the right-moving soliton ($v_0 > 0$) incidents on the linear ramp potential ($k > 0$), it is velocity gradually slows down to zero, and then moves leftwards with linearly increasing velocity. The maximum distance the soliton can reach is $v_0^2/(2k)$, according to the conservation law of energy. This is confirmed by the direct numerical simulation of Eq.(1) by use of the standard split-step Fourier method (Fig.1), where the maximum distance the soliton reached is 1, as predicted.

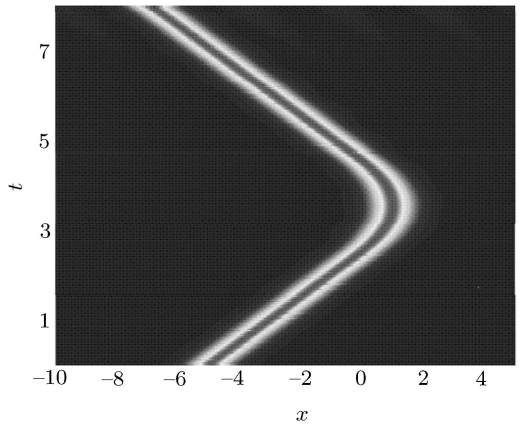


Fig.1. Total reflection of bright soliton when collides with a linear ramp potential $V(x) = kx$ for $x \geq 0$ and $V(x) = 0$ for $x < 0$. The parameters are $A = 3$, $v_0 = 1$, $k = 0.5$, $x_0 = -5$.

Strictly speaking, the above analysis is valid only when the linear ramp potential is infinite or the whole body of the bright soliton is inside the liner ramp potential. In other words, the dynamics of bright soliton will be different when it is interacted with a finite size potential. Now we consider the collision dynamics of BEC bright soliton with a trapezoid potential:

$$V(x) = \begin{cases} kx, & \text{for} \quad 0 \leq x < a, \\ ka, & \text{for} \quad a \leq x < a + b, \\ k(2a + b - x), & \text{for} \quad a + b \leq x < 2a + b, \\ 0, & \text{otherwise}, \end{cases} \quad (3)$$

where k, a and b are positive constants. For convenience, hereafter we name Region I for $x < 0$, Region II for $0 \leq x < a$, Region III for $a \leq x < a + b$, Region IV for $a+b \leq x < 2a+b$ and Region V for $x \geq 2a+b$. It is easy to conclude that, the bright soliton moves with a constant velocity if its whole body lies in Regions I, III or V, and experiences a constant deceleration and acceleration when the whole body lies in Regions II and IV, respectively.

Apparently, the BEC bright soliton will be totally reflected for $a \gg v_0^2/2k$ (Fig.1), and totally transmitted for $a \ll v_0^2/(2k)$ (Fig.2) when it is interacted with the trapezoid potential. From Fig.2 we see that the width of the soliton is broadened when entering into Region III, which is different from the results of Eqs.(1) and (2), which shows that the soliton shape does not change in the presence of linear ramp potential. This is because the whole soliton lies in both Regions II and III, and we can no longer treat the bright soliton as a quasi-particle. The broadening effect can be explained as follows: when the soliton is in both Regions II and III (since b is so large in comparison with the soliton width, we assume that the soliton is

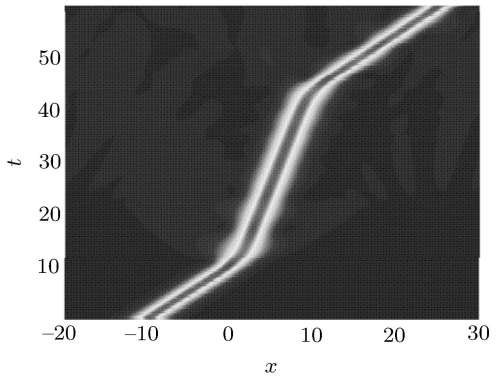


Fig.2. Total transition of bright soliton when collides with a trapezoid potential. The parameters are $A = 1$, $v_0 = 0.5$, $k = 0.5$, $x_0 = -10$, $a = 0.9v_0^2/(2k)$, $b = 10$.

not in Region IV at the same time), one part of soliton which lies in Region III moves with constant velocity while the other part experiences a backwards force which slows down its velocity, so the soliton broadens. Actually, when the soliton enters into Region II from Region I, it is compressed and this effect cannot be seen clearly in Fig.2; we shall demonstrate such effect in Fig.3. And due to the symmetry of the trapezoid potential, when the soliton enters into Region IV from Region III, it narrows, while when the soliton enters into Region V from Region IV, it broadens. In the whole process, only a little radiation is observed.

The deforming (broadening and narrowing) effect of the BEC bright solitons implies that, if we decrease the width b to make the bright soliton be located in Regions II, III and IV at the same time, the soliton will experience a pair of outwards forces, which will split the soliton into two parts and make them move in opposite directions. Technically, in order to split the BEC bright soliton, the soliton's velocity when it is in Regions II, III and IV should be small enough to avoid the transmission. Therefore, we should select a such that $a \approx v_0^2/(2k)$ to ensure the soliton moves very slowly in Region III, and select b such that b is smaller than the extent of the bright soliton to insure that the soliton can be located in Regions II, III and IV.

