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Propagation of Gauss–Bessel beams in turbulent atmosphere*

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This paper studies the propagation properties of Gauss–Bessel beams in a turbulent atmosphere. Based on the extended Huygens–Fresnel principle, it derives the intensity distribution expression for such beams propagating in a turbulent atmosphere. Then the influence of turbulence and source beam parameters on the beam propagation is studied in great detail. It finds that the intensity distribution of Gauss–Bessel beams will change into Gaussian profile in a turbulent atmosphere, and that stronger turbulence and smaller topological charges will lead to a faster changing.

Keywords: Gauss–Bessel beams, turbulent atmosphere, propagation properties
PACC: 4225B, 4225F, 9265R

1. Introduction

The propagation of laser beams in a turbulent atmosphere is a subject of considerable importance in optical communication.[1,2] Therefore, there has been much research on the propagation of different kinds of laser beams in a turbulent atmosphere.[3−9] In 2002, Gbur and Wolf gave out the condition in which partially coherent beams are less influenced by turbulent medium, which was later experimentally demonstrated by Dogariu and Amarande.[10,11] Furthermore, Shirai, Dogariu and Wolf studied the beam spread of Gaussian–Schell beams in a turbulent atmosphere in 2003.[12]

Recently, the propagation of Bessel beams, called non-diffraction beams, and Gauss–Bessel beams in free space has been extensively studied.[13−15] However, to the best of our knowledge, the propagation of these kinds of beams in a turbulent atmosphere has little been studied.[16] The model of Gauss–Bessel beam was first proposed by Gori *et al* in 1987.[17] It is known that high-order Gauss–Bessel beams have wide applications due to that the beam carries a vortex phase factor $\exp(in\phi)$, which results in a hollow intensity distribution.[18−20] Cai and his co-workers have done much research on different kinds of dark hollow beams.[21−23] Furthermore, since Gauss–Bessel beam is confined by a Gaussian exponential which makes it physically realizable, it has attracted much attention of researchers.[24,25] Therefore, in this paper we want to investigate the propagation of Gauss–Bessel beams in a turbulent atmosphere.

2. Formulation

Let us first consider a Gauss–Bessel beam propagating along the z axis in a turbulent atmosphere. The cross-spectral density of the Gauss–Bessel beam at $z = 0$ plane may be written as

$$W(\boldsymbol{r}_1, \boldsymbol{r}_2, 0, \omega) = \langle E(\boldsymbol{r}_1, 0, \omega) E^*(\boldsymbol{r}_2, 0, \omega) \rangle , \qquad (1)$$

where $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are the position vectors located in the transverse plane $z = 0$, ω is the angular frequency, and $E(\boldsymbol{r}, 0, \omega)$ is the electric field of the beam. The angular brackets indicate an ensemble average.

According to the extended Huygens–Fresnel principle, the cross-spectral density of the beam after propagating for a certain distance z in a turbulent atmosphere is given by[26−28]

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$$W(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z,\omega)=\frac{k^2}{4\pi^2z^2}\int\int\langle E(\boldsymbol{r}_1,0,\omega)E^*(\boldsymbol{r}_2,0,\omega)\rangle\times\exp\bigg[-\frac{\mathrm{i}k}{2z}(\boldsymbol{r}_1-\boldsymbol{\rho}_1)^2+\frac{\mathrm{i}k}{2z}(\boldsymbol{r}_2-\boldsymbol{\rho}_2)^2\bigg]$$
$$\times\langle\exp[\psi(\boldsymbol{r}_1,\boldsymbol{\rho}_1,z,\omega)+\psi(\boldsymbol{r}_2,\boldsymbol{\rho}_2,z,\omega)]\rangle\mathrm{d}\boldsymbol{r}_1\mathrm{d}\boldsymbol{r}_2, \tag{2}$$

where $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ are the position vectors at the receiver plane, k is the wave number, and z is the propagation distance. The first exponential term expresses the diffraction that the propagating beam is subjected to, which is also common in free space propagation. The second exponential term, on the other hand, describes the effects of turbulence on propagation, which can be expressed as[29]

