Vol 18 No 3, March 2009
1674-1056/2009/18(03)/1123-04
Chinese Physics B
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and IOP Publishing Ltd

Acoustic elliptical cylindrical cloaks*

Ma Hua(马 华)[a)†], Qu Shao-Bo(屈绍波)[a)], Xu Zhuo(徐 卓)[b)], and Wang Jia-Fu(王甲富)[a)]

[a)] *The College of Science, Air Force University of Engineering, Xi'an 710051, China*
[b)] *Electronic Materials Research Laboratory, Key Laboratory of Ministry of Education, Xi'an Jiaotong University, Xi'an* 710049, *China*

(Received 26 June 2008; revised manuscript received 21 August 2008)

By making a comparison between the acoustic equations and the 2-dimensional (2D) Maxwell equations, we obtain the material parameter equations (MPE) for acoustic elliptical cylindrical cloaks. Both the theoretical results and the numerical results indicate that an elliptical cylindrical cloak can realize perfect acoustic invisibility when the spatial distributions of mass density and bulk modulus are exactly configured according to the proposed equations. The present work is the meaningful exploration of designing acoustic cloaks that are neither sphere nor circular cylinder in shape, and opens up possibilities for making complex and multiplex acoustic cloaks with simple models such as spheres, circular or elliptic cylinders.

Keywords: invisibility, cloaks, metamaterials, acoustic waves
PACC: 4328, 4330, 4360

1. Introduction

Pendry *et al*[1] proposed the concept of electromagnetic (EM) perfect invisibility based on metamaterial techniques, and stated a general coordinate transformation approach to obtaining the specific distributions of permittivity and permeability in designing EM cloaks. The same concept was discussed by Leonhardt[2,3] while the optical conformal mapping method was adopted subject to the ray tracing limit. Later, the simulations and experiments of the EM cloaks,[4–6] the interactions between EM waves and the cloaks, and scattering of EM waves on the cloaks[7–9] were also presented. As an exploration of designing cloaks that are neither spheres nor circular cylinders in shape, the authors of this paper have obtained the material parameter equations (MPE) for elliptical cylindrical cloaks in the EM wave case.[10] Milton *et al*[11] investigated how to change the form of the conventional elastodynamic equations under curvilinear transformations and proposed modified motion equations that are invariant under the transformation. As a result, their work opens up possibilities for achieving the perfect invisibility for acoustic waves. Then, the researches into cylindrical acoustic cloaks[12] and spherical acoustic cloaks[13] were conducted.

In this paper, by comparing the acoustic equations with 2D Maxwell equations in the transverse electric (TE) polarization, we deduce the acoustic MPE for elliptical cylindrical cloaks, and carry out numerical simulation to verify this result.

2. Deduction of acoustic parameter equations

The considered objects are elliptic cylinders and the associated elliptic cylinder coordinate $S(\xi, \eta, z)$ is selected as a new coordinate. The relationship between Cartesian coordinate $C(x, y, z)$ and elliptic cylinder coordinate $S(\xi, \eta, z)$ is expressed as follows:

$$x = c\xi\eta, \quad y = c\sqrt{(\xi^2 - 1)(1 - \eta^2)}, \quad z = z, \qquad (1)$$

where c is the focal length of elliptic cylinder and is assumed to be a constant. Under such an assumption, elliptical cylindrical shell with an immovable focus can be identified by ξ, and an arbitrary spatial point located on the shell is described by (ξ, η, z). In this way, the spatial distortion can be described as a coordinate transformation between the original Cartesian mesh and the new elliptical cylindrical mesh, and also the spatial compression from the elliptical region

*Project supported by the National Natural Science Foundation of China (Grant Nos 50632030, 10474077 and 60871027), the National Basic Research Program of China (Grant No 2009CB613306), and The Innovation Funds of the College of Science, Air Force University of Engineering, China.
†Corresponding author. E-mail: mahuar@163.com

$[1, \xi_2]$ into the annular region $[\xi_1, \xi_2]$ can be therefore expressed mathematically.

