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The propagation and deposition of fast muons in dense DT mixture*

Li Xue-Mei(李雪梅)[a)b)†], Shen Bai-Fei(沈百飞)[a)], Zhang Xiao-Mei(张晓梅)[a)], Jin Zhang-Ying(金张英)[a)], and Wang Feng-Chao(王凤超)[a)]

[a)] *State Key Laboratory of High Field Laser Physics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai* 201800, *China*

[b)] *Department of Physics, School of Sciences, Guangxi University for Nationalities, Nanning* 530006, *China*

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The propagation of the fast muon population mainly due to collisional effect in a dense deuterium–tritium (DT for short) mixture is investigated and analysed within the framework of the relativistic Fokker–Planck equation. Without the approximation that the muons propagate straightly in the DT mixture, the muon penetration length, the straggling length, and the mean transverse dispersion radius are calculated for different initial energies, and especially for different densities of the densely compressed DT mixture in our suggested muon-driven fast ignition (FI). Unlike laser-driven FI requiring super-high temperature, muons can catalyze DT fusion at lower temperatures and may generate an ignition sparkle before the self-heating fusion follows. Our calculation is important for the feasibility and the experimental study of muon-driven FI.

Keywords: fast muons, propagation, deposition
PACC: 5225D, 5220H

1. Introduction

For both magnetic confinement fusion (MCF) and inertial confinement fusion (ICF),[1] deuterium and tritium (DT) ions are assumed free ions with Maxwellian velocity distribution function and any effect due to plasma electrons and other ions is neglected in most theories, so the fusions can be considered as the collision of DT ions. The realization of the fusion needs super-high temperature according to the electromagnetic principles and the quantum barrier transparency theory. However, the muon with 207 times the mass of and the same charge as the electron is suggested[2,3] as a catalyzer of the fusion, so that DT fusion can happen more easily. The theoretical study on the feasibility of muon catalyzed fusion[2,3] proceeded later. Then it is proved experimentally that the muons can catalyze the fusion in 1957.[4] Up to now, one muon costing the energy of 8 GeV can catalyse more than 100 fusions before it leaves the fusion cycle because of its decay[5] or capture by Helium,[6,7] and then release the energy of 2 GeV and neutrons.[8] However, it means that the spent energy still cannot be paid off with the released energy. So Petrov[8] suggested the hybrid muon-catalysis, in which the produced neutrons of initial energy of 14.1 MeV from the muon catalysis reaction can be used in obtaining fission energy and nuclear breeding in the following fission reactions, then extra energy can be obtained. For the same purpose, in this paper we will suggest the muon-driven fast ignition (FI).

After the compression of the DT mixture, in the laser-driven FI for ICF, an electron or ion beam produced by high-power-laser may generate a sparkle at the surface of the high density compressed fuel and then the fusion energy can induce the self-heating fusion of the surrounding mixture. Before the ignition happens, in order to heat the hot spot to high temperature, the current of the fast electron beam for FI must be much higher than Alfven limit so that it may be difficult for the electron beam to move in. While the way to catalyse more reactors with one muon may be found in the future, we suggest the muon-driven FI in which the short ignition laser pulse is entirely or partly replaced by a muon beam. Although the energy spent for producing muons cannot be paid off

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†E-mail: xmlee0102@yahoo.com.cn

with the released energy in the hot spot, the fusion energy released by the whole fusion fuel may still be larger than the total input energy including the energy for muons production and the compressing laser pulse.

In this kind of ignition, the requirement for the high temperature may be low because of the catalysis of the muons for the fusion. In order to explore the feasibility of this method, we study the propagation and deposition of the energetic muons as the very first step.

The propagation and the deposition of the muons are very important for the fusion process and have been studied a lot[9−11] (not for FI), where the density of the liquid DT mixture is about $5\times10^{28}\,\mathrm{m}^{-3}$. In the case of the muon-driven FI, the DT mixture is compressed to much higher densities than that of the liquid one. So here we study the propagating of the fast muons in such a very denser DT mixture using the relativistic Fokker–Planck equation[12] without the approximation[9−11] that the muons propagate in the DT mixture straightly.

