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The design of metamaterial cloaks embedded in anisotropic medium*

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By using coordinate transformation method, this paper obtains an useful equation of designing meta-material cloaks embedded in anisotropic medium. This equation is the generalization of what was introduced early by Pendry *et al* (2006 *Science* **312** 1780) and can be more widely used. As an example of its applications, this paper deduces the material parameter equation for cylinder cloaks embedded in anisotropic medium, and then offers the numerical simulation. The results show that such a cylinder cloak has perfect cloaking performance and therefore verifies the method proposed in this paper.

Keywords: invisibility, cloaks, metamaterials
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1. Introduction

The reports in *Science* by Pendry *et al*[1] and Leonhardt[2] in 2006 suggested an interesting application of meta-materials in controlling electromagnetic waves, which introduced a new concept of the perfect invisibility. According to their reports, an exotic lens called cloak shell can be designed to exclude electromagnetic fields from a transparent object without perturbing any external fields, thus an object hidden in it is perfectly invisible to the outside. The reports by Cummer and Schurig *et al*[3,4] verified this theory by means of both simulations and experiments. Zhou and Hu[5] have extended this application to acoustic fields by analysing a multilayered sphere constructed with acoustic meta-materials. More recently, Chen *et al*[6] have analytically established the interactions of electromagnetic wave with a general class of spherical cloaks based on a full wave Mie scattering model.

Most theoretical researches into meta-material invisibility are based on the coordinate transformation method, and a general equation (called Pendry's equation) resulted from this method is adopted to deduce material parameter equations for some simple models, including sphere, circular and elliptic cylinder cloaks.[7,8] The accompanying online material for Ref.[1] has also presented a detailed analysis about this equation. However, we notice that, the applicability of such an equation is limited to designing cloaks that are embedded in the isotropic medium. Both the form and the theoretical analysis of this equation are all based on the assumption that the cloaks are embedded in isotropic medium, whether they are space-dependent or not. As a result, when the invisibilities of objects embedded in anisotropic medium are considered, Pendry's equation has to be generalized.

Using the same method, we consider the situation of anisotropic medium and obtain the generalizing form of Pendry's equation. In the present paper, we will report our researches. Furthermore, for the sake of verification, we present an example of this generalization by the numerical simulation.

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2. Deduction using form-invariant coordinate transformation of Maxwell's equation

First, we define three coordinates: Cartesian system $C(x,\ y,\ z)$, curvilinear system $S(q_1,\ q_2,\ q_3)$ and its compressed form $S'(q_1',\ q_2',\ q_3')$, where S and S' are orthogonal systems, the latter represents a spatial compression of the former. Note that, only in the considered region Ω which is embedded in a medium, can there be such compression transformation along axial orientations between S and S'. Suppose that the notations h_i (i=1, 2, 3) and $h_i'(i=1,2,3)$ are the Lame coefficients from C to S and to S' respectively; μ and ε represent the relative permeability and permittivity tensors in system C, respectively.

Then the components of Maxwell's equation can be written respectively in systems C, S and S' as follows:

$$\frac{\partial}{\partial y}E_3-\frac{\partial}{\partial z}E_2=-\frac{\partial}{\partial t}\mu_0(\mu_{11}H_1+\mu_{12}H_2+\mu_{13}H_3),\quad \text{etc. (in } C), \tag{1}$$

$$\frac{\partial}{\partial q_2}h_3\hat{E}_3-\frac{\partial}{\partial q_3}h_2\hat{E}_2=-\frac{\partial}{\partial t}\mu_0(h_1h_2h_3)\left(\frac{\hat{\mu}_{11}}{h_1h_1}h_1\hat{H}_1+\frac{\hat{\mu}_{12}}{h_1h_2}h_2\hat{H}_2+\frac{\hat{\mu}_{13}}{h_1h_3}h_3\hat{H}_3\right),\quad \text{etc. (in } S), \tag{2}$$

$$\frac{\partial}{\partial q_2'}h_3'\hat{E}_3'-\frac{\partial}{\partial q_3'}h_2'\hat{E}_2'=-\frac{\partial}{\partial t}\mu_0(h_1'h_2'h_3')\left(\frac{\hat{\mu}_{11}'}{h_1'h_1'}h_1'\hat{H}_1'+\frac{\hat{\mu}_{12}'}{h_1'h_2'}h_2'\hat{H}_2'+\frac{\hat{\mu}_{13}'}{h_1'h_3'}h_3'\hat{H}_3'\right),\quad \text{etc. (in } S'). \tag{3}$$

