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Optical bistability and multistability in a three-level Δ-type atomic system under the nonresonant condition*

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Under a nonresonant condition, we theoretically investigate hybrid absorptive-dispersive optical bistability and multistability behaviours in a three-level Δ-type system by using a microwave field to drive a hyperfine transition between two upper excited states inside a unidirectional ring cavity. We find that the optical bistability and multistability behaviours can be controlled by adjusting the intensity of the microwave field or the intensity of the coherent coupling field. Furthermore, our studies show an interesting phenomenon of the transition from the optical bistability to the optical multistability only by changing the negative detuning of the coupling field into the positive detuning of the coupling field.

Keywords: optical bistability, multistability, detuning
PACC: 4250, 4265P

1. Introduction

In a multilevel atomic system with a strong coherent field, the absorption and the dispersion of a probe field have aroused the interest of investigators. Studies show that atomic coherent and quantum correlations have exhibited many interesting and unexpected consequences.[1−15] For example, coherent population trapping, lasing without inversion, and electromagnetically induced transparency.

The principle of optical bistability (OB) was first introduced by Szöke *et al* [16] in 1969. In 1976, Gibbs *et al* [17] observed the phenomenon of OB in experiment. The OB in two-level systems[18−23] has attracted great attention for more than a decade due to its potential applications in many fields, such as optical transistors, memory elements, all optical switches, and so on. The effects of both the squeezed state fields and the phase fluctuation on the OB have subsequently been studied.[24,25] Recently, the OB in three-level systems has also been studied theoretically and experimentally.[26−31] Walls and Zoller[26] proposed a novel mechanism for the OB by utilizing narrow nonabsorption resonances which occur in a coherently driven three-level system. Joshi *et al* [29] experimentally studied the OB behaviour in an optical ring cavity filled with a collection of three-level rubidium atoms coupled by a strong coherent field and a weak probe field, and their studies showed that the OB could easily be controlled by changing the intensity and the frequency detuning of the coupling field. Cheng *et al* [30] investigated the steady-state OB behaviour in a three-level $\wedge$-type atomic system enclosed by a microwave field, on condition that the applied fields are nonresonant with corresponding atomic transitions. Under the resonant condition, Li[31] studied the OB behaviour of a three-level $\vee$-type atomic system driven by a microwave field, and found that the OB could be controlled by adjusting the intensity of the microwave field and relative phase. The OB and the optical multistability (OM) in a four-level atomic system have been studied in Refs.[32, 33].

In the present paper, we choose a three-level Δ-type system in a unidirectional ring cavity. Under the nonresonant condition, we investigate the OB and the OM by adjusting the intensity of the microwave field

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or the intensity and the detuning of the coherent coupling field. The OB or OM in our scheme essentially is of a hybrid type that is formed by combining absorptive type with dispersive type. The remainder of the present paper is organized as follows. In Section 2, we present the theoretical model and establish the corresponding Maxwell–Bloch equations. Our numerical results and physical analysis are shown in Section 3. The conclusions are briefly given in Section 4.

2. Theoretical model

The schematic diagram of studied system is shown in Fig.1(a), and it comprises two upper excited states $|3\rangle$ and $|2\rangle$ and one ground state $|1\rangle$. The transition $|2\rangle \leftrightarrow |1\rangle$ with frequency ω_{21} is driven by a coherent coupling field, $E_{\rm c}$ and $\omega_{\rm c}$ are amplitude and angular frequency, respectively.