$$\begin{aligned}&\langle\exp[\psi(\boldsymbol{r}_1,\boldsymbol{\rho}_1,z,\omega)+\psi(\boldsymbol{r}_2,\boldsymbol{\rho}_2,z,\omega)]\rangle\\&=\exp\bigg[-\frac{(\boldsymbol{r}_1-\boldsymbol{r}_2)^2+(\boldsymbol{r}_1-\boldsymbol{r}_2)(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)+(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2}{\rho_0^2}\bigg]\\&=\exp\bigg[-\frac{(\boldsymbol{r}_1-\boldsymbol{r}_2)^2}{\rho_0^2}\bigg]\quad(\text{if}\quad\boldsymbol{\rho}_1=\boldsymbol{\rho}_2=\boldsymbol{\rho}),\end{aligned} \tag{3}$$

where $\rho_0=(0.545C_n^2k^2z)^{-3/5}$ is the coherent length of a spherical wave propagating in the turbulent medium, and C_n^2 is the structure constant which indicates the atmospheric turbulence.

The field of the Gauss–Bessel beam at the source plane $z=0$ can be written as

$$E(r,0)=A_0J_n(\alpha r)\exp(\mathrm{i}n\theta)\exp(-r^2/w_0^2), \tag{4}$$

where n denotes the topological charge, J_n is the Bessel function, and α and w_0 are the width parameters, $\exp(\mathrm{i}n\theta)$ is the vortex phase factor, and $\exp(-r^2/w_0^2)$ is the Gaussian exponential that confines the Bessel function.

Substituting Eqs.(3) and (4) into Eq.(2), we obtain the cross-spectral density expression for Gauss–Bessel beam after propagating in turbulent atmosphere as follows

$$W(\rho_1,\phi_1,\rho_2,\phi_2,z)=\frac{A_0^2k^2}{4\pi^2z^2}\int\int\int\int J_n(\alpha r_1)J_n(\alpha r_2)\exp(\mathrm{i}n\theta_1)\exp(-\mathrm{i}n\theta_2)\exp\bigg[-\frac{(r_1^2+r_2^2)}{w_0^2}\bigg]$$
$$\times\exp\bigg[-\frac{\mathrm{i}k}{2z}(r_1^2+\rho_1^2)+\frac{\mathrm{i}k}{z}r_1\rho_1\cos(\theta_1-\phi_1)+\frac{\mathrm{i}k}{2z}(r_2^2+\rho_2^2)-\frac{\mathrm{i}k}{z}r_2\rho_2\cos(\theta_2-\phi_2)\bigg]$$
$$\times\exp\bigg[-\frac{1}{\rho_0^2}(r_1^2+r_2^2)+\frac{2}{\rho_0^2}r_1r_2\cos(\theta_1-\theta_2)\bigg]r_1r_2\mathrm{d}r_1\mathrm{d}r_2\mathrm{d}\theta_1\mathrm{d}\theta_2. \tag{5}$$

Using the following formulae:[30]

$$\exp\bigg[\frac{\mathrm{i}kr_1\rho}{z}\cos(\theta_1-\phi)\bigg]=\sum_{l=-\infty}^{+\infty}\mathrm{i}^lJ_l\bigg(\frac{kr_1\rho}{z}\bigg)\exp[\mathrm{i}l(\theta_1-\phi)], \tag{6}$$

$$\int_0^{2\pi}\exp\bigg[-\mathrm{i}n\theta_1+\frac{2r_1r_2}{\rho_0^2}\cos(\theta_1-\theta_2)\bigg]\mathrm{d}\theta_1=2\pi\exp(-\mathrm{i}n\theta_2)I_n\bigg(\frac{2r_1r_2}{\rho_0^2}\bigg), \tag{7}$$

$$\int_0^{2\pi}\exp(\mathrm{i}m\phi)\mathrm{d}\phi=\begin{cases}2\pi & \text{if } m=0,\\0 & \text{if } m\neq0,\end{cases} \tag{8}$$

$$\begin{aligned}J_{-l}(\alpha r)&=(-1)^lJ_l(\alpha r),\\J_l(-\alpha r)&=(-1)^lJ_l(\alpha r),\end{aligned} \tag{9}$$

and setting $\phi_1=\phi_2=\phi$, $\rho_1=\rho_2=\rho$, we obtain the intensity distribution in the received plane as

$$I(\rho,z)=W(\rho,\rho,z)=\frac{A_0^2k^2}{z^2}\sum_{l=-\infty}^{+\infty}\iint J_n(\alpha r_1)J_n(\alpha r_2)J_l\bigg(\frac{kr_1\rho}{z}\bigg)J_l\bigg(\frac{kr_2\rho}{z}\bigg)I_{(n+l)}\bigg(\frac{2r_1r_2}{\rho_0^2}\bigg)$$
$$\times\exp\bigg[-\frac{\mathrm{i}k}{2z}(r_1^2-r_2^2)\bigg]\exp\bigg[-\frac{1}{\rho_0^2}(r_1^2+r_2^2)\bigg]\exp\bigg[-\frac{(r_1^2+r_2^2)}{w_0^2}\bigg]r_1r_2\mathrm{d}r_1\mathrm{d}r_2. \tag{10}$$

Based on the above derived expressions, we can perform some numerical calculations in the following section.