This paper is organized as follows. In the next section, after we describe the various model assumptions, the Belieav and Budker relativistic extension of the Landau collision kernel is used to set up the relativistic Fokker–Planck equation for fast muons dynamics in a dense DT mixture. Then, in Section 3, we solve this equation with the use of the Green-function method. In Sections 4, we calculate the mean penetration length, the straggling length, and the mean transverse dispersion radius of the muon beam using the explicit analytical calculation. In the final section, we give discussions.

2. The relativistic Fokker–Planck equation of the fast muons

A muon interacting with a DT mixture loses its energy and momentum through two main channels: (i) collisions and (ii) radiation. Collisional energy loss leads to a continuous slowing down while bremsstrahlung causes large and sudden energy losses. Bremsstrahlung[13−15] is the radiation during collision especially the Coulomb collision, which is the main energy loss scheme for the ultra-relativistic particle, while it is not important for nonrelativistic particles. Because the radiation power is proportional to the square of the particle charges and inversely proportional to the mass of the incident particles, the effect of the bremsstrahlung does not play an important role for heavy incident particles. In this paper, for muons of several MeV propagating in the dense DT mixture, the energy loss from bremsstrahlung can be neglected. Furthermore, provided that the beam density is small compared to the background DT mixture density, we can neglect the interaction between the muons of the beam.

The slowing down time scale of a fast muon is $\tau = 207\times[4\pi c n_\mathrm{e} r_\mathrm{e}^2 \ln\Lambda_\mathrm{me}]^{-1}$, having the dimension of time, where n_e is the electron density of the background DT mixture, r_e is the classical electron radius, c is the velocity of light, and $\ln\Lambda_\mathrm{me}$ is the Coulomb logarithm. Throughout this paper, in order to simplify this study, we will use τ as the unit of time, $m_\mathrm{e}c$ as the unit of electron momentum where m_e is the mass of the electron, and Mc as the unit of muon momentum where M is the mass of the muon. Thus, the unit of length is $\lambda = c\tau$

$$\left[\frac{\lambda}{\mathrm{cm}}\right] = \frac{207}{\ln\Lambda_\mathrm{me}}\left[\frac{10^{30}\mathrm{m}^{-3}}{n_\mathrm{e}}\right]. \tag{1}$$

The relativistic momentum is denoted as $p=\gamma\nu$ and the relativistic energy as $\gamma=\sqrt{1+p^2}$.

Densities of the background electrons and ions are n_e and n_i respectively. Because $n_\mathrm{e} = Zn_\mathrm{i}$ is required for the quasineutrality, we have Z=1 in the DT mixture. In order to study the interaction between a fast muon and the background DT mixture, we set up the kinetic equation for the fast muon momentum distribution function $f(p,t)$, including both the collisional effect and the angular scattering:[17]

$$\partial f/\partial t = C_\mathrm{me}(f,F_\mathrm{e}) + C_\mathrm{mi}(f,F_\mathrm{i}) \tag{2}$$

with the two collisional operators C_me and C_mi describing muon–electron and muon–ion collisional interactions. F_e and F_i are distribution functions of electrons and ions in the background DT mixture respectively.

In this paper, the initial energy of the muon is several MeV and the velocity of the muon is much larger than that of the background particles. Therefore, it is reasonable to denote the background electron and ion distributions by two 'cold' distributions

$$F_\mathrm{s}(P_\mathrm{s}) = n_\mathrm{s}\frac{\delta(p_\mathrm{s})}{4\pi P_\mathrm{s}^2}, \tag{3}$$

where s=e, i represents the background species and $p_\mathrm{s}=|P_\mathrm{s}|$. So we have

$$\int F_\mathrm{s}(p)\mathrm{d}^3P = n_\mathrm{s}. \tag{4}$$

Using the expressions (2), (3), and (4) and the Belieav and Budker relativistic extension of the Landau collision kernel, we set up the relativistic kinetic equation: the relativistic Fokker–Planck equation[12,16−21] in the set of spherical momentum coordinates as shown in Fig.1.