Milton *et al*[11] investigated how to change the form of the conventional elastodynamic equations under curvilinear transformations and proposed the modified motion equations that are invariant under transformations. By noting the analogy between acoustic equations and Maxwell equations in the 2D geometry, Cummer and Schurig[12] demonstrated the cylindrical acoustic cloaks. According to their methods, in elliptical cylindrical coordinate with an invariant z, we simplify the time harmonic acoustic equations $\left(\text{in this case, } \frac{\partial}{\partial t} = \mathrm{j}\omega\right)$ into

$$\mathrm{j}\omega\rho_\xi v_\xi = -\frac{1}{c}\sqrt{\frac{\xi^2-1}{\xi^2-\eta^2}}\cdot\frac{\partial P}{\partial \xi}, \quad \mathrm{j}\omega\rho_\eta v_\eta = -\frac{1}{c}\sqrt{\frac{1-\eta^2}{\xi^2-\eta^2}}\cdot\frac{\partial P}{\partial \eta},$$
$$\mathrm{j}\omega\lambda^{-1}P = -\frac{1}{c}\frac{\sqrt{(\xi^2-1)(1-\eta^2)}}{\xi^2-\eta^2}\cdot\left[\frac{\partial}{\partial\xi}\left(\sqrt{\frac{\xi^2-\eta^2}{1-\eta^2}}\cdot v_\xi\right)+\frac{\partial}{\partial\eta}\left(\sqrt{\frac{\xi^2-\eta^2}{\xi^2-1}}\cdot v_\eta\right)\right]. \tag{2}$$

Then, simplifying the z-invariant 2D Maxwell equations in TE polarization in elliptical cylindrical coordinate yields

$$\mathrm{j}\omega\mu_\eta H_\eta = -\frac{1}{c}\sqrt{\frac{\xi^2-1}{\xi^2-\eta^2}}\cdot\frac{\partial}{\partial\xi}(-E_z), \quad \mathrm{j}\omega\mu_\xi(-H_\xi) = -\frac{1}{c}\sqrt{\frac{1-\eta^2}{\xi^2-\eta^2}}\cdot\frac{\partial}{\partial\eta}(-E_z),$$
$$\mathrm{j}\omega\varepsilon_z(-E_z) = -\frac{1}{c}\frac{\sqrt{(\xi^2-1)(1-\eta^2)}}{\xi^2-\eta^2}\cdot\left[\frac{\partial}{\partial\xi}\left(\sqrt{\frac{\xi^2-\eta^2}{1-\eta^2}}\cdot H_\eta\right)+\frac{\partial}{\partial\eta}\left(\sqrt{\frac{\xi^2-\eta^2}{\xi^2-1}}\cdot(-H_\xi)\right)\right]. \tag{3}$$

Comparing expressions (2) and (3), we find that the exact variable exchanges between these two sets of equations as follows:

$$[P, v_\xi, v_\eta, \rho_\xi, \rho_\eta, \lambda^{-1}] \leftrightarrow [-E_z, H_\eta, -H_\xi, \mu_\eta, \mu_\xi, \varepsilon_z]. \tag{4}$$

In the EM wave case, we have verified the perfect invisibility of an elliptical cylindrical cloak that contains specific spatial distributions of permittivity and permeability.[10] The analogy between expressions (2) and (3) implies that an elliptical cylindrical cloak can possess perfect acoustic invisibility when its spatial distributions of mass density and bulk modulus are exactly configured according to the spatial distributions of permittivity and permeability in the EM wave case with the variable exchanges in accordance with relationship (4). By this comparison, for 2D elliptical cylindrical acoustic cloaks, the required relative mass density elements and bulk modulus are

$$\rho_\xi = \mu_\eta = \sqrt{\frac{\xi^2-1}{\alpha^2-1}}\cdot\frac{\xi_2-1}{\xi_2-\xi_1},$$
$$\rho_\eta = \mu_\xi = \sqrt{\frac{\alpha^2-1}{\xi^2-1}}\cdot\frac{\xi_2-\xi_1}{\xi_2-1},$$
$$\lambda^{-1} = \varepsilon_z = \sqrt{\frac{\xi^2-1}{\alpha^2-1}}\cdot\frac{\alpha^2-\eta^2}{\xi^2-\eta^2}\cdot\frac{\xi_2-1}{\xi_2-\xi_1}, \tag{5}$$

where $\alpha = (\xi-\xi_1)\dfrac{\xi_2-1}{\xi_2-\xi_1}+1$, and

$$\xi_1 \le \xi \le \xi_2, -1 \le \eta \le 1. \tag{6}$$

3. Simulation and discussion

To verify the above equations, we carried out the numerical simulation by using the commercial software, COMSOL multiphysics, on a two-processor personal computer (PC), and by configuring a computational domain. In the computational domain, a thin elliptic sound soft boundary (SSB) surrounded by an elliptic cloaking shell with the same focus was built and a time-harmonic uniform plane acoustic pressure wave was selected. Theoretically, the inner region of the cloak is shielded from external waves and consequently an object of any shape or material inside it has no influence on the external waves. But practically, especially in computation, the number of the computational elements may not be too large, so the SSB was chosen to ensure that the acoustic pressure

inside it was ideally zero, and thus a similar cloaking performance was achieved. On the other hand, compared with circular cylinders, elliptic cylinders have lower symmetries and therefore it is useful to determine whether the acoustic scattering relies on the incident angles with respect to the elliptic major axis. Under this consideration, we performed the simulations with four incident angles 0°, 30°, 45°, and 90°, separately.