3. Results and discussions

In this section, we present some numerical results to show the propagation properties of Gauss–Bessel beams in turbulent atmosphere. The change of the intensity distribution against different topological charges and different atmospheric turbulence is studied in great detail. For easier comparison, the normalized intensity $(I(x,0,z)/I(x,0,z)_{\max})$ is used here.

Figure 1 presents some contour plots of zero-order Gauss–Bessel beams propagating in a turbulent atmosphere at several different propagation distances (a) z=0 km, (b) z=0.5 km, (c) z=1.2 km, (d) z=1.8 km, (e) z=3 km, (f) z=4 km. In this figure, the other parameters are chosen as w_0=0.01 m, λ=632.8 nm, $\alpha = 500$, $n = 0$, $C_n^2 = 5 \times 10^{-14}\,\mathrm{m}^{-2/3}$. It is clearly indicated that the Gauss–Bessel beam transforms into hollow shape after propagating in a turbulent atmosphere (for example, z=0.5 km), and finally turns into Gaussian profile and keeps this Gaussian shape invariant. Meanwhile, the beam spreads as it propagates in the turbulent atmosphere.

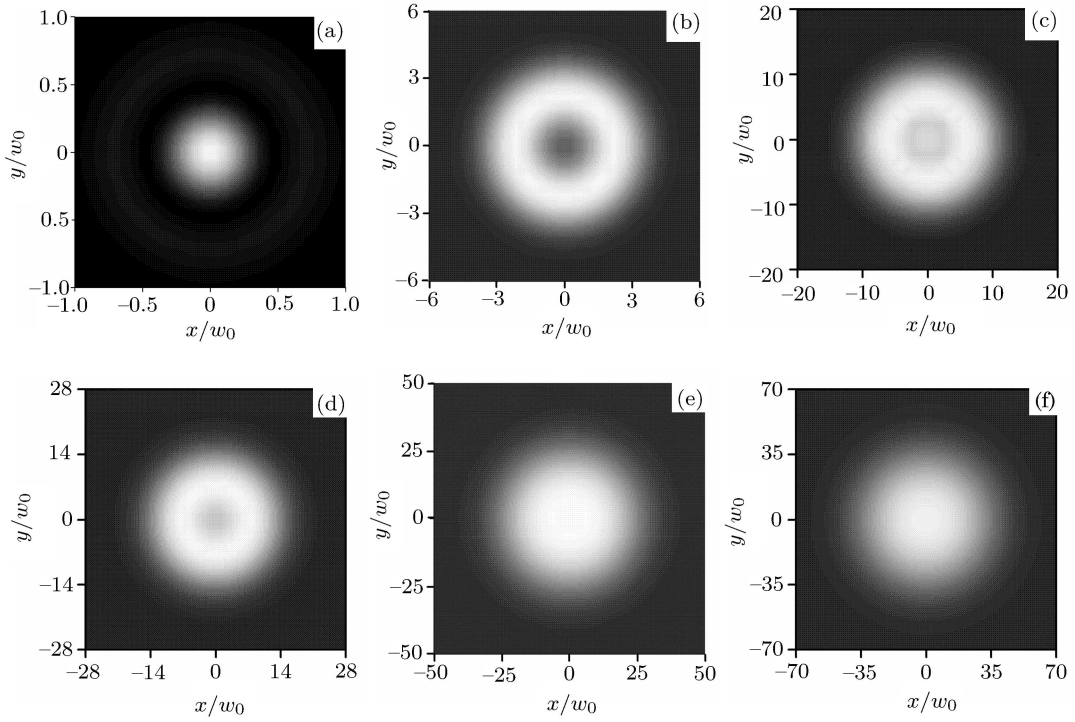


Fig.1. The intensity distributions of zero-order Gauss–Bessel beams propagating in a turbulent atmosphere at several different propagation distances: (a) z=0 km, (b) z=0.5 km, (c) z=1.2 km, (d) z=1.8 km, (e) z=3 km, and (f) z=4 km. The other parameters are chosen as w_0=0.01 m, λ=632.8 nm, $\alpha = 500$, $n = 0$, $C_n^2 = 5\times10^{-14}\,\mathrm{m}^{-2/3}$.