Comparing Eqs.(1)–(3), we find that the coordinate transformation does not change the form of Maxwell's equation, it simply changes the definitions of the fields and the corresponding constitutive parameters, i.e. permeability and permittivity. This means that we can solve Eqs.(2, 3) by using the same form adopted in Eq.(1). As a result, the field $h_i'\hat{H}_i'$ and $h_i'\hat{E}_i'$ in Eq.(3) should have similar behaviour as the fields E_i and H_i in Eq.(1) if we take $\hat{\mu}_{ij}'(h_1'h_2'h_3')/h_i'h_j'$ and $\hat{\varepsilon}_{ij}'(h_1'h_2'h_3')/h_i'h_j'$ as the newly defined constitutive parameters and resolve them correctly. Based on this consideration, we define $\hat{\mu}_{ij}''$ and $\hat{\varepsilon}_{ij}''$ as the new relative permeability and permittivity components associated to fields $h_i'\hat{H}_i'$ and $h_i'\hat{E}_i'$, respectively. The expressions of them are as follows:

$$\hat{\mu}_{ij}''=\hat{\mu}_{ij}'\frac{h_1'h_2'h_3'}{h_i'h_j'}\quad (i,j=1,2,3),$$

$$\hat{\varepsilon}_{ij}''=\hat{\varepsilon}_{ij}'\frac{h_1'h_2'h_3'}{h_i'h_j'}\quad (i,j=1,2,3). \tag{4}$$

Obviously, Eq.(4) is a formalized result and its strict expression depends on $\hat{\mu}_{ij}'$ and $\hat{\varepsilon}_{ij}'$. Here our goal is to build a perfect cloaking system, so we can base on the physical requirements and adopt the coordinate transformation to deduce the expressions of $\hat{\mu}_{ij}'$ and $\hat{\varepsilon}_{ij}'$, and thereby to obtain the strict expressions of $\hat{\mu}_{ij}''$ and $\hat{\varepsilon}_{ij}''$.

As defined at the beginning, the system S' stands for a compression of system S in a special region Ω, which means that Eq.(3) should be reduced to Eq.(2) when no spatial compression occurs. In such case, the following equations must be satisfied:

$$\hat{\mu}_{ij}'\frac{h_1h_2h_3}{h_ih_j}=\hat{\mu}_{ij},\quad \hat{\varepsilon}_{ij}'\frac{h_1h_2h_3}{h_ih_j}=\hat{\varepsilon}_{ij}, \tag{5}$$

and

$$\hat{\mu}=A\mu A^{\mathrm{T}},\quad \hat{\varepsilon}=A\varepsilon A^{\mathrm{T}}, \tag{6}$$

where $\hat{\mu}_{ij}$ and $\hat{\varepsilon}_{ij}$ are respectively the components of tensor $\hat{\boldsymbol{\mu}}$ and $\hat{\boldsymbol{\varepsilon}}$, and A is the normalized transform matrix from C to S. Solving Eq.(2), we obtain $\hat{\mu}_{ij}'$ and $\hat{\varepsilon}_{ij}'$ in region Ω,

$$\hat{\mu}_{ij}'=\hat{\mu}_{ij}\frac{h_ih_j}{h_1h_2h_3},\quad \hat{\varepsilon}_{ij}'=\hat{\varepsilon}_{ij}\frac{h_ih_j}{h_1h_2h_3}. \tag{7}$$

Substituting Eq.(7) into Eq.(4), we obtain the exact expression of Eq.(4) and can give it a physical explanation. Region Ω is compressed spatially, but the changes of electromagnetic properties resulted from such compression are perfectly compensated by the permeability and permittivity determined by Eq.(4). As a result, no field scatterings or phase front changes occur around the region as if no objects exist. Such region can be used to cloak electromagnetic waves without disturbing the outer fields, and thus perfect invisibility is achieved.

