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Deterministic generation of entangled coherent states for two atomic samples*

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This paper proposes an efficient scheme for deterministic generation of entangled coherent states for two atomic samples. In the scheme two collections of atoms are trapped in an optical cavity and driven by a classical field. Under certain conditions the two atomic samples evolve from an coherent state to an entangled coherent state. During the interaction the cavity mode is always in the vacuum state and the atoms have no probability of being populated in the excited state. Thus, the scheme is insensitive to both the cavity decay and atomic spontaneous emission.

Keywords: entangled coherent state, atomic sample, Raman coupling
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Entanglement is one of the most striking features of quantum mechanics, which is not only an essential ingredient for testing quantum mechanics against local hidden variable theories,[1] but also a key resource for the implementation of quantum information processing tasks, such as quantum cryptography[2] and quantum teleportation.[3] Because atoms are ideal for the storage of quantum information and photons are suitable for the transportation of quantum information, the cavity QED system is a qualified candidate for entanglement engineering and quantum information processing. Numerous schemes have been proposed for the generation of entangled states for two or more atoms via interaction with a cavity mode.[4−7] Recent advances in microwave cavity QED techniques, along with Rydberg atoms interacting with a superconducting cavity, allow the test of many interesting quantum effects arising from the interaction of atoms with a quantized cavity field. Up to now, entangled states involving two or three atoms have been produced in experiment.[8,9]

The above mentioned researches concentrate on entanglement of individual atoms. The creation and measurement of macroscopic entanglement are important for exploring the fuzzy quantum-classical boundary. In addition to fundamental interest, the entanglement between two atomic samples has applications in quantum communication. Schemes have been proposed to entangle two atomic samples with linear optics.[10,11] However these schemes are probabilistic and the success probability depends upon the efficiency of the photodetectors.

This paper presents an alternative scheme to generate entangled coherent states for two collections of atoms in an optical cavity. In the scheme the atomic samples interact with the cavity mode and a classical field. Under the large detuning condition the cavity mode and classical field induce Raman coupling between two atomic ground states. When the difference between the detunings of the cavity mode and classical field is much larger than the Raman coupling strength, the atomic samples do not exchange energy with the fields. However, the dispersive Raman interaction results in the coupling between the two atomic samples. After an appropriate interaction time, the two atomic samples evolve to an entangled coherent state due to the atom–atom coupling. During the procedure the cavity mode is always in the vacuum state and the atoms are always populated in two ground states. Thus both the cavity decay and atomic spontaneous emission are suppressed. Furthermore, the scheme is deterministic.

We consider two collections of identical three-level atoms trapped in a single-mode optical cavity. The two atomic samples involve N_1 and N_2 atoms, respectively. Each atom has two ground states $|e\rangle$ and $|g\rangle$

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and an excited state $|r\rangle$. The transition $|e\rangle \rightarrow |r\rangle$ is driven by a classical laser field with a Rabi frequency Ω, while the transition $|g\rangle \rightarrow |r\rangle$ is driven by the cavity mode with the coupling constant g. Assume that the classical field and cavity mode are detuned from the respective transitions by Δ_1 and Δ_2, as shown in Fig.1.

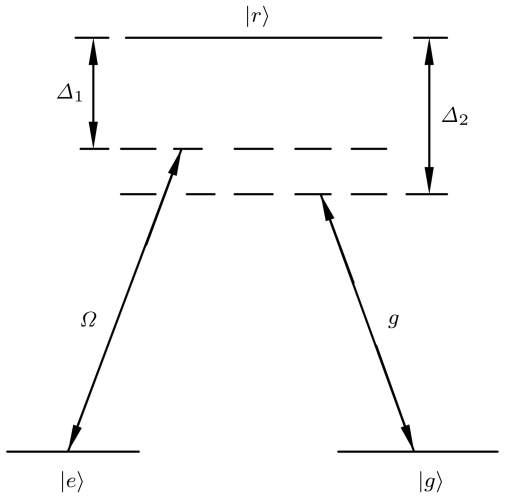


Fig.1. The atomic level configuration and transitions. The transition $|e\rangle \rightarrow |r\rangle$ is driven by a classical laser field with the Rabi frequency Ω and detuning Δ_1, and the transition $|g\rangle \rightarrow |r\rangle$ is coupled to the cavity mode with the coupling constant g and detuning Δ_2.