Figure 2 shows the intensity distributions of first-order Gauss–Bessel beams propagating in a turbulent atmosphere at several propagation distances: (a) $z = 0$ km, (b) $z = 0.5$ km, (c) $z = 1.2$ km, (d) $z = 1.8$ km, (e) $z = 3$ km, and (f) $z = 4$ km. It shows that the first-order Gauss–Bessel beam has a doughnut shape initially. With the increase of propagation distance, the dark core will become smaller and then gradually disappear. Finally, the intensity profile changes into Gaussian shape, which is the same as the case of zero-order Gauss–Bessel beams propagating in a turbulent atmosphere. Comparing Fig.1 with Fig.2, we can see that the difference between zero-order and higher order Gauss–Bessel beam lies in the intensity distribution of the source plane.

Figure 3 is plotted to illustrate the influence of atmospheric turbulence on the propagation of Gauss–Bessel beams in a turbulent atmosphere. In this figure, the solid curve refers to $C_n^2 = 0$ (i.e. free space), the dash curve refers to $C_n^2 = 5 \times 10^{-15}\,\mathrm{m}^{-2/3}$, and the dot curve refers to $C_n^2 = 10^{-14}\,\mathrm{m}^{-2/3}$. The propagation distances are chosen as (a) $z = 0.5$ km, (b) $z = 5$ km, (c) $z = 15$ km, (d) $z = 30$ km, and (e) $z = 35$ km. As shown in Fig.3, the first-order Gauss–Bessel beam will keep its hollow shape unchanged while propagating in free space. However, when prop-

agating in a turbulent atmosphere, the dark core of the first-order Gauss–Bessel beam will gradually disappear and finally changes into a Gaussian intensity distribution. Furthermore, the stronger is the turbulence, the higher is the changing rate.