$$\begin{aligned} &C_{\rm me}(f,F_{\rm e})+C_{\rm mi}(f,F_{\rm i})\\ &=-\frac{1}{p^2}\frac{\partial}{\partial p}\gamma^2 f-\frac{Z\Lambda}{M_1p^2}\frac{\partial}{\partial p}\gamma^2 f-\frac{1+Z\Lambda}{2\times\frac{M}{m}}\frac{\gamma}{p^3}\\ &\times\left[\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial}{\partial\mu}+\frac{1}{1-\mu^2}\frac{\partial^2}{\partial\phi^2}\right]f. \end{aligned} \tag{5}$$

Here M_1 is the mass number of the DT mixture. Also we have introduced the Coulomb logarithm ratio $\Lambda=\ln\Lambda_{\rm me}/\ln\Lambda_{\rm mi}$. For the case in this paper, Z=1 and the energy of the fast muon is not very high, so Coulomb logarithm ratio is about one.[5]

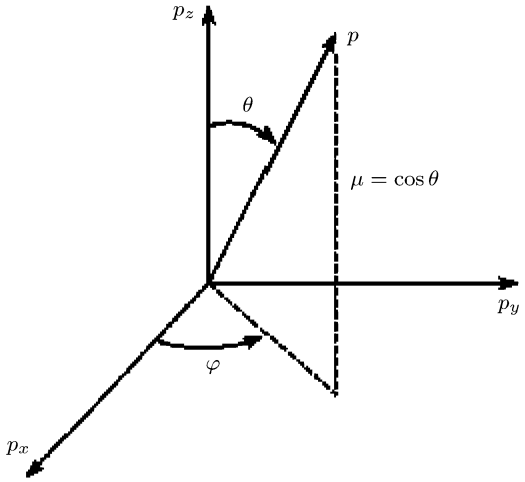


Fig.1. Definition of the set of spherical momentum coordinates ($p,\ \theta,\ \varphi$).

The first part on the right-hand side of Eq.(5) describes the effect of collisional drag due to muon–electron collisions and the second part describes the effect of collisional drag due to muon–ion collisions. We find that the energy loss due to the background ions is much smaller than that due to the background electrons, so the second part can be neglected. The third part describes the pitch angle scattering due to both muon–electron and muon–ion collisions. In this article, $Z\Lambda$=1 and $M/m=207>>1$, then the mean transverse dispersion radius corresponding to the angle deflection is much smaller than the muon propagating length while the mean transverse dispersion radius and the propagating length are on the same order for the electrons[17] propagating in the plasmas. But the transverse dispersion does change the theoretical propagating length as we will show later in Fig.6.

From the comparison of the first part with the second part in Eq.(5), it is found that the energy of a muon is mostly delivered to the lighter particles.[5] This is because the transferred momentum $\Delta p\propto(q_aq_be^2/l^2)l/v_p$, depends only on the charges Z_1 and Z_2 of the colliding partners, the impact parameter b, and the collision time $b/v_{\rm p}$, but not on the mass. However, the transferred energy $\Delta\varepsilon=(\Delta p)^2/2m$ is inversely proportional to its mass, so low-mass electrons pick up most of the energy.

3. Green function of the Belieav–Budker operator

The initial condition for the fast muon in the DT mixture is described in the following: the momentum is $\boldsymbol{p}_0$ and the position is $\boldsymbol{r}_0$=(0, 0, 0) at time t_0=0. So the initial momentum distribution is a shifted Dirac function:

$$f(P,t_0)=\delta(P-P_0). \tag{6}$$

The evolution of $f(p,t)$ is the Green function of the Beliaev–Budker operator:

$$f(P,t)=G(P,P_0,t,t_0) \tag{7}$$

and G is the solution of

$$\begin{aligned} &\frac{\partial G}{\partial t}-\frac{1}{p^2}\frac{\partial}{\partial p}\gamma^2 G-\frac{1+Z\Lambda}{2\times\dfrac{M}{m}}\frac{\gamma}{p^3}\\ &\times\left[\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial}{\partial\mu}+\frac{1}{1-\mu^2}\frac{\partial^2}{\partial\phi^2}\right]G\\ &=\frac{\delta\,(p-p_0)}{p^2}\delta(\mu-\mu_0)\delta\,(\phi-\phi_0)\,\delta\,(t-t_0)\,, \end{aligned} \tag{8}$$