Obviously, Eqs.(4) and (7) are the generalization of Pendry's equation[1] and can be adopted to design cloaks that are embedded in medium having any properties: isotropic or anisotropic, uniform or space-dependent. Note that, the coordinate systems C, S and S' must be orthogonal when they are used into Eqs.(4) and (7), in other words, the non-orthogonal systems and the related coordinate transformations are not applicable to these equations.

3. Design of cylinder cloaks embedded in anisotropic media

As a simple example for the applications of the method, we use Eqs.(4) and (7) to deduce the material parameter equations for cylinder cloaks that are embedded in uniform anisotropic media. In Cartesian system, the relative permeability tensor of the selected medium is assumed to be

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_1 & & \\ & \mu_2 & \\ & & \mu_3 \end{bmatrix}. \tag{8}$$

Suppose that system S represents cylindrical coordinates and S' is its compressed form in a cylinder region, then the transformation between S and S' can be described as

$$r' = R_1 + r\frac{R_2 - R_1}{R_2}, \quad \phi' = \phi, \quad z' = z, \tag{9}$$

where R_2 and R_1 are the outer and inner radius of cylinder cloak shells, respectively.

By applying the transformations described by Eqs.(4), (6) and (7), we obtain the expression of $\hat{\mu}''$ in system S' as follows:

$$\hat{\boldsymbol{\mu}}'' = \begin{bmatrix} (\mu_1\cos^2\phi' + \mu_2\sin^2\phi')\dfrac{r'-R_1}{r'} & \sin\phi'\cos\phi'(\mu_2-\mu_1) & 0 \\ \sin\phi'\cos\phi'(\mu_2-\mu_1) & (\mu_1\sin^2\phi' + \mu_2\cos^2\phi')\dfrac{r'}{r'-R_1} & 0 \\ 0 & 0 & \mu_3\left(\dfrac{R_2}{R_2-R_1}\right)^2\dfrac{r'-R_1}{r'} \end{bmatrix}. \tag{10}$$

From Eq.(10), in the considered cylinder shell with $R_1 < r' < R_2$, the components $\hat{\mu}''_{12}$ and $\hat{\mu}''_{21}$ are no longer zero, which is determined by the anisotropic properties of the medium (in this case, $\mu_1 \neq \mu_2$). As expected, Eq.(10) is a more general equation for cylinder cloaks and can be reduced to the equation considering the isotropic medium with $\mu_1 = \mu_2$.[3,4] The expression of $\hat{\boldsymbol{\varepsilon}}''$ also can be obtained by the same method. In fact, $\hat{\boldsymbol{\varepsilon}}''$ has the similar form with $\hat{\boldsymbol{\mu}}''$.

Now, we present our numerical simulation of a cylinder cloak to verify our method. In the simulation, we use a commercial simulation software COMSOL$^{\mathrm{TM}}$ on a two-processor personal computer workstation and configure a computational domain, in which a cylinder cloak shell is embedded in an anisotropic medium and a 1 GHz transverse-electric (TE) polarized time-harmonic uniform plane wave is put on the shell.

Figure 1 shows the simulation results, where two cases were considered. Firstly, we put a TE wave along one principal axis (i.e. x axis) of the medium when no cloak is embedded. In this case, the wave propagates along the principal axis regularly without any abruptions, as seen in Fig.1(a). However, when a cloak is embedded in the same medium, as shown in Fig.1(b), the phase fronts of the wave bend around the inner shell, and remain planar out of the cloak as if no objects exist. In the second case, we put the TE wave along a direction of 45° angle with respect to x axis. Figure 1(c) and 1(d) give a comparison in terms of cloaking performance. Different from the first case that the wave propagates along the principal-axis-aligned orientation, the incident wave splits and intervenes in the second case. Nevertheless, Figs.1(c) and 1(d) still show that, the fields out of cloaking region are completely the same, thus, the cloaking performance of the propagation in non-axis-aligned orientations is evident.