In the interaction picture, the Hamiltonian is

$$\begin{aligned} H = & \sum_{j=1}^{N_1}(\Omega \mathrm{e}^{\mathrm{i}\Delta_1 t}\left|r_{1,j}\right\rangle\left\langle e_{1,j}\right| + g a \mathrm{e}^{\mathrm{i}\Delta_2 t}\left|r_{1,j}\right\rangle\left\langle g_{1,j}\right|) \\ & + \sum_{j=1}^{N_2}(\Omega \mathrm{e}^{\mathrm{i}\Delta_1 t}\left|r_{2,j}\right\rangle\left\langle e_{2,j}\right| + g a \mathrm{e}^{\mathrm{i}\Delta_2 t}\left|r_{2,j}\right\rangle\left\langle g_{2,j}\right|) \\ & + \mathrm{H.c.}, \end{aligned} \tag{1}$$

where a is the annihilation operator of the cavity mode. Under the condition Δ_1, $\Delta_2 \gg \Omega$, g, the upper level $|r\rangle$ can be adiabatically eliminated, hence we obtain the effective Hamiltonian[12,13]

$$H_{\mathrm{e}} = H_{\mathrm{e},1} + H_{\mathrm{e},2}, \tag{2}$$

where

$$\begin{aligned} H_{\mathrm{e},1} = & -\sum_{j=1}^{N_1}\left[\frac{\Omega^2}{\Delta_1}\left|e_{1,j}\right\rangle\left\langle e_{1,j}\right| + \frac{g^2}{\Delta_2} a^+ a \left|g_{1,j}\right\rangle\left\langle g_{1,j}\right|\right] \\ & -\sum_{j=1}^{N_2}\left[\frac{\Omega^2}{\Delta_1}\left|e_{2,j}\right\rangle\left\langle e_{2,j}\right| + \frac{g^2}{\Delta_2} a^+ a \left|g_{2,j}\right\rangle\left\langle g_{2,j}\right|\right], \end{aligned} \tag{3}$$

$$\begin{aligned} H_{\mathrm{e},2} = & -\sum_{j=1}^{N_1}\lambda(a S_{1,j}^{+}\mathrm{e}^{\mathrm{i}\delta t} + a^+ S_{1,j}^{-}\mathrm{e}^{-\mathrm{i}\delta t}) \\ & -\sum_{j=1}^{N_2}\lambda(a S_{2,j}^{+}\mathrm{e}^{\mathrm{i}\delta t} + a^+ S_{2,j}^{-}\mathrm{e}^{-\mathrm{i}\delta t}), \end{aligned} \tag{4}$$

$\lambda = \dfrac{\Omega g}{2}\left(\dfrac{1}{\Delta_1} + \dfrac{1}{\Delta_2}\right)$, $\delta = \Delta_2 - \Delta_1$, $S_{l,j}^{+} = \left|e_{l,j}\right\rangle\left\langle g_{l,j}\right|$ and $S_{l,j}^{-} = \left|g_{l,j}\right\rangle\left\langle e_{l,j}\right|$. $H_{\mathrm{e},1}$ describes the Stark shifts for the two ground states $|e_j\rangle$ and $|g_j\rangle$, induced by the classical field and cavity fields, and $H_{\mathrm{e},2}$ describes the Raman coupling between the two ground states.