where ($p_0,\ \mu_0,\ \phi_0$) is the initial momentum in the set of spherical momentum coordinates at time t_0=0. Because the spherical harmonics are the eigenfunctions of the angular part of the collision operator, the solution of Eq.(8) can be written as:[16,17,19,22]

$$G = \frac{\delta\left(\arctan\left(p\right) - p - \arctan\left(p_0\right) + p_0 - \left(t - t_0\right)\right)}{\gamma^2} H\left(t - t_0\right) \times \sum_{l=0}^{l=+\infty} \sum_{m=-l}^{m=+l} Y_l^m\left(\mu, \varphi\right) Y_l^{m^*}\left(\mu_0, \varphi_0\right) \left[\frac{p\left(\gamma_0 + 1\right)}{p_0\left(\gamma + 1\right)}\right]^{[l(l+1)(Z\Lambda+1)/2/(M/m)]}, \quad (9)$$

where the function Y_{lm} is the spherical harmonics and H is the Heaviside function.

Clearly, Eq.(9) describes the evolution of the fast muon propagating in the DT mixture. The part before the sum symbol describes the process of slowing down while the part after that describes the process of the angle deflection.

This Green function $G(P,\ P_0,\ t,\ t_0)$ is the transition probability in the physical interpretation: a muon starting at time t_0 with momentum $\boldsymbol{p}_0$ will be found with the probability $G(P,\ P_0,\ t,\ t_0)$ at later time t in the momentum volume element d$\boldsymbol{p}$ centred around $\boldsymbol{p}$. The expression of the evolution of any value is[23]

$$\langle g(P,t)\rangle = \int \mathrm{d}p g(p,t) G(P,P_0,t,t_0). \quad (10)$$

Using Eq.(10), we can calculate the muon propagating length, the straggling length, and the mean transverse dispersion radius in the dense DT mixture.

4. Propagation and deposition of the fast muon

To evaluate the size of the DT mixture volume deposited of the fast muons, we have to calculate the penetration length, as well as the straggling length, and the mean transverse dispersion radius for the muons.[17] The three characteristic lengths are defined as follows: (1) the muon penetration length is $\langle z\rangle$, where the brackets $\langle\rangle$ stands for an average over the various realizations of the collisional noise. (2) The straggling length is $s = \sqrt{\langle z^2\rangle - \langle z\rangle^2}$, representing the statistical spreading of a set of muons with the same initial condition. (3) The mean transverse dispersion radius is $\sqrt{\langle r^2\rangle}$. The detailed calculations can be referred to Ref.[12], with M/m=270 in some expressions. Considering an equivalence DT mixture with Z=1, n=$10^{32}\,\mathrm{m}^{-3}$ and the unit length $\lambda = 2070\,\mu$m, the muon penetration length, the straggling length and the mean transverse dispersion radius of the muons with initial energy of 18 MeV increasing with decreasing energy are shown in Fig.2 with the energy unit of MeV and length unit of μm. As shown in the figure, both the straggling length and the mean transverse dispersion radius are much smaller than the penetration length, on the same order. The stopping length, the straggling length and the mean transverse dispersion radius are 53 μm, 1.45 μm, and 2.6 μm respectively. The size of the DT mixture volume deposited of the fast muons is a column centred at the distance of 53 μm, with the full length of 2.9 μm and the radius of 2.6 μm.