Figures 3 and 4 show the splitting dynamics of BEC bright soliton when it interacts with the trapezoid potential. The solid curve in Fig.3(b) is the initial envelope of the BEC bright soliton located in Region I, and the dashed curve corresponds to the result after the first collision with the trapezoid potential, which happened in Regions I and II. It can be easily seen that the bright soliton is compressed. This is because the front part of the soliton lying in Region II suffers a backwards force while the rest part moves with the constant velocity in Region I, so the soliton width narrows, and its amplitude increases to keep the total number of atoms unchanged. Similarly, the envelope is broadened when it enters into Region III from Region II, and after a short time it almost recovers to its initial shape, but with a little deformation in the rear of the soliton, as shown by the dotted line. Then, as the time goes on, the envelope is further deformed (dash-dot curve) and finally divided into two parts moving towards opposite directions, as we can see from Figs.3(a) and 4(a). In comparison with Fig.4, we see that the amplitudes of each part are different for different values of b, which implies that the number of atoms allocated to each part can be controlled by the parameter b.

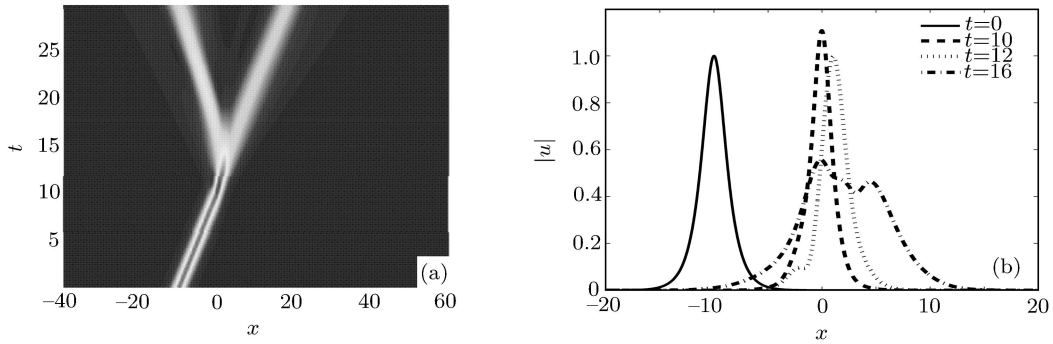


Fig.3. Splitting of bright soliton when collides with a trapezoid potential. The parameters are $A = 1,\ x_0 = -10,\ v_0 = 1,\ k = 0.5,\ a = v_0^2/(2k),\ b = 2$.

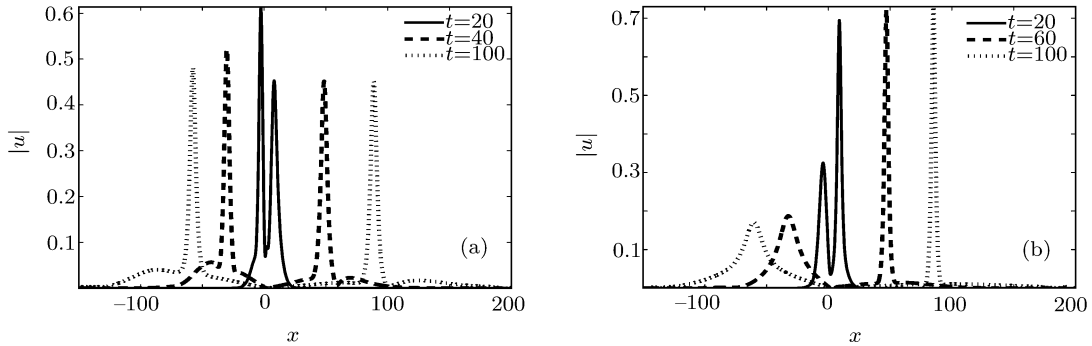


Fig.4. Splitting of bright soliton when collides with a trapezoid potential. The parameters are the same with Fig.3 except for (a) $b = 2$, and (b) $b = 0.5$.

Further numerical simulations (Fig.5) confirm the conjecture. From Fig.5 we see that the number of atoms in the reflection part (solid line) increases with b, while the number of atoms allocated to the transmission part (dotted line) decreases with b. This can be explained as follows: the smaller the width b is, the more earlier the soliton can move into the right side of trapezoid potential, and so the more atoms can be allocated to the transmission part, provided that other parameters are fixed.

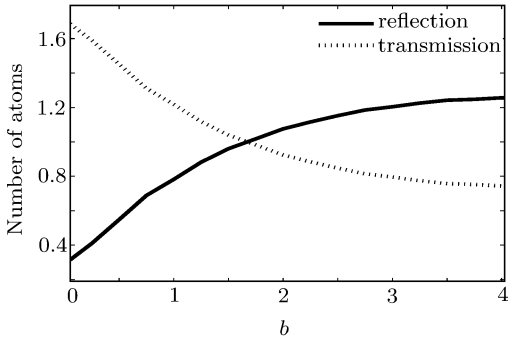


Fig.5. Atom numbers in the reflection and transmission part. The parameters are the same with Fig.3 except b.

In summary, we have investigated the collision dynamics of a BEC bright soliton with a trapezoid potential. It was found that, besides the known effect such as total reflection and total transmission, the bright soliton can be even divided into two bright solitons with different amplitudes. It was also found that the trapezoid potential parameter b can be used to control the numbers of atoms allocated in each soliton. Also, the BEC bright soliton can be split into two solitons; in contrast, the two BEC bright solitons can merge and fuse if they approach each other and collide in the region of the trapezoid potential. This can be clearly concluded from the fact that if we take the conjugation to Eq.(1) and replace t by $t_0 - t$, where t_0 is a constant, the obtained equation is the same as Eq.(1). That is, if we take the conjugation of the two bright soliton solutions which is obtained after collision as the initial condition, the two bright solitons will emerge fused.

Acknowledgments

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