Figure 1 shows the acoustic pressure distributions when a plane pressure wave is launched into the cloak separately along the four directions. In the figure, we find that in the four cases, the phase fronts remain planar outside the cloak and the scattering is quite small; but inside the cloaking shell, the phase fronts are warped, which causes power-flow to bend around the inner shell. We also find that differences of scattering exist in the four cases and that these differences can be reduced by optimizing the computational configuration and by increasing the computational elements, which means small reliance of scattering on incident angle. In short, when elliptical cylindrical cloaks are made according to expressions (5), the incident acoustic waves are smoothly excluded from the interior region with minimal scattering, and so the cloaking effect is evident.

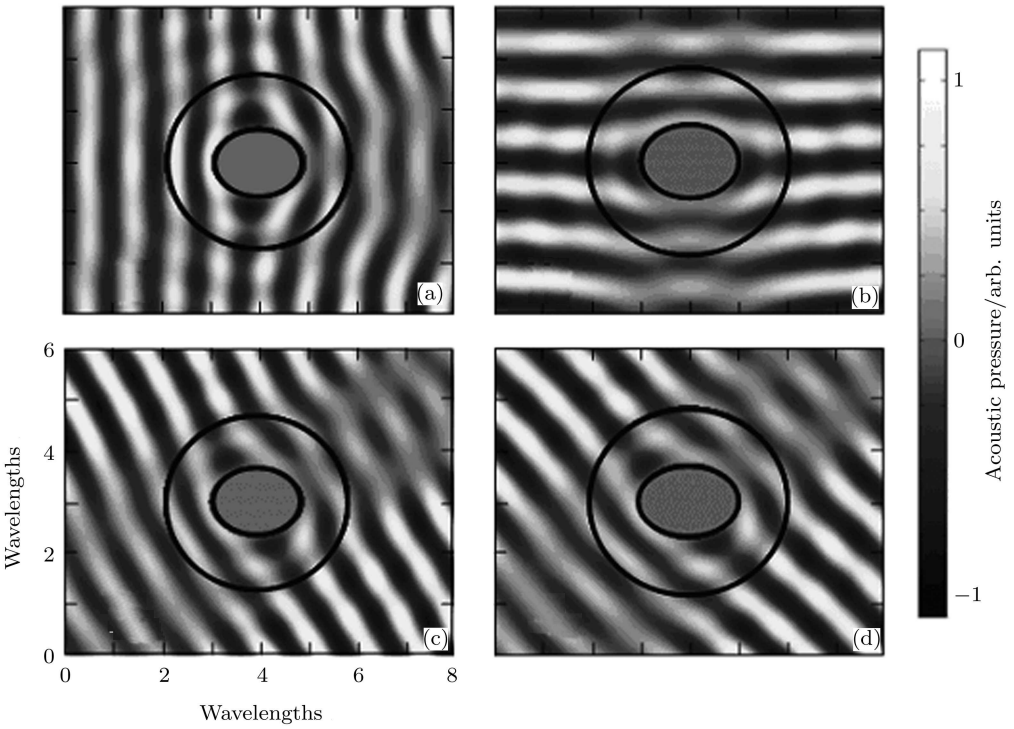


Fig.1. The pressure distributions with acoustic waves being incident at angles: 0°(a), 90°(b), 30°(c) and 45°(d) with respect to the elliptic major axis.

4. Conclusion

By making a comparison between the acoustic equations and 2D Maxwell equations and by adopting the coordinate transformation method, we obtain the MPE for elliptical cylindrical acoustic cloaks. What should be mentioned here is that acoustic waves essentially differ from EM waves, therefore such a comparison method is only based on the form-invariant property that both the acoustic and the EM wave equations have when the coordinate transformation is performed. From this point of view, such a comparison is essentially only an analogue, which offers a comparatively simple method of obtaining the acoustic MPE on condition that the MPE of a similarly shaped EM cloak has already been known. The present study is the meaningful exploration of designing acoustic cloaks that are not spheres or circular cylinders in shape, and opens up possibilities for making complex and multiplex acoustic cloaks with simple models such as spheres, circular or elliptic cylinders.

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