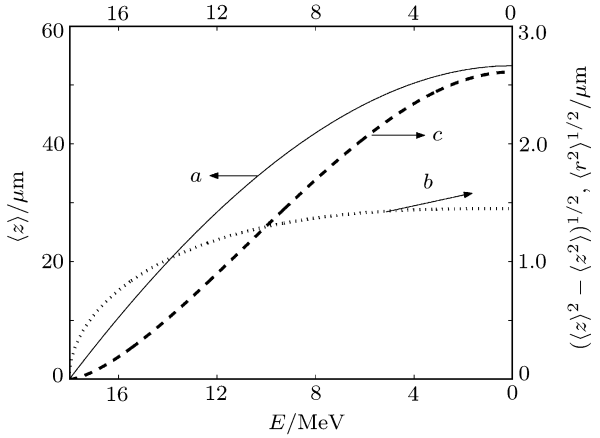


Fig.2. The muon penetration length (curve a, the solid line), the straggling length (curve b, the dotted line) and the mean transverse dispersion radius (curve c, the dashed line) for the muon with initial energy of 18 MeV propagating in the compressed dense DT mixture of the density n=$10^{32}\,\mathrm{m}^{-3}$.

We have calculated the deposited size of the 18 MeV muons in the DT mixture of density of n=$10^{32}\,\mathrm{m}^{-3}$. As the initial energy of the muons increases, the deposited size is enlarged. We can choose the different energies of muons and the number of the muon beams to make the deposited size of muons equal to the size required. For our suggested muon-driven FI, the DT mixture is compressed to higher densities than the liquid one, so the three lengths are calculated for different densities. The stopping length, the straggling length and the mean transverse dispersion radius (with the length unit of μm) are shown in Figs.3, 4 and 5 respectively with different initial

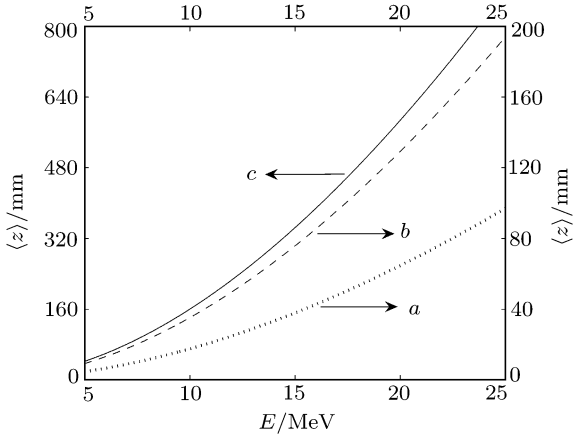


Fig.3. The stopping length for the muon with initial energy from 5 to 25 MeV in the compressed dense DT mixture of the density n=10^{32} m^{-3} (curve a, the dotted line), n=5×10^{31} m^{-3} (curve b, the dashed line) and n=10^{31} m^{-3} (curve c, the solid line).

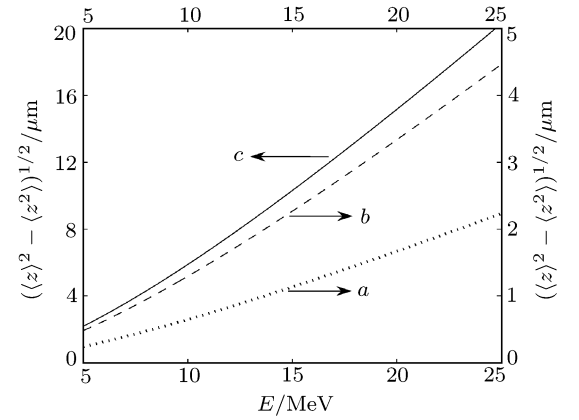


Fig.4. The straggling length for the muon with initial energy from 5 to 25 MeV in the compressed dense DT mixture of the density n=10^{32} m^{-3} (curve a, the dotted line), n=5×10^{31} m^{-3} (curve b, the dashed line) and n=10^{31} m^{-3} (curve c, the solid line).