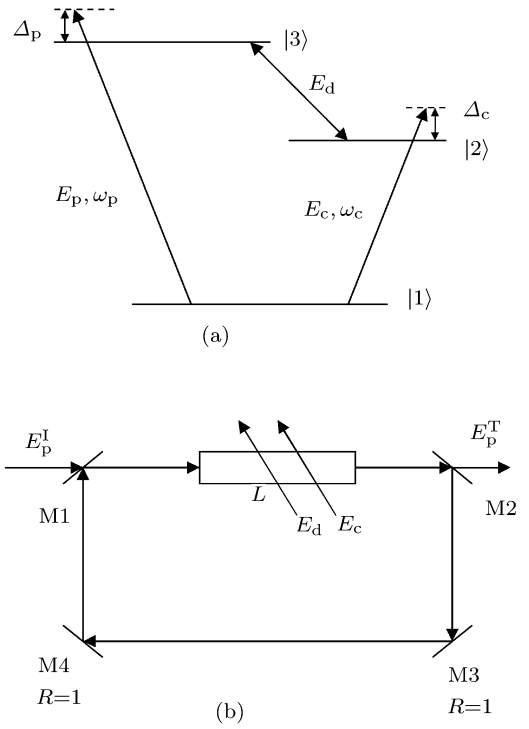


Fig.1. Relevant energy diagram of a three-level △-type atoms (a) and schematic of a unidirectional ring cavity (b). (a) Relevant energy diagram of a three-level △-type atoms in a coherent medium interacting with a coherent coupling field $E_{\rm c}$, a microwave field $E_{\rm d}$, and a probe field $E_{\rm p}$. (b) Schematic of a unidirectional ring cavity with four mirrors and an atomic vapour cell of length L, where M3 and M4 are perfect mirrors, $E_{\rm c}$ and $E_{\rm d}$ are the noncirculating coupling field and microwave field, respectively and $E_{\rm p}^{\rm I}$ and $E_{\rm p}^{\rm T}$ are the incident field and transmitted field, respectively.

A probe field with amplitude $E_{\rm p}$ and angular frequency $\omega_{\rm p}$ is applied to driving the transition $|3\rangle \leftrightarrow |1\rangle$ of frequency ω_{31}, and a microwave field with amplitude $E_{\rm d}$ is added to the transition $|3\rangle \leftrightarrow |2\rangle$. We denote by γ_{mn} (m=3, 2, and n=1) the decay rates of the populations from $|m\rangle$ to $|n\rangle$. For three radiation fields, the coherent coupling and microwave field do not circulate in the ring cavity, and the probe field circulates in the ring cavity. The Hamiltonian of this system in the Schrödinger picture in the rotating-wave approximation is given by (we let h =1)

$$\begin{aligned} H =&\omega_{31}|3\rangle\langle 3| + \omega_{21}|2\rangle\langle 2| \\ &- (\Omega_{\rm p}{\rm e}^{-{\rm i}\omega_{\rm p}t}|3\rangle\langle 1| + \Omega_{\rm c}{\rm e}^{-{\rm i}\omega_{\rm c}t}|2\rangle\langle 1| \\ &+ \Omega_{\rm d}{\rm e}^{-{\rm i}\omega_{\rm d}t}|3\rangle\langle 2| + {\rm H.c}), \end{aligned} \tag{1}$$

where we let $E_1 = \hbar\omega_1 = 0$. If we choose $H_0 = \omega_{\rm p}|3\rangle\langle 3| + \omega_{\rm c}|2\rangle\langle 2|$, in the interaction picture, the Hamiltonian of this system becomes

$$\begin{aligned} H_{\rm int} =& -\Delta_{\rm p}|3\rangle\langle 3| - \Delta_{\rm c}|2\rangle\langle 2| \\ &- (\Omega_{\rm p}|3\rangle\langle 1| + \Omega_{\rm c}|2\rangle\langle 1| \\ &+ \Omega_{\rm d}|3\rangle\langle 2| + {\rm H.c}), \end{aligned} \tag{2}$$