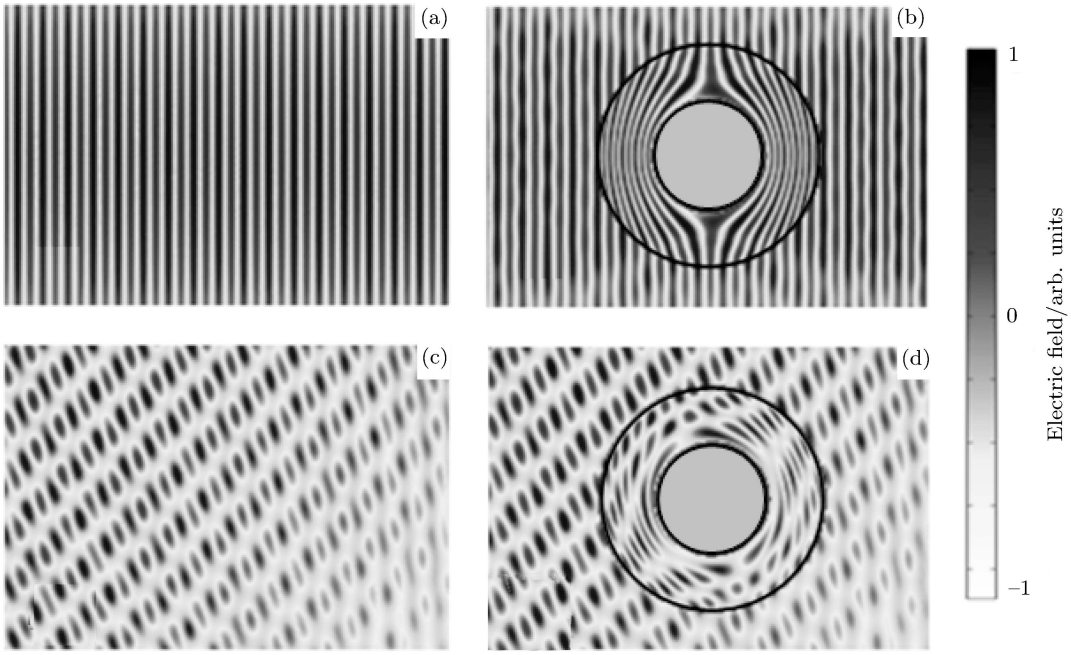


Fig.1. A cylinder cloak with outer radius R_2=0.6 m and inner radius R_1=0.3 m is built, and an anisotropic media, whose relative permeability and permittivity are described in Cartesian system: $\mu_1 = 2,\ \mu_2 = 5,\ \mu_3 = 1$ and $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$, is selected. We use a 1 GHz TE electromagnetic (EM) wave as the incident EM wave and two cases are considered. In the first case, EM wave propagates along one principal axis of the media and the electric field distribution is shown: (a), no cloak is embedded in the media; and (b), the cylinder cloak is embedded in the media. In the second case, EM wave propagates along a direction that has a 45° angle with respect to one principal axis of the media and the electric field distribution is shown: (c), no cloak is embedded in the media; and (d), the cylinder cloak is embedded in the media.

4. Conclusion

By analogy, we generalize the Pendry's method to design a cloak that can be invisible when it is embedded in anisotropic medium. The equations we obtained are thus the generalizations of Pendry's method. Though the results are not strictly theoretical analysis, their rationality is verified by our simulation. This research offers a powerful method to design devices in cloaking objects that are embedded in complicated medium.

References

[1] Pendry J B, Schurig D and Smith D R 2006 *Science* **312** 1780
[2] Leonhardt U 2006 *Science* **312** 1777
[3] Cummer S A, Popa B I, Schurig D, Smith D R and Pendry J 2006 *Phys. Rev.* E **74** 036621
[4] Schurig D, Mock J J, Justice B J, Cummer S A Pendry J B, Starr A F and Smith D R 2006 *Science* **314** 977
[5] Zhou X M and Hu G K 2007 *Phys. Rev.* E **75** 046606
[6] Chen H S, Wu B I, Zhang B L and Kong J A 2007 *Phys. Rev. Lett.* **99** 063903
[7] Ma H, Qu S B, Xu Z, Zhang J Q, Chen B W and Wang J F 2008 *Phys. Rev.* A **77** 013825
[8] Jiang W X, Cui T J, Yu G X, Lin X Q, Cheng Q and Chin J Y 2008 *J. Phys.* D: *Appl. Phys.* **41** 085504