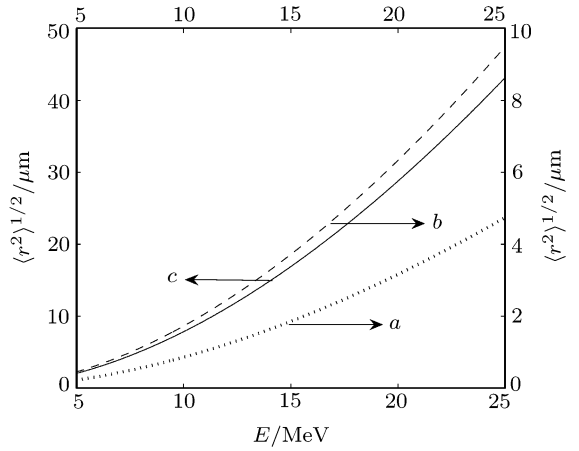


Fig.5. The mean transverse dispersion radius for the muon with initial energy from 5 to 25 MeV in the compressed dense DT mixture of the density n=10^{32}m^{-3} (curve a, the dotted line), n=5×10^{31} m^{-3} (curve b, the dashed line) and n=10^{31} m^{-3} (curve c, the solid line).

energies (from 5 to 25 MeV) and background densities (n=10^{31} m^{-3}, 5×10^{31} m^{-3}, and 10^{32} m^{-3}). With these calculations, we can choose the suitable energy of the muon for the specific density and the deposited muon size required.

5. Discussion

In order to heat the hot spot to high temperature, the current of the fast electron beam for FI must be much higher than Alfven limit so that it may be difficult for the electron beam to move in. So, we suggest muon-driven FI, in which muons may generate an ignition sparkle at lower temperature because of their catalysis for fusion, and then self-heating fusion follows. The fusion energy released by the whole fusion fuel may still be larger than the total input energy including the energy for muons production and the compressing laser pulse. In this paper, we have mainly studied the problem of the characteristic size of the muon deposition volume in the dense DT mixture as the very first step to study the feasibility of the suggested muon-driven FI.

The muon deposited spot is characterized by three lengths: the stopping length, the straggling length, and the mean transverse dispersion radius. Solving the relativistic Fokker–Planck equation, we have calculated the three kinds of length of different initial energies for different densities without the approximation that the muons propagate in the DT mixture straightly. With these calculations, we can choose the suitable energy of the muon for the specific density and the deposited muon size required. For example, the initial energy of muons required is about 18 MeV for the compressed DT mixture core with density of n=10^{32} m^{-3}and radius of 50 μm.

The comparison of the penetration lengths for muons with the initial energy of 18 MeV by the previous Ref.[5] (the solid line) and our paper (the dotted line) is shown in Fig.6. We will find that the length in Ref.[5] is longer than that in our result and the reason for the difference between them is that we have considered the angular scattering. With the approximation that the muons propagate in the DT mixture straightly, the initial energy of muons required for the compressed DT mixture core also with density of n=10^{32} m^{-3} and radius of 50 μm is about 17 MeV, which is smaller than 18 MeV when we consider the angular scattering. So muons cannot be considered as

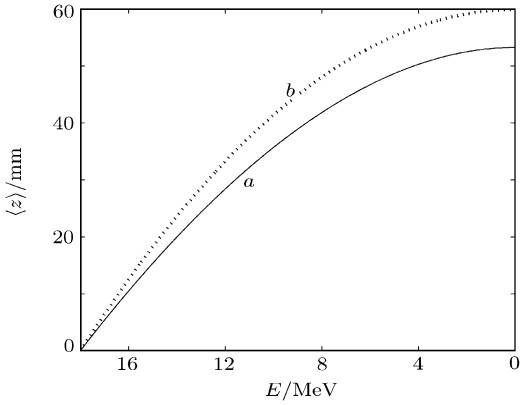


Fig.6. The comparison of the penetration length for the muons with initial energy of 18 MeV propagating in the compressed dense DT mixture of the density n=10^{32}ṁ$^{-3}$ by the ways in Ref.[5] (curve b, the dotted line) and our paper (curve a, the solid line).

propagating in the DT mixture straightly when exact accuracy is needed.

Our calculation of deposited sizes is important for understanding the mechanism of the catalysis for the fusion, and it may give some theoretical guidance for the feasibility of our suggested muon-driven FI. As we know, high energy electron or proton beams can be used for FI. At present, it is hard to say the FI driven by muons has more advantage or it is easier for generating energetic muons than generating energetic electrons and protons. But at least it provides a possible alternative since the traditional FI methods are still not successful.

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