In the case $\delta \gg \lambda$, there is no energy exchange between the atomic systems and the cavity. The energy conversing transitions are between $|e_{l,j}g_{m,k}n\rangle$ and $|g_{l,j}e_{m,k}n\rangle$. The effective Rabi frequency for the transitions between these states, mediated by $|g_{l,j}g_{m,k}n+1\rangle$ and $|e_{l,j}e_{m,k}n-1\rangle$, is given by[6,13]

$$\begin{aligned} \lambda' = & \frac{\left\langle e_{l,j}g_{m,k}n\right| H_i \left|g_{l,j}g_{m,k}n+1\right\rangle\left\langle g_{l,j}g_{m,k}n+1\right| H_i \left|g_{l,j}e_{m,k}n\right\rangle}{\delta} \\ & + \frac{\left\langle e_{l,j}g_{m,k}n\right| H_i \left|e_{l,j}e_{m,k}n-1\right\rangle\left\langle e_{l,j}e_{m,k}n-1\right| H_i \left|g_{l,j}e_{m,k}n\right\rangle}{-\delta} \\ = & \frac{\lambda^2}{\delta}. \end{aligned} \tag{5}$$

Since the two transition paths interfere destructively, the Rabi frequency is independent of the photon-number of the cavity mode. Then the nonresonant Raman coupling leads to the Stark shifts and atom–atom coupling, hence $H_{\mathrm{e},2}$ can be replaced by[6,13]

$$\begin{aligned} H'_{\mathrm{e},2} = & \lambda'\left[\sum_{j=1}^{N_1}(\left|e_{1,j}\right\rangle\left\langle e_{1,j}\right| a a^+ - \left|g_{1,j}\right\rangle\left\langle g_{1,j}\right| a^+ a) \right. \\ & \left. + \sum_{j=1}^{N_2}(\left|e_{2,j}\right\rangle\left\langle e_{2,j}\right| a a^+ - \left|g_{2,j}\right\rangle\left\langle g_{2,j}\right| a^+ a)\right. \end{aligned}$$

$$+\sum_{j,k=1}^{N_1} S_{1,j}^{+}S_{1,k}^{-}+\sum_{j,k=1}^{N_2} S_{2,j}^{+}S_{2,k}^{-} +\sum_{\mu=1}^{N_1}\sum_{\nu=1}^{N_2} S_{1,\mu}^{+}S_{2,\nu}^{-}+\sum_{\mu=1}^{N_1}\sum_{\nu=1}^{N_2} S_{1,\mu}^{-}S_{2,\nu}^{+}\Bigg], \quad j\neq k. \tag{6}$$

The entire process is a four-photon transition.[13,14] Since $a^{+}a$ commutes with the total effective Hamiltonian $H_{\mathrm{e},1}+H'_{\mathrm{e},2}$, the photon-number does not change during the process. When the cavity mode is initially in the vacuum state, it will remain in the vacuum state throughout the procedure. Then the Hamiltonians $H_{\mathrm{e},1}$ and $H'_{\mathrm{e},2}$ reduce to

$$H_{\mathrm{e},1}=\sum_{j=1}^{N_1}\frac{\Omega^2}{\Delta_1}\left|e_{1,j}\right\rangle\left\langle e_{1,j}\right|+\sum_{j=1}^{N_2}\frac{\Omega^2}{\Delta_1}\left|e_{2,j}\right\rangle\left\langle e_{2,j}\right| \tag{7}$$

and

$$H'_{\mathrm{e},2}=\lambda'\Bigg[\sum_{j=1}^{N_1}\left|e_{1,j}\right\rangle\left\langle e_{1,j}\right|+\sum_{j=1}^{N_2}\left|e_{2,j}\right\rangle\left\langle e_{2,j}\right| +\sum_{j,k=1}^{N_1} S_{1,j}^{+}S_{1,k}^{-}+\sum_{j,k=1}^{N_2} S_{2,j}^{+}S_{2,k}^{-} +\sum_{\mu=1}^{N_1}\sum_{\nu=1}^{N_2} S_{1,\mu}^{+}S_{2,\nu}^{-}+\sum_{\mu=1}^{N_1}\sum_{\nu=1}^{N_2} S_{1,\mu}^{-}S_{2,\nu}^{+}\Bigg], \quad j\neq k. \tag{8}$$

We can rewrite $H'_{\mathrm{e},2}$ as

$$H'_{\mathrm{e},2}=\lambda' S^{+}S^{-}, \tag{9}$$

where

$$S^{+}=\sum_{j=1}^{N_1} S_{1,j}^{+}+\sum_{j=1}^{N_2} S_{2,j}^{+}, \qquad S^{-}=\sum_{j=1}^{N_1} S_{1,j}^{-}+\sum_{j=1}^{N_2} S_{2,j}^{-}. \tag{10}$$

The operators S^{+} and S^{-} obey the commutation relations of the angular-momentum operators.