where H.c means Hermitian conjugation, $\Delta_{\rm p}$ is the detuning of the probe field $E_{\rm p}$ from the atomic transition $|3\rangle \leftrightarrow |1\rangle$, $\Delta_{\rm c}$ is the detuning of the coherent coupling field $E_{\rm c}$ from the atomic transition $|2\rangle \leftrightarrow |1\rangle$, $\Omega_{\rm p} = \delta_{31}E_{\rm p}/2\hbar$ and $\Omega_{\rm c} = \delta_{21}E_{\rm c}/2\hbar$ are the one-half Rabi frequencies $\Omega_{\rm p}$ and $\Omega_{\rm c}$ for the corresponding transitions, $\Omega_{\rm d} = \delta_{32}E_{\rm d}/2\hbar$ is the one-half Lamor frequency $\Omega_{\rm D}$ for the transition $|3\rangle \leftrightarrow |2\rangle$, and δ_{31}, δ_{21} and δ_{32} are the relevant dipole moments.

We study the three-level Δ-type system by using a density-matrix approach. From the Liouville equation we can easily obtain the following equations for the density-matrix elements:

$$\begin{aligned} {\rm i}\dot{\rho}_{11} =&\Omega_{\rm c}\rho_{12} + \Omega_{\rm p}\rho_{13} - \Omega_{\rm p}^*\rho_{31} \\ &- \Omega_{\rm c}^*\rho_{21} + {\rm i}\gamma_{21}\rho_{22} + {\rm i}\gamma_{31}\rho_{33}, \\ {\rm i}\dot{\rho}_{22} =&\Omega_{\rm d}\rho_{23} + \Omega_{\rm c}^*\rho_{21} - \Omega_{\rm d}^*\rho_{32} \\ &- \Omega_{\rm c}\rho_{12} - {\rm i}\gamma_{21}\rho_{22}, \\ {\rm i}\dot{\rho}_{33} =&\Omega_{\rm d}^*\rho_{32} + \Omega_{\rm p}^*\rho_{31} - \Omega_{\rm p}\rho_{13} \\ &- \Omega_{\rm d}\rho_{23} - {\rm i}\gamma_{31}\rho_{33}, \\ {\rm i}\dot{\rho}_{21} =&\Omega_{\rm c}\rho_{22} + \Omega_{\rm p}\rho_{23} - \Omega_{\rm c}\rho_{11} \\ &- \Omega_{\rm d}^*\rho_{31} - \Delta_{\rm c}\rho_{21} - \frac{\rm i}{2}\gamma_{21}\rho_{21}, \\ {\rm i}\dot{\rho}_{31} =&\Omega_{\rm c}\rho_{32} + \Omega_{\rm p}\rho_{33} - \Omega_{\rm p}\rho_{11} \\ &- \Omega_{\rm d}\rho_{21} - \Delta_{\rm p}\rho_{31} - \frac{\rm i}{2}\gamma_{31}\rho_{31}, \\ {\rm i}\dot{\rho}_{32} =&\Omega_{\rm d}\rho_{33} + \Omega_{\rm c}^*\rho_{31} + \Delta_{\rm c}\rho_{32} - \Omega_{\rm p}\rho_{12} - \Omega_{\rm d}\rho_{22} \\ &- \Delta_{\rm p}\rho_{32} - \frac{\rm i}{2}(\gamma_{21} + \gamma_{31})\rho_{32}, \end{aligned} \tag{3}$$

where the over-dots represent the derivative with respect to time t, $\sum_{m=1}^{3}\rho_{mm}=1$, and $\rho_{mn}=\rho_{nm}^{*}$ (m=3, 2, n=1, 2, and $m\neq n$). For simplicity, we assume that decay rates $\gamma_{31=}\gamma_{21=}\gamma$ and all electric dipole matrix elements are also the same. In the unidirectional ring cavity, we assume that the atomic system is a collection of homogeneously broadened three-level atoms, the mirrors 3 and 4 have 100% reflectivity, and the reflection and the transmission coefficients of mirrors M1 and M2 are R and T, respectively (for details see Fig.1(b)).