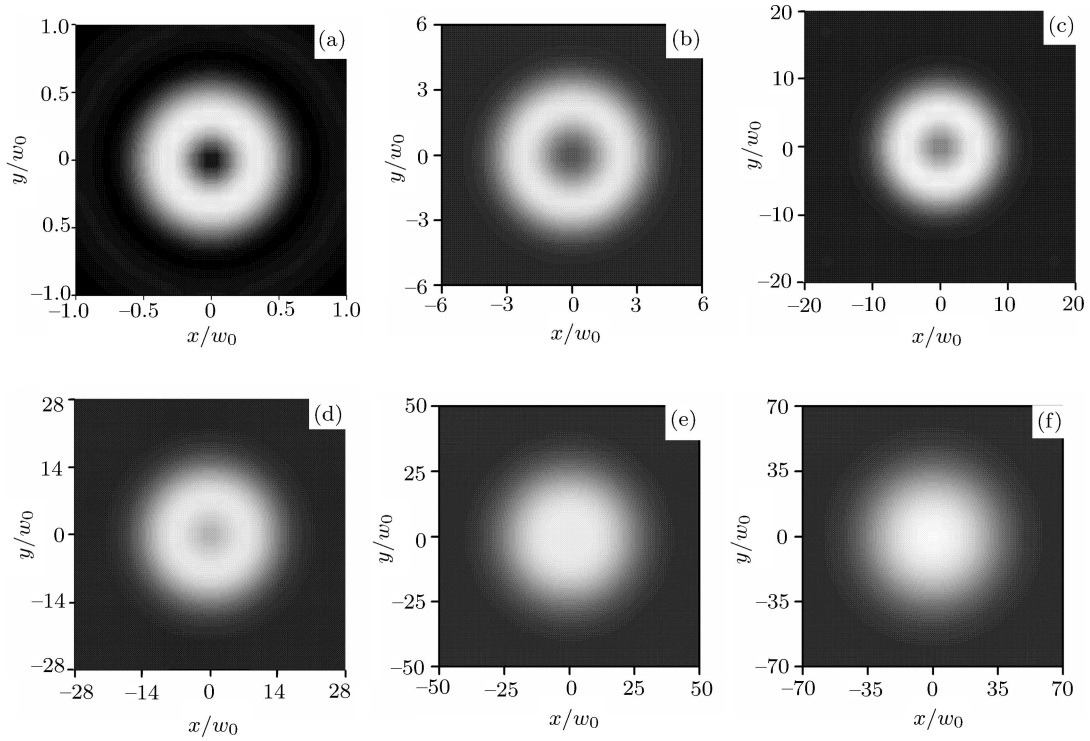
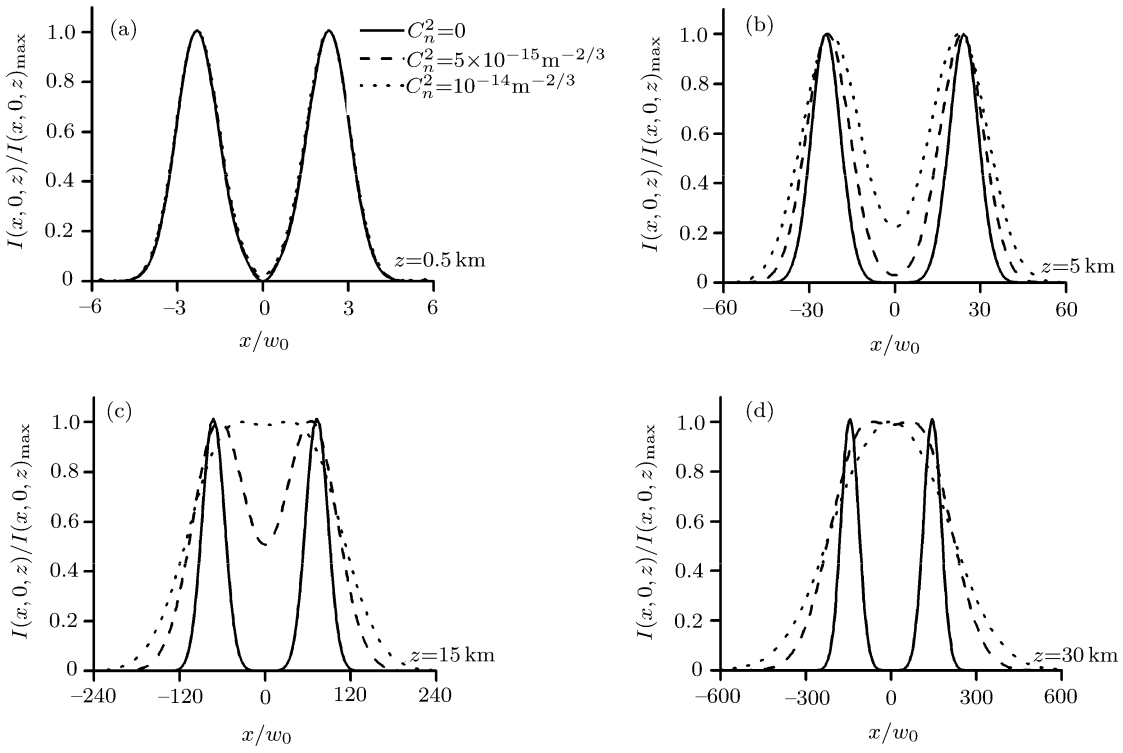


Fig.2. Contour plots of first-order Gauss–Bessel beams propagating in a turbulent atmosphere at several different propagation distances: (a) $z = 0$ km, (b)$z = 0.5$ km, (c) $z = 1.2$ km, (d) $z = 1.8$ km, (e) $z = 3$ km, and (f) $z = 4$ km. The other parameters are the same as those in Fig.1.