Assume that each atom is initially in the state $(1+|\xi|^2)^{-1/2}(|g_{l,j}\rangle+\xi\,|e_{l,j}\rangle)$. In this case each atomic system is initially in the coherent state[15]

$$|\xi,N_l\rangle=(1+|\xi|^2)^{-N_l/2}\sum_{k=0}^{N_l}\xi^{k_l}\,|k_l\rangle\,, \tag{11}$$

where $|k_l\rangle$ are the symmetric states with k_l atoms in the state $|e\rangle$,

$$|k_l\rangle=\sum_{j} P_j(|e_{l,1},e_{l,2},\ldots,e_{l,k},g_{l,k+1},g_{l,k+2},\ldots g_{l,N_l}\rangle), \tag{12}$$

where $\{P_j\}$ denotes the set of all distinct permutations of the N_l atoms. The initial state for the whole system can be written as

$$\begin{aligned}&|\xi,N_1\rangle\,|\xi,N_2\rangle\\ &=(1+|\xi|^2)^{-N/2}\sum_{k_1=0}^{N_1}\sum_{k_2=0}^{N_2}\xi^{k_1+k_2}\,|k_1\rangle\,|k_2\rangle\\ &=(1+|\xi|^2)^{-N/2}\sum_{k=0}^{N}\xi^{k}\,|\phi_k\rangle\,,\end{aligned} \tag{13}$$

where

$$|\phi_k\rangle=\sum_{k_2=0}^{k}|k-k_2\rangle\,|k_2\rangle\,, \qquad N=N_1+N_2. \tag{14}$$

The application of the collective raising and lowering operators to the state $|\phi_k\rangle$ leads to[16]

$$S^{+}\,|\phi_k\rangle=\sqrt{(k+1)(N-k)}\,|\phi_{k+1}\rangle\,, \qquad S^{-}\,|\phi_k\rangle=\sqrt{k(N-k+1)}\,|\phi_{k-1}\rangle\,. \tag{15}$$

Thus we have

$$(H_{\mathrm{e},1}+H'_{\mathrm{e},2})\,|\phi_k\rangle=\left[\frac{\Omega^2}{\Delta_1}k+\lambda' k(N-k+1)\right]|\phi_k\rangle\,. \tag{16}$$

The time evolution of the system is given by

$$|\psi(t)\rangle=(1+|\xi'|^2)^{-N/2}\sum_{k=0}^{N}\xi'^{k}\mathrm{e}^{\mathrm{i}\lambda' k^2 t}\,|\phi_k\rangle\,, \tag{17}$$

where $\xi'=\xi\mathrm{e}^{-\mathrm{i}\left[\frac{\Omega^2}{\Delta_1}+\lambda'(N+1)\right]t}$. With the choice $\lambda' t=\dfrac{\pi}{2}$ we obtain

$$|\psi(t)\rangle=(1+|\xi'|^2)^{-N/2}\sum_{k=0}^{N}\xi'^{k}\mathrm{i}^{k^2}\,|\phi_k\rangle\,. \tag{18}$$

We can rewrite $|\psi(t)\rangle$ as

$$|\psi(t)\rangle=\frac{1}{\sqrt{2}}(1+|\xi'|^2)^{-N/2}\times\sum_{k=0}^{N}\left[\mathrm{e}^{\mathrm{i}\pi/4}\xi'^{k}\,|\phi_k\rangle+\mathrm{e}^{-\mathrm{i}\pi/4}(-\xi')^{k}\,|\phi_k\rangle\right]. \tag{19}$$