In our system, the probe field which circulates in the optical cavity is governed by Maxwell's equation, then in the slowly varying envelope approximation, the dynamic equation of the probe field is given by

$$\frac{\partial E_{\rm p}}{\partial t}+c\frac{\partial E_{\rm p}}{\partial z}=\mathrm{i}\frac{\omega_{\rm p}}{2\varepsilon_0}P(\omega_{\rm p}), \tag{4}$$

where c and ε_0 are the light speed and permittivity in free space, respectively, and $P(\omega_{\rm p})$ is the induced polarization in transition $|3\rangle\leftrightarrow|1\rangle$ and it is in slow oscillation and given by $P(\omega_{\rm p})=N\delta\rho_{31}$, where N is the number density of atoms in the sample.

In the steady state, the term $\dfrac{\partial E_{\rm p}}{\partial t}=0$, Eq.(4) can be given by

$$\frac{\partial E_{\rm p}}{\partial z}=\mathrm{i}\frac{N\omega_{\rm p}\delta}{2c\varepsilon_0}\rho_{31}. \tag{5}$$

For a perfectly tuned ring cavity, in the steady state limit, the boundary conditions impose the following conditions between the incident field $E_{\rm p}^{\rm I}$ and the transmitted field $E_{\rm p}^{\rm T}$

$$E_{\rm p}(L)=E_{\rm p}^{\rm T}/\sqrt{T}, \tag{6a}$$

$$E_{\rm p}(0)=\sqrt{T}E_{\rm p}^{\rm I}+RE_{\rm p}(L), \tag{6b}$$

where L is the length of the atomic sample, and the second term on the right-hand side of Eq.(6b) describes a feedback mechanism due to the mirror, which essentially gives rise to the OB or the OM, that is to say, there is no OB or OM when R=0.

In the mean-field limit, normalizing the fields by letting $y=\dfrac{\delta E_{\rm p}^{\rm I}}{\hbar\sqrt{T}}$ and $x=\dfrac{\delta E_{\rm p}^{\rm T}}{\hbar\sqrt{T}}$ and using the boundary conditions, we can obtain the input–output relationship

$$y=x-\mathrm{i}C\gamma\rho_{31}, \tag{7}$$

where $C=\dfrac{N\omega_{\rm p}L\delta^2}{2\hbar\varepsilon_{\rm p}cT\gamma}$ is the usual cooperation parameter, and the second term on the right-hand side of expression (7) is very important for achieving the OB or OM.

3. Numerical results

Now we show some numerical studies of steady state under the conditions of nonresonance. In the following numerical calculations, all the parameters are reduced to being dimensionless by scaling γ. In Fig.2(a), we plot the curves for output field versus input field at different intensities of the coherent coupling field $\Omega_{\rm C}$ on condition that the applied fields do not resonate with corresponding atomic transitions. Under the condition of the large coupling-field detuning $\Delta_{\rm c}$ (where $\Delta_{\rm c}=6\gamma$) and the small intensity of coupling field $\Omega_{\rm C}$, we can obtain the OM, for which the main reason is that the order of the variable x in expression (7) goes higher for three-level atoms, and it depends on the relative strengths of various parameters associated with the three-level atoms, that is to say, the observed OM depends on the complicated form of the variable x. With the increase of $\Omega_{\rm C}$, the OM gradually disappears, and the OB behaviour occurs for which the main reason is that the increase of the coupling field can reduce the Kerr nonlinearity of atomic medium. In order to gain a deeper understanding of the problem, we plot the curves for absorption [Im(ρ_{31})] and dispersion [Re(ρ_{31})] versus coupling field $\Omega_{\rm C}$ in Fig.2(b). We can easily see that the change of the dispersion associated with the Kerr nonlinearity of atomic medium is very small and the change of the absorption is also very small when $\Omega_{\rm C}>30\gamma$. Therefore, we can realize the transition between the OB and the OM through the changing of the intensity of the coupling field under the condition of nonresonance. If we consider $\Omega_{\rm C}=0$, levels $|1\rangle$, $|2\rangle$ and $|3\rangle$ constitute a $\wedge$-type atomic system whose optical bistability behaviour has been investigated in Ref.[29]. Choosing appropriate physical parameters, we can also obtain the OB under the condition of $\Omega_{\rm C}=0$.