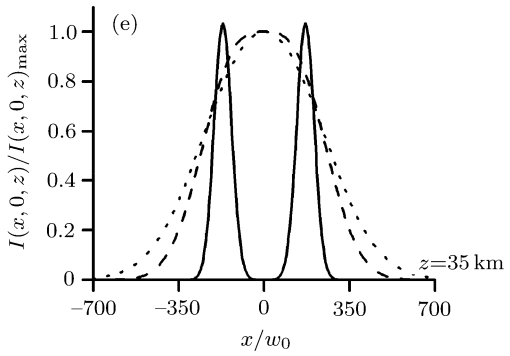
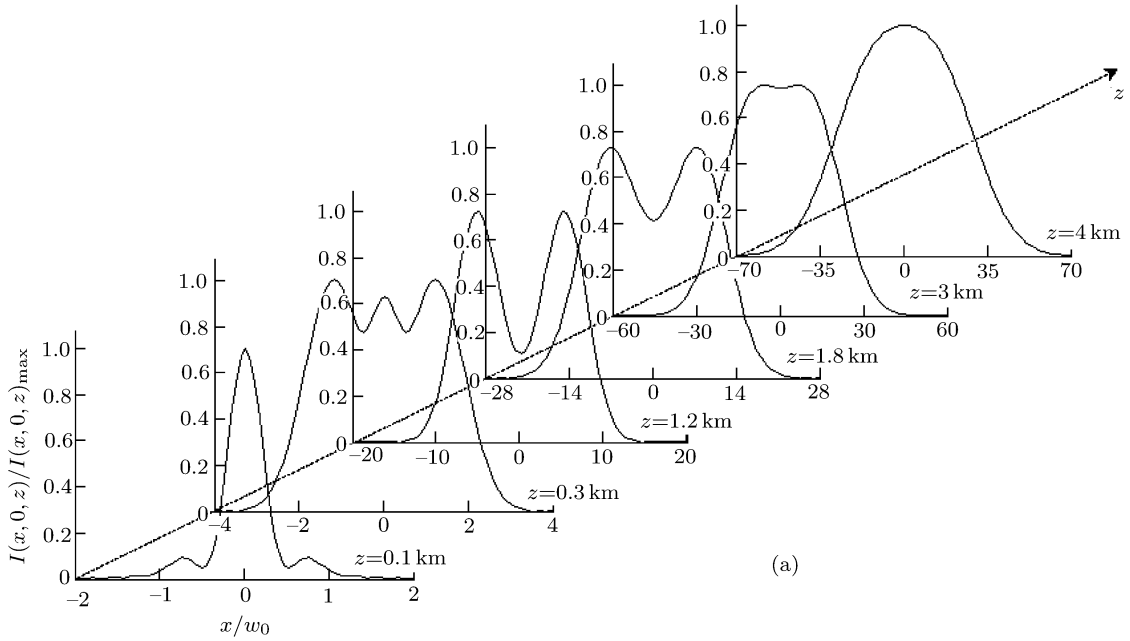


Fig.3. The influence of atmospheric turbulence on the propagation properties of first-order Gauss–Bessel beam propagating in a turbulent atmosphere. The propagation distances are chosen as (a) $z = 0.5$ km, (b) $z = 5$ km, (c) $z = 15$ km, (d) $z = 30$ km, and (e) $z = 35$ km. The solid curve refers to $C_n^2 = 0$ (i.e. free space), the dash curve refers to $C_n^2 = 5 \times 10^{-15}\,\mathrm{m}^{-2/3}$, and the dot curve refers to $C_n^2 = 10^{-14}\,\mathrm{m}^{-2/3}$. The other parameters are the same as those in Fig.2.

To illustrate the influence of topological charges on the propagation properties of Gauss–Bessel beams propagating in a turbulent atmosphere, Fig.4 is presented. Three different kinds of topological charges are chosen as (a) $n = 0$, (b) $n = 1$, (c) $n = 10$. Figure 4 shows that the Gauss–Bessel beam will finally change into Gaussian intensity distribution while propagating in a turbulent atmosphere. It is found that in the source plane, Gauss–Bessel beam of zero-order has a peak intensity in the centre, while higher order Gauss–Bessel beams have a doughnut intensity distribution, and that the larger is the topological charge, the larger is the dark core. It is also found that the intensity distribution changes faster in a turbulent atmosphere for the beam with a smaller topological charge. Furthermore, the beam spreads less for the beam with a larger topological charge, which indicates that the vortex phase factor $\exp(in\phi)$ may constrain the beam spreading.



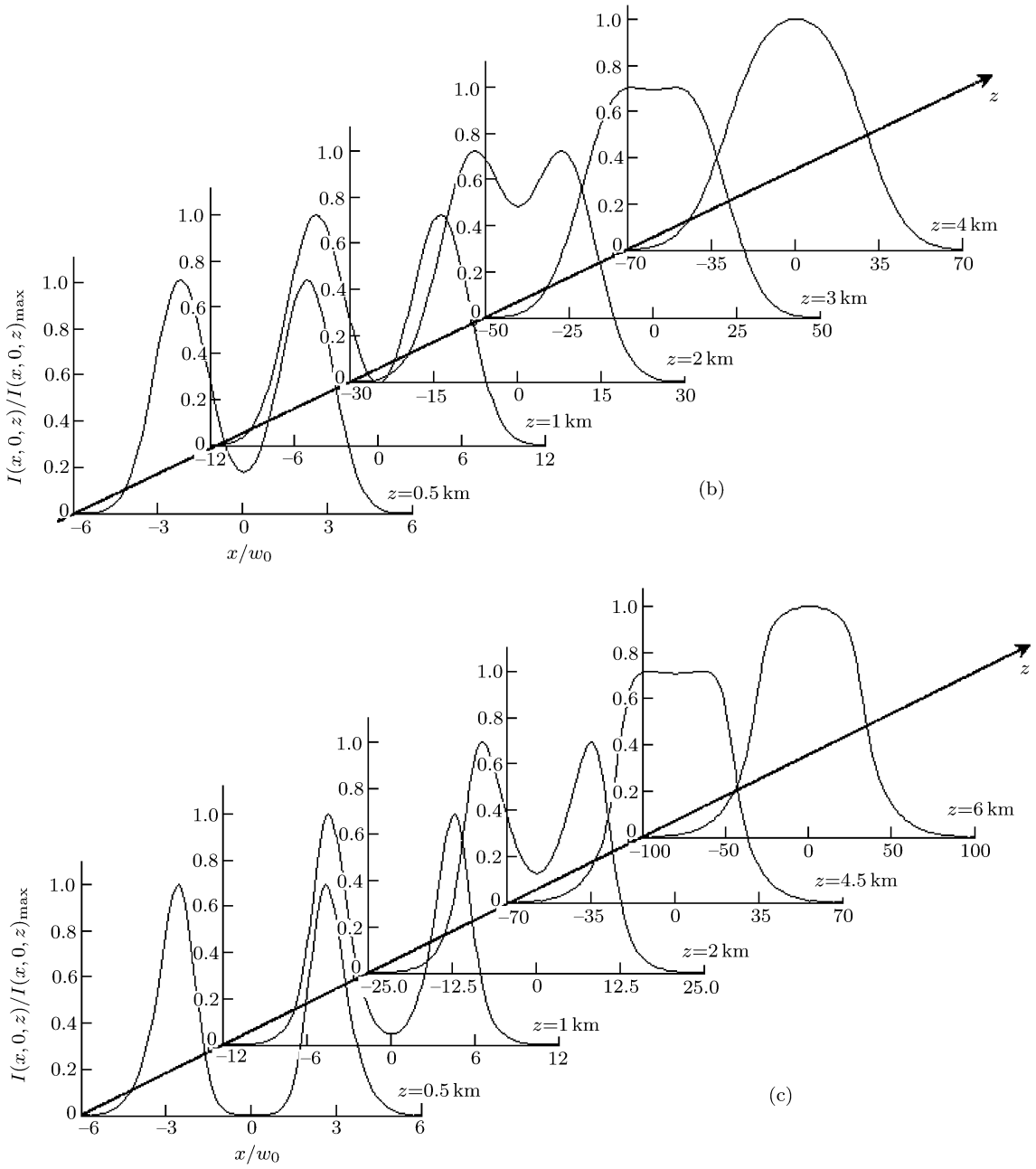


Fig.4. The influence of topological charges on the propagation properties of Gauss–Bessel beams propagating in a turbulent atmosphere. Three different kinds of topological charges are chosen as (a) $n = 0$, (b) $n = 1$, (c) $n = 10$. The other parameters are the same as those in Fig.1.

4. Conclusions

This paper studies the propagation properties of Gauss–Bessel beams in a turbulent atmosphere. For this purpose, the intensity distribution expression for this kind of beam propagating in a turbulent atmosphere was derived and solved down to a double integral stage by using the extended Huygens–Fresnel principle. Based on this expression, some numerical calculations were performed and the results were discussed. It is found that the Gauss–Bessel beam will change into a Gaussian profile after propagating in a turbulent atmosphere. It is also found that both the turbulence and topological charges will influence the propagation properties of Gauss–Bessel beams in turbulence. The intensity distribution changes faster with a stronger turbulence or a smaller topological charge. These results are very important for further applications of Gauss–Bessel beams in optical communication in a turbulent atmosphere.

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