Using Eq.(13) we can also rewrite $|\psi(t)\rangle$ as

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(1+|\xi'|^2)^{-N/2}(\mathrm{e}^{\mathrm{i}\pi/4}|\xi,N_1\rangle|\xi,N_2\rangle + \mathrm{e}^{-\mathrm{i}\pi/4}|-\xi,N_1\rangle|-\xi,N_2\rangle). \quad (20)$$

By this way we obtain an entangled coherent states for the two atomic samples. It should be noted that the two atomic samples are entangled via coherent evolution and no conditional measurement is required, which is in contrast with the previous schemes.[10,11]

We now give a brief discussion on the experimental implementation of the proposed model. Set $\Delta_1 = 20\,g$, $\delta = \Omega = g$, $\Gamma = \kappa = 0.01\,g$,[17] where Γ and κ are the decay rates for the atomic excited state and the cavity mode respectively. Then we obtain $\lambda \simeq g/20$ and $\lambda' \simeq g/400$. In this case the conditions Δ_1, $\Delta_2 \gg \Omega$, g and $\delta \gg \lambda$ are satisfied. The required interaction time is $t = 200\,\pi/g$. The atomic samples and the cavity mode are virtually excited. The effective decoherence rates due to the atomic spontaneous emission and cavity decay are $\Gamma' = \Gamma g^2/\Delta_1^2 = 2.5\times10^{-5}\,g$ and $\kappa' = \kappa\lambda^2/\delta^2 = 2.5\times10^{-5}\,g$,[18] respectively. The infidelity induced by the decoherence has the order of $(\Gamma'+\kappa')t \simeq 3.14\times10^{-2}$. In order to individually address the two atomic samples we assume that they are located at two separate antinodes of the cavity mode.

In summary we have proposed a scheme for the preparation of entangled coherent states for two atomic samples in an optical cavity. There are two distinct advantages of the scheme, one is that the cavity mode is always in the vacuum state, the other is that the atoms have no probability of being populated in the excited state. Therefore, the scheme is insensitive to the cavity decay and the atomic spontaneous emission. The scheme does not involve conditional measurement and the entangled state is deterministically obtained.

References

[1] Bell J S 1965 *Physics* **1** 195
[2] Ekert A K 1991 *Phys. Rev. Lett.* **67** 661
[3] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wootters W 1993 *Phys. Rev. Lett.* **70** 1895
[4] Cirac J I and Zoller P 1994 *Phys. Rev.* A **50** R2799
[5] Gerry C C 1996 *Phys. Rev.* A **53** 2857
[6] Zheng S B and Guo G C 2000 *Phys. Rev. Lett.* **85** 2392
[7] Zhang J 2008 *Chin. Phys.* B **17** 1674
[8] Hagley E, Maitre X, Nogues G, Wunderlich C, Brune M, Raimond J M and Haroche S 1997 *Phys. Rev. Lett.* **79** 1
[9] Rauschenbeutel A, Nogues G, Osnaghi S, Bertet P, Brune M, Raimond J M and Haroche S 2000 *Science* **288** 2024
[10] Duan L M, Lukin M D, Cirac J I and Zoller P 2001 *Nature* **414** 413
[11] Zheng S B 2007 *Chin. Phys. Lett.* **24** 3297
[12] Wu Y 1996 *Phys. Rev.* A **54** 1586
[13] Imamoglu A, Auschalom D D, Burkard G, DiVincenzo D P, Loss D, Sherwin M and Small A 1999 *Phys. Rev. Lett.* **83** 4204
[14] Sackett C A, Kielpinski D, King B E, Langer C, Meyer V, Myatt C J, Rowe M, Turchette Q A, Itano W M, Wineland D J and Monroe C 2000 *Nature* **404** 256
[15] Arecchi F T, Courtens E, Giklmore R and Thomas H 1972 *Phys. Rev.* A **6** 2211
[16] Delgado A, Klimov A B, Retamal J C and Saavedra C 1998 *Phys. Rev.* A **58** 655
[17] Serafini A, Macini S and Bose S 2006 *Phys. Rev. Lett.* **96** 010503
[18] Parkins A S, Solano E and Cirac J I 2006 *Phys. Rev. Lett.* **96** 053602