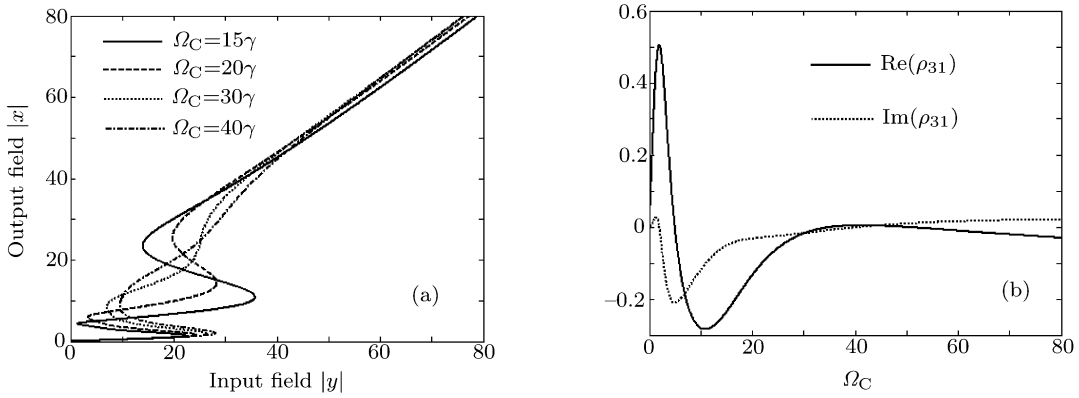


Fig.2. Curves for output field versus input field (a) and absorption and dispersion versus coupling field (b). (a) Curves for output field versus input field at different values of coupling-field intensity $\Omega_{\rm C}$, with other parameters being $C = 150$, $\Delta_{\rm c} = 6\gamma$, $\Delta_{\rm p} = 3\gamma$, $\gamma_{31} = \gamma_{21} = \gamma$, and $2\Omega_{\rm d} = 3\gamma$. (b) The absorption [$\mathrm{Im}(\rho_{31})$] and the dispersion [$\mathrm{Re}(\rho_{31})$] versus coupling field $\Omega_{\rm C}$, with the parameters being the same as those in Fig.2(a) except for $\Omega_{\rm p} = 5\gamma$.

Figure 3 demonstrates the dependences of the OB and OM on the intensity of the microwave field $\Omega_{\rm D}$ under the nonresonant conditions. From Fig.3, if we chose the detunings $\Delta_{\rm c} = 4\gamma$ and $\Delta_{\rm p} = 2\gamma$, we can find the OB takes place only for a small value of $\Omega_{\rm D}$. When $\Omega_{\rm D} = 0$, that is, the microwave field does not exist, the system is a $\vee$-type atomic system where we can obtain the OB(see dash-dot line in Fig.3). While increasing the intensity of the microwave field, we can observe the OM, which can be qualitatively explained as follows. When a microwave field with an appropriate intensity is applied, the transition $|3\rangle \leftrightarrow |2\rangle$ takes place, which leads to an increase in the Kerr nonlinearity of the atomic medium and the occurrence of the OM behaviour. When the $\Omega_{\rm D}$ becomes too large, the OM will disappear, which is also due to the reduction in nonlinearity in such a case.

An interesting effect of the detuning of the coupling field on OM and OB can be easily seen in Fig.4. Figure 4 shows that we can convert the OM into the OB when we change the positive detuning of the coupling field into the negative detuning of the coupling field. A qualitative explanation for this phenomenon is as follows. We have assumed $\omega_{\rm p} = \omega_{\rm d} + \omega_{\rm c}$ in process of deriving Eq.(2), and then we can easily obtain the relation among the detunings of the three fields as $\Delta_{\rm p} = \Delta_{\rm c} + \Delta_{\rm d}$. When the $\Delta_{\rm c}$ is positive, the detuning of the microwave field is not very large, which enhances the coherence of the atomic medium, and we can see the OM. When the $\Delta_{\rm c}$ is negative, the detun-

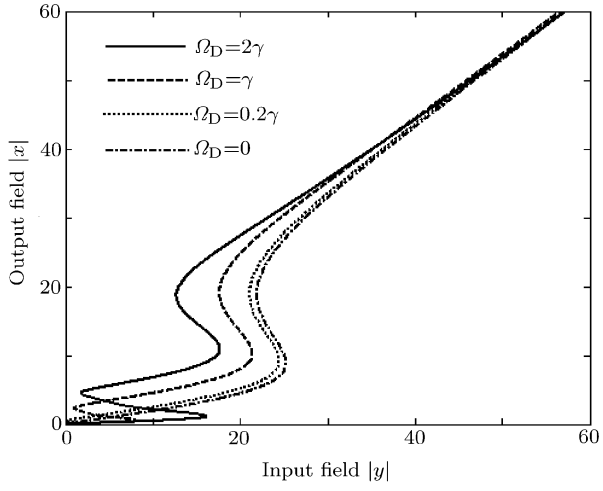


Fig.3. Curves for output field versus input field at different values of microwave-field intensity $\Omega_{\rm D}$, with other parameters being $C = 100$, $\Delta_{\rm p} = 2\gamma$, $\Delta_{\rm c} = 4\gamma$, $\gamma_{31} = \gamma_{21} = \gamma$, and $2\Omega_{\rm c} = 15\gamma$.

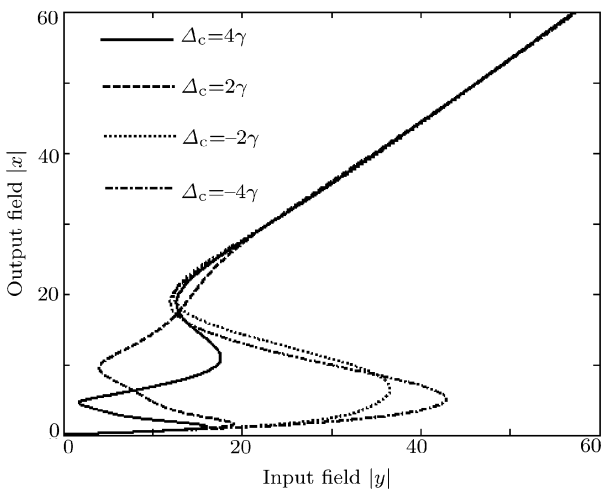


Fig.4. Curves for output field versus input field at different values of detuning of coupling-field$\Delta_{\rm c}$, with other parameters being $C = 100$, $\Delta_{\rm p} = 2\gamma$, $\gamma_{31} = \gamma_{21} = \gamma$, and $2\Omega_{\rm d} = 2\gamma$, $2\Omega_{\rm c} = 15\gamma$.

ing of the microwave field becomes very large, which weakens the coherence of the atomic medium, then we can see only the OB, that is to say, the change of the Δ_c can modify the absorption and change the nonlinearity of atomic medium. So, we can control the OM and the OB through adjusting the detuning of the coupling field.

4. Conclusions

We theoretically investigate a hybrid absorptive-dispersive optical bistability and multistability behaviours in a three-level Δ-type system by using a microwave field to drive a hyperfine transition between two upper excited states inside a unidirectional ring cavity under the nonresonant condition. We find that the optical bistability and multistability behaviours can be controlled by adjusting the intensity of microwave field or the intensity of the coherent coupling field. What is more, our study shows a new way the transition from the optical bistability to the multistability takes place only by changing the negative detuning of the coupling field into the positive detuning of the coupling field. Such a system can be used for making efficient and fast all-optical switching devices and optical